Using the Complex Variable Boundary Element Method (CVBEM) to Approximate Three-Dimensional Potential Problems

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Math 597
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My task was to investigate how increasing the number of planes improves the accuracy of the CVBEM solution. I also investigated the effect of increase the nodes.

I modeled the surface by computing points that are approximately equally spaced on the surface. Figure 1 shows the model after "dusting" the surface with points.

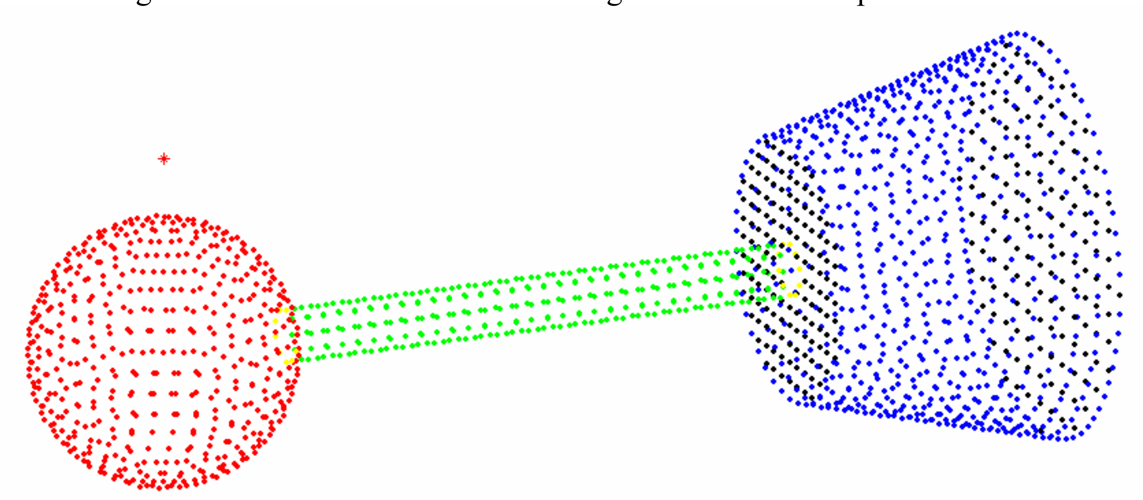


Figure 1: Model with points on the surface.

I developed a Matlab program that created points with equal arc length between points. So as the radius increases so does the number of points on the surfaces. Figure 2 shows that the points are equally spaced.

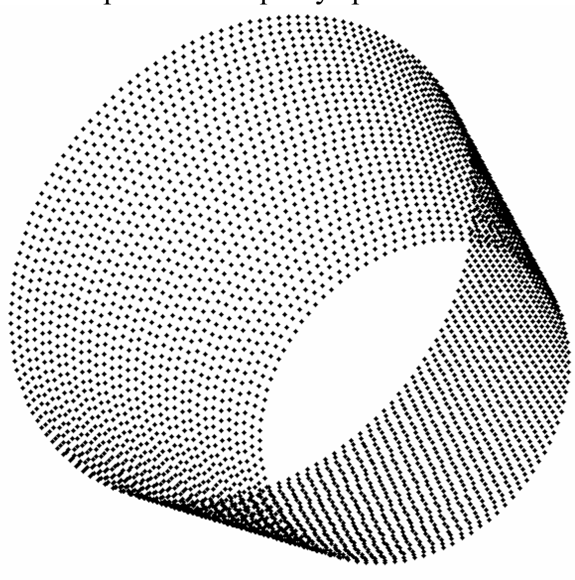


Figure 2: Arc length method

Next I created test points that are a slice through the model. Figure 3 shows the test points.

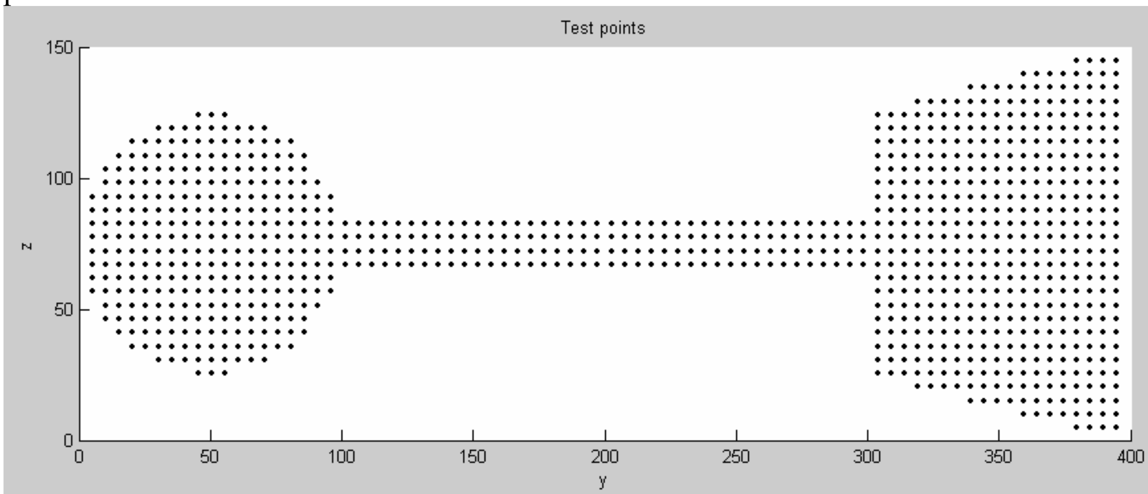


Figure 3: Test points

<u>Summary of Results</u>

I found that by increasing the number of planes and increasing the number of nodes per plane the approximate solution approached the exact solution. Tables 1 and 2 summarizes the results.

Table 1: Norm of Absolute Errors[1]

	3 planes	5 planes	7 planes	9 planes
4 Nodes	2.50	1.83	1.35	1.10
8 Nodes	1.64	0.79	0.50	0.36
12 Nodes	1.49	0.58	0.22	0.15
16 Nodes	1.46	0.51	0.17	0.15
20 Nodes	1.45	0.48	0.17	0.15
24 Nodes	1.44	0.50	0.17	0.15

Table 2: Norm of Relative Errors[1]

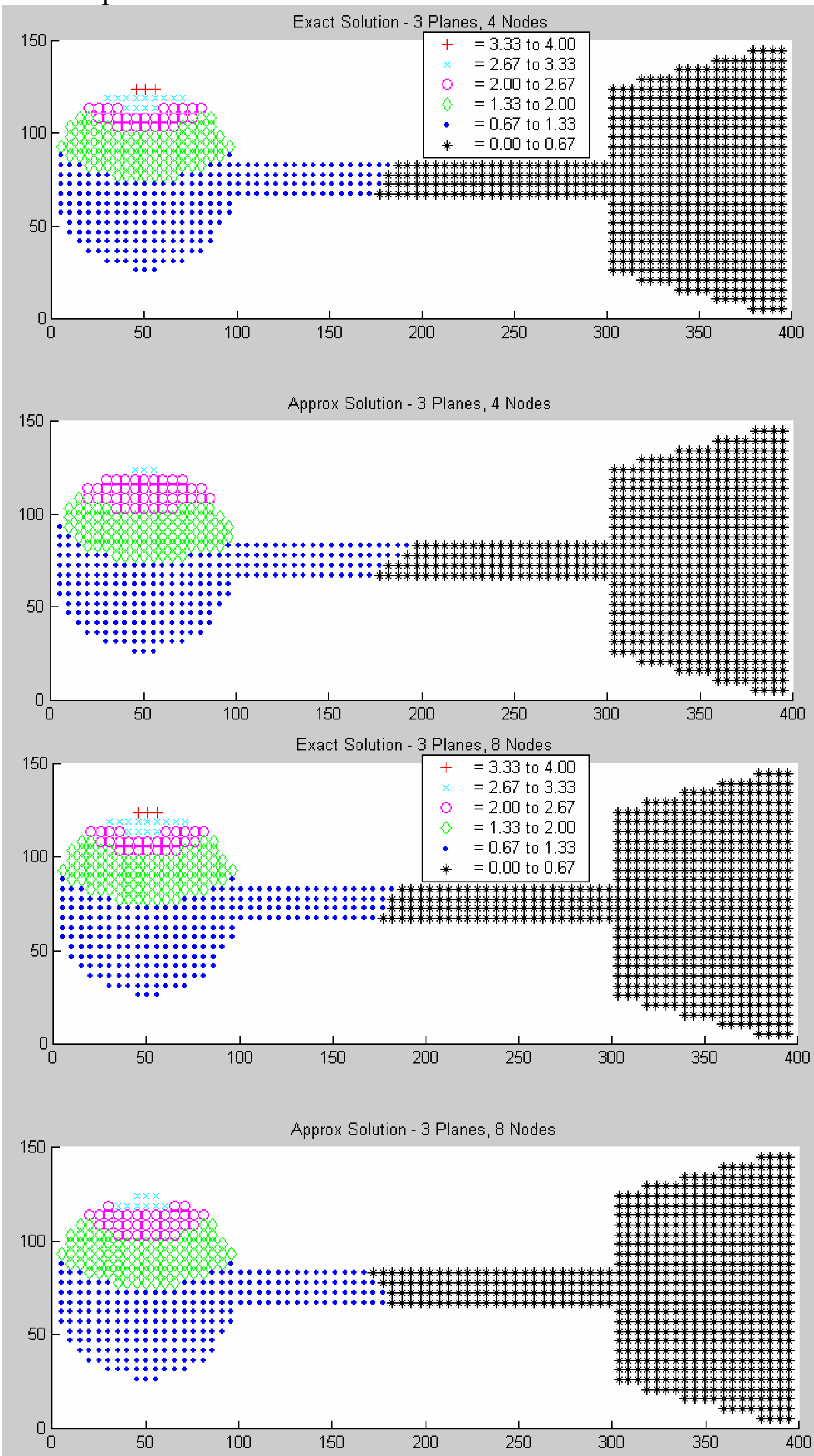
	3 planes	5 planes	7 planes	9 planes
4 Nodes	1.82	1.14	0.90	1.03
8 Nodes	1.46	0.64	0.58	0.39
12 Nodes	1.50	0.46	0.17	0.10
16 Nodes	1.50	0.31	0.14	0.10
20 Nodes	1.46	0.34	0.14	0.10
24 Nodes	1.48	0.37	0.14	0.10

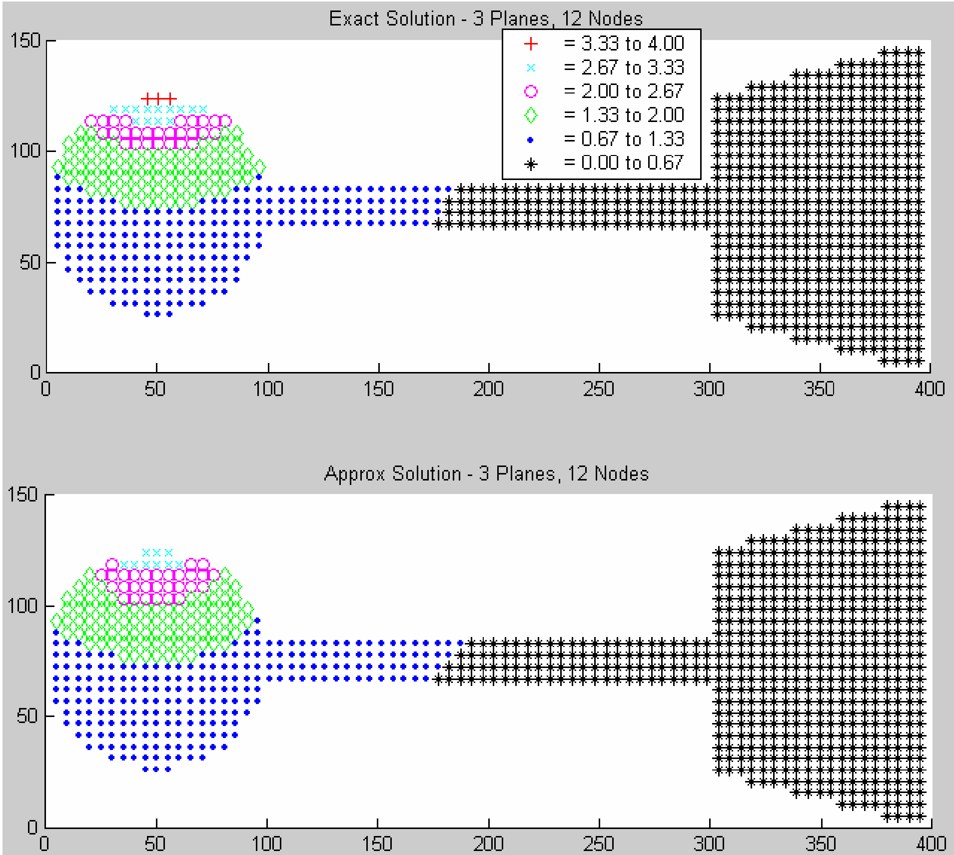
[1] Matlab method used to compute norms:

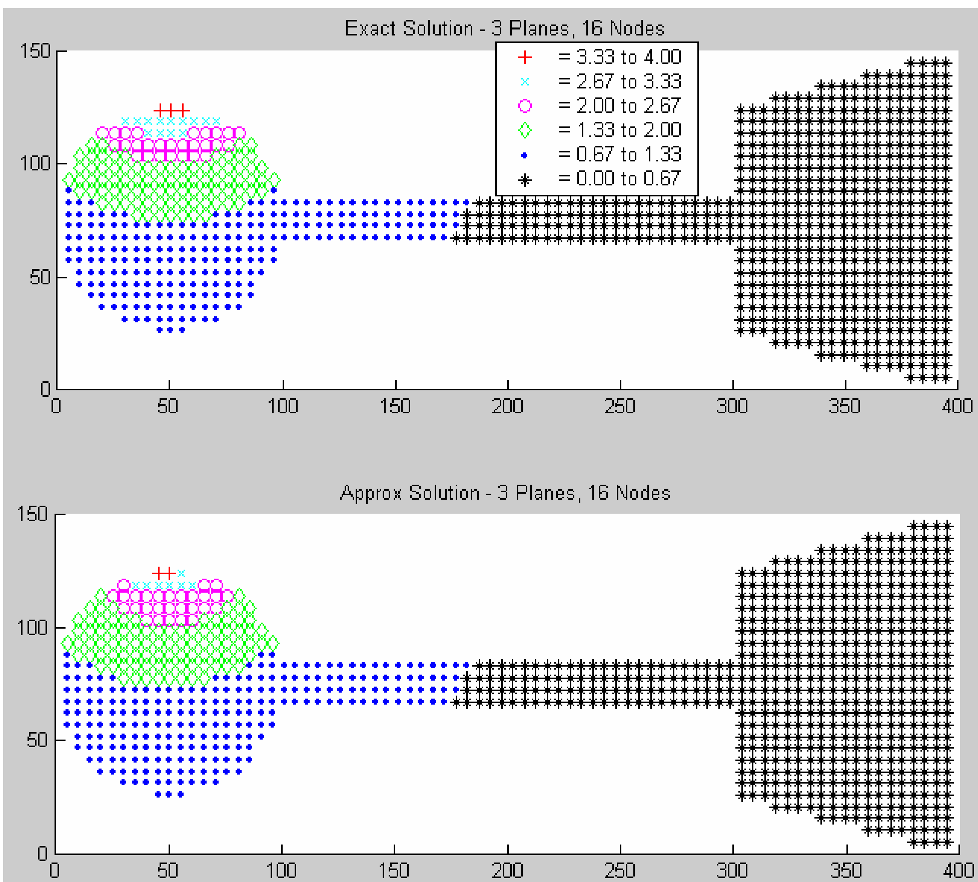
```
A=dlmread('test.txt',' ');
norm_abs = norm(A(:,7));
norm_rel = norm(A(:,8));
```

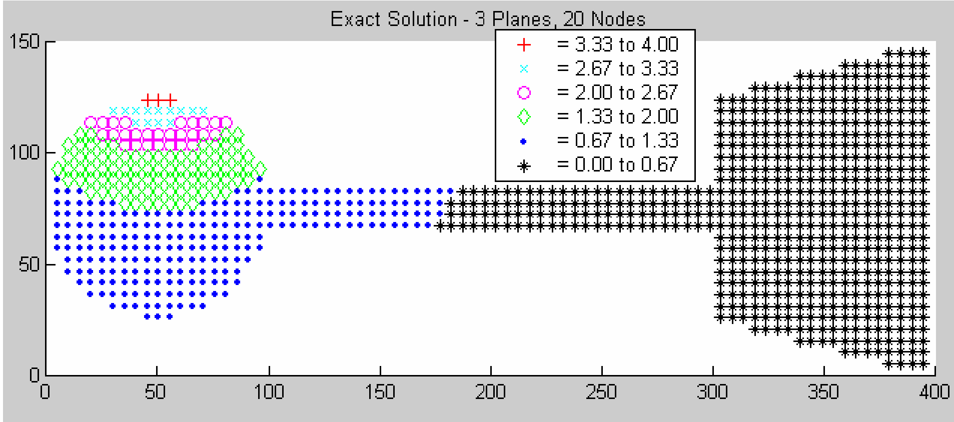
Detailed Results

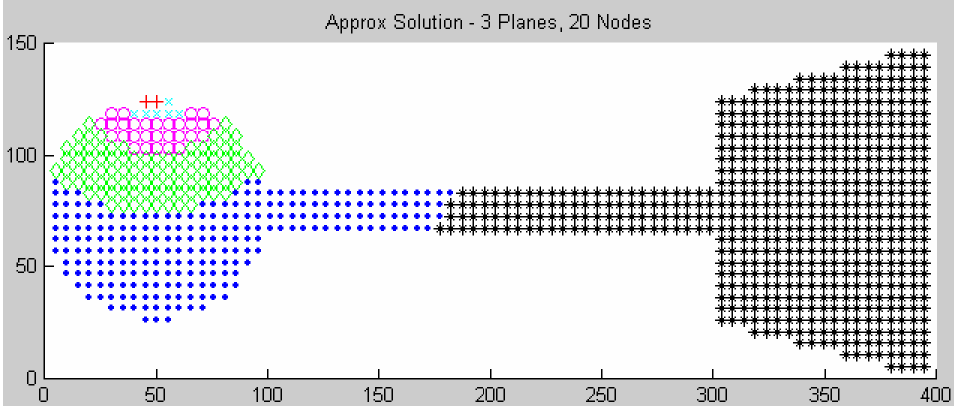
The graphs on pages 3-14 shows a representation of the exact and approximate temperature for each test point. From these graphs we can see that the solution improves as the number of planes and nodes increase.

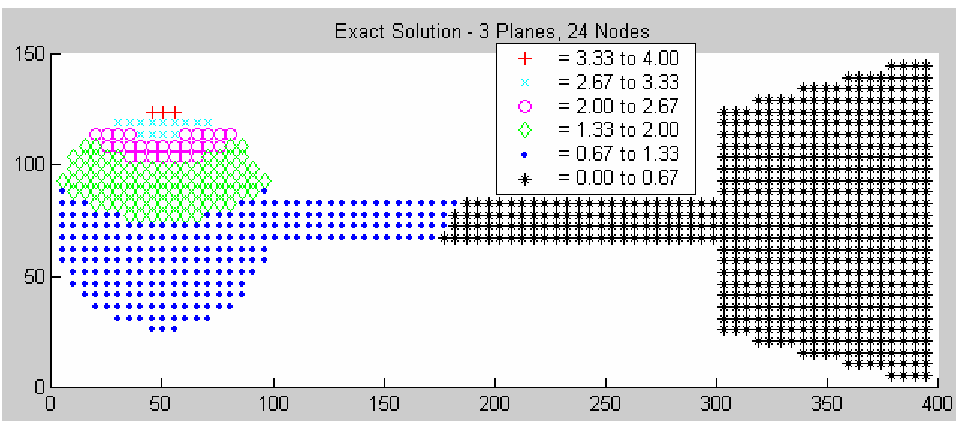


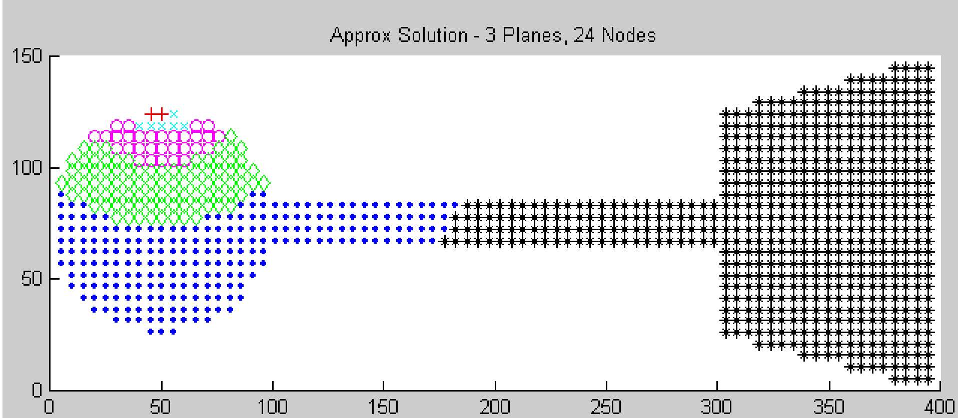

Exact Solution - 3 Planes, 12 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 3 Planes, 12 Nodes

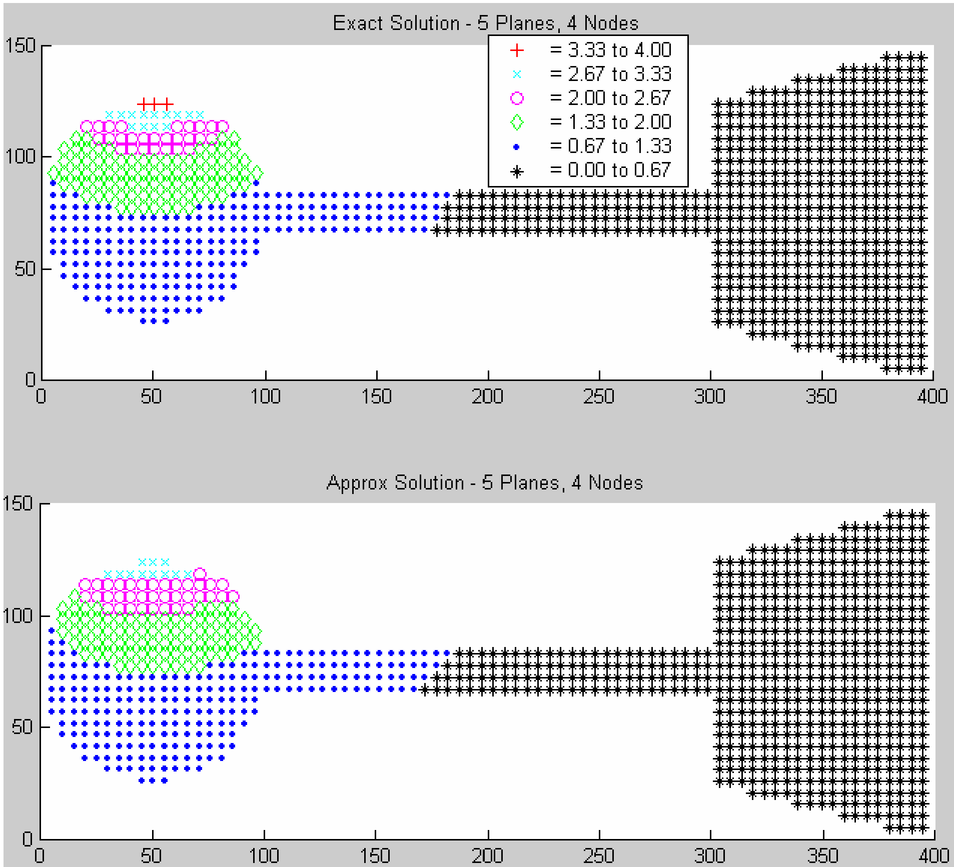

Exact Solution - 3 Planes, 16 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 3 Planes, 16 Nodes

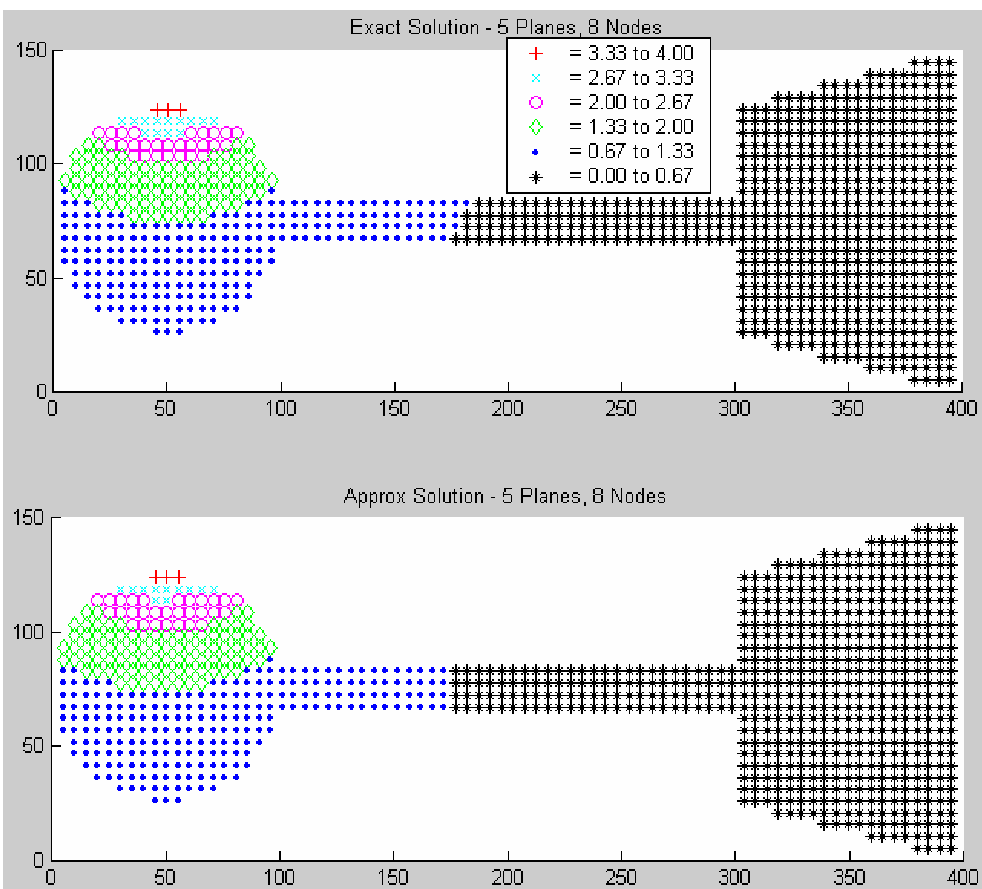

Exact Solution - 3 Planes, 20 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
150
100
50
0
0 50 100 150 200 250 300 350 400

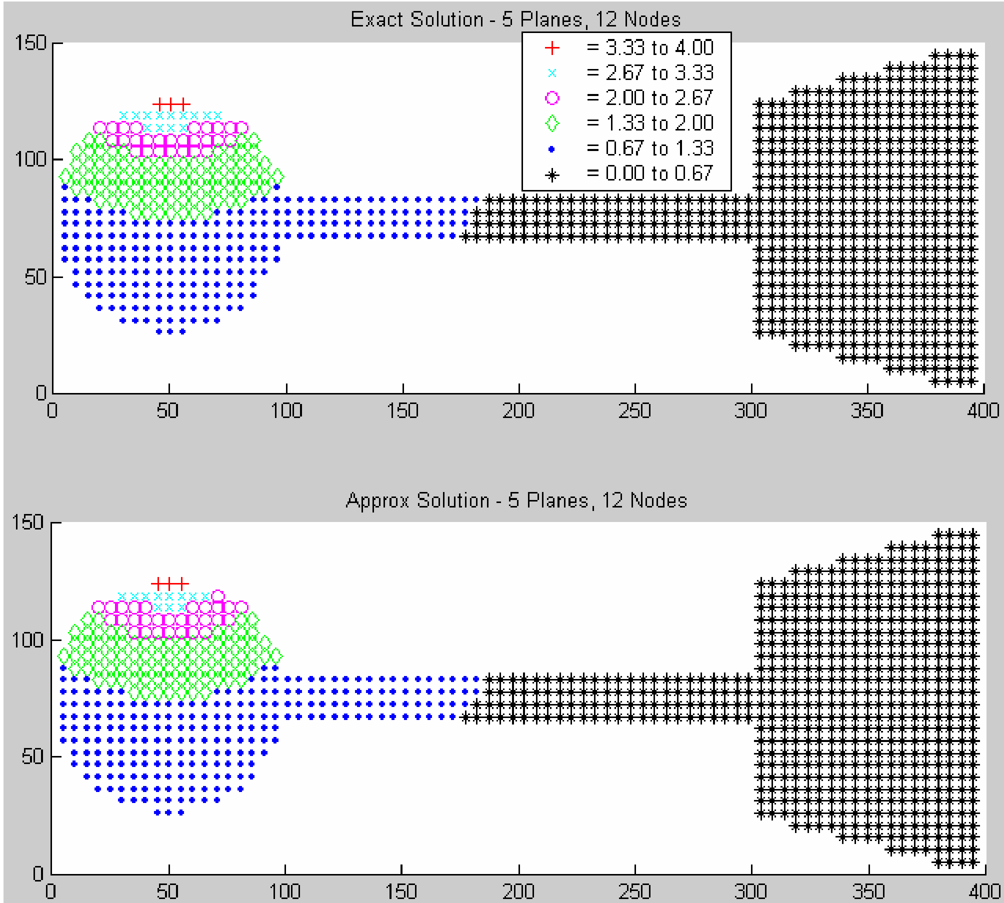

Approx Solution - 3 Planes, 20 Nodes
150
100
50
0
0 50 100 150 200 250 300 350 400

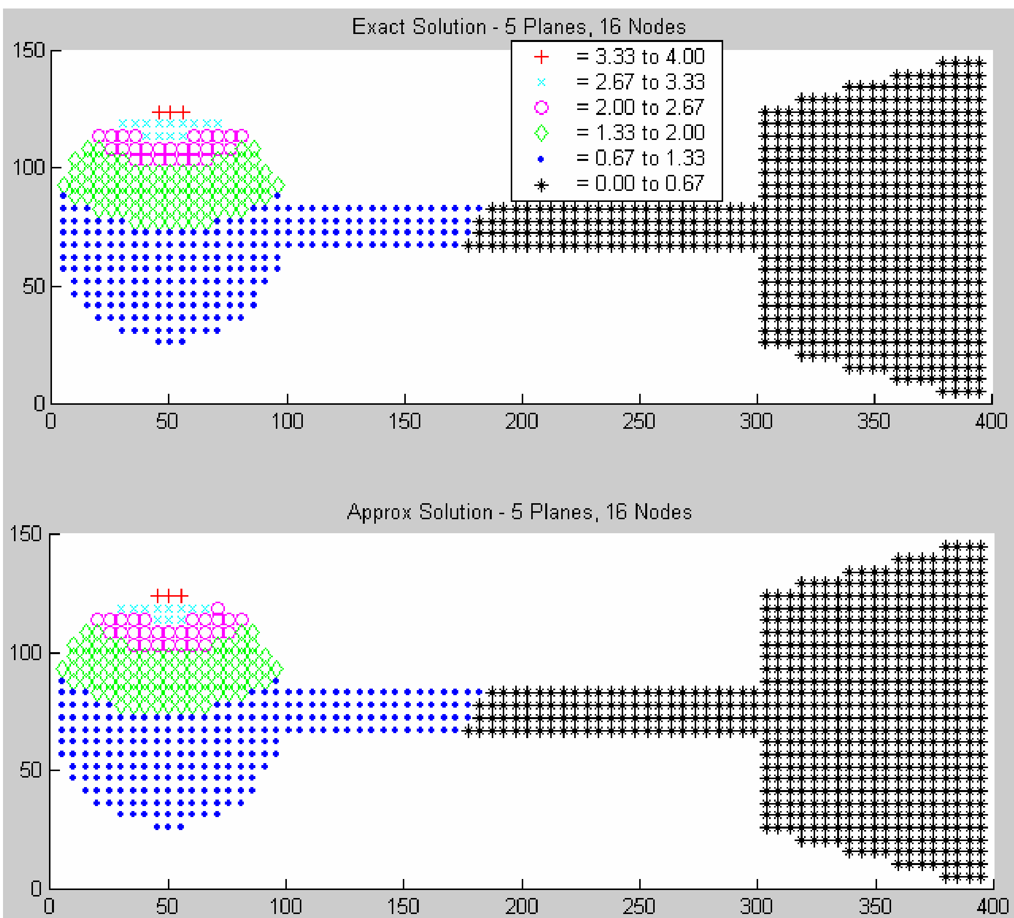

Exact Solution - 3 Planes, 24 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
150
100
50
0
0 50 100 150 200 250 300 350 400

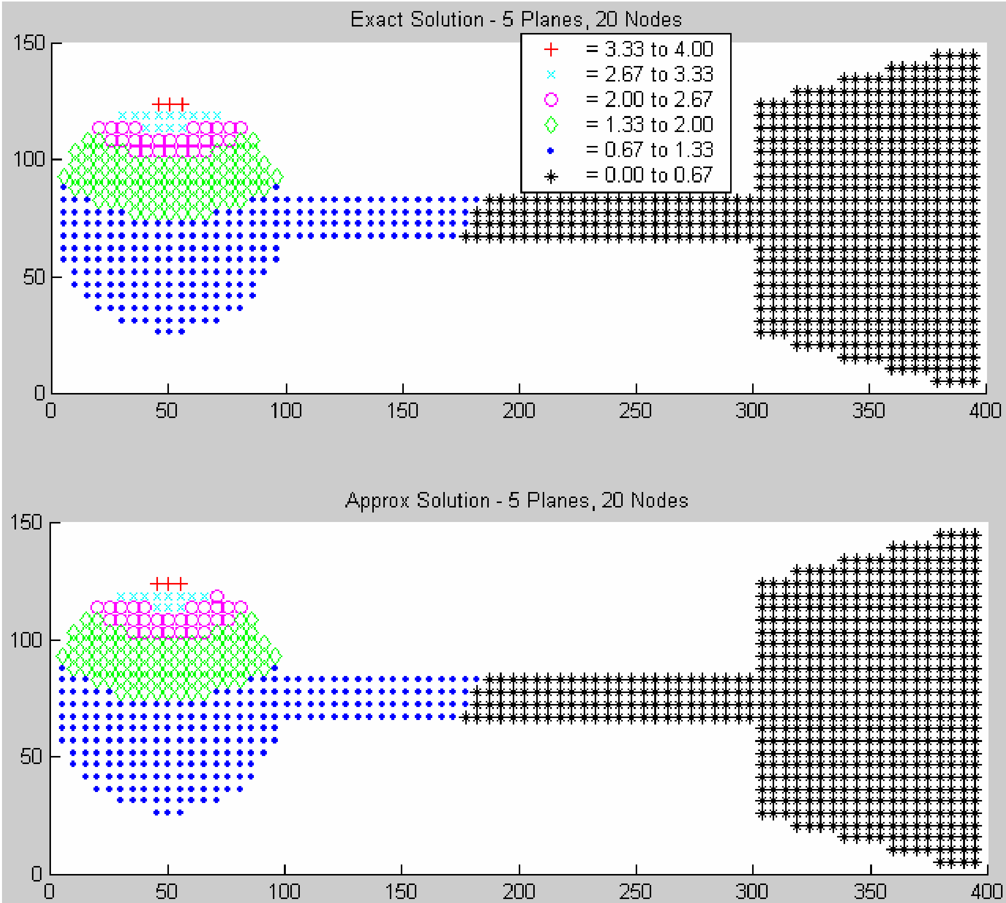

Approx Solution - 3 Planes, 24 Nodes
150
100
50
0
0 50 100 150 200 250 300 350 400

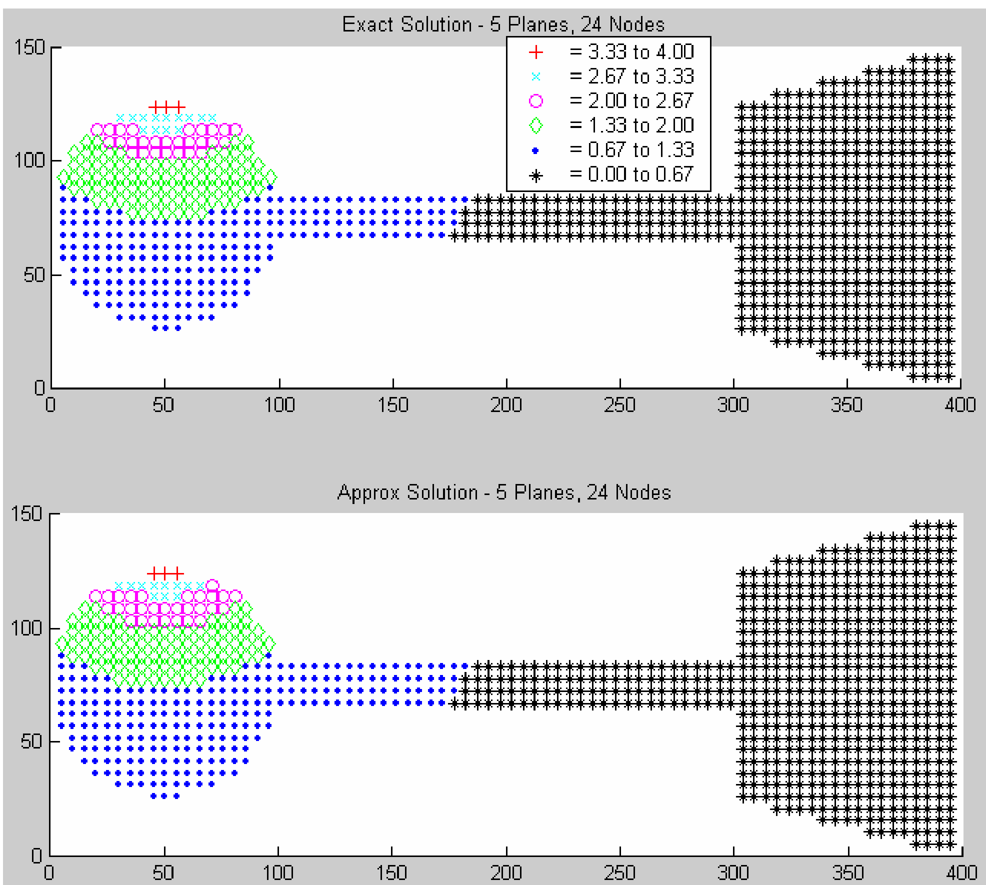

Exact Solution - 5 Planes, 4 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 5 Planes, 4 Nodes

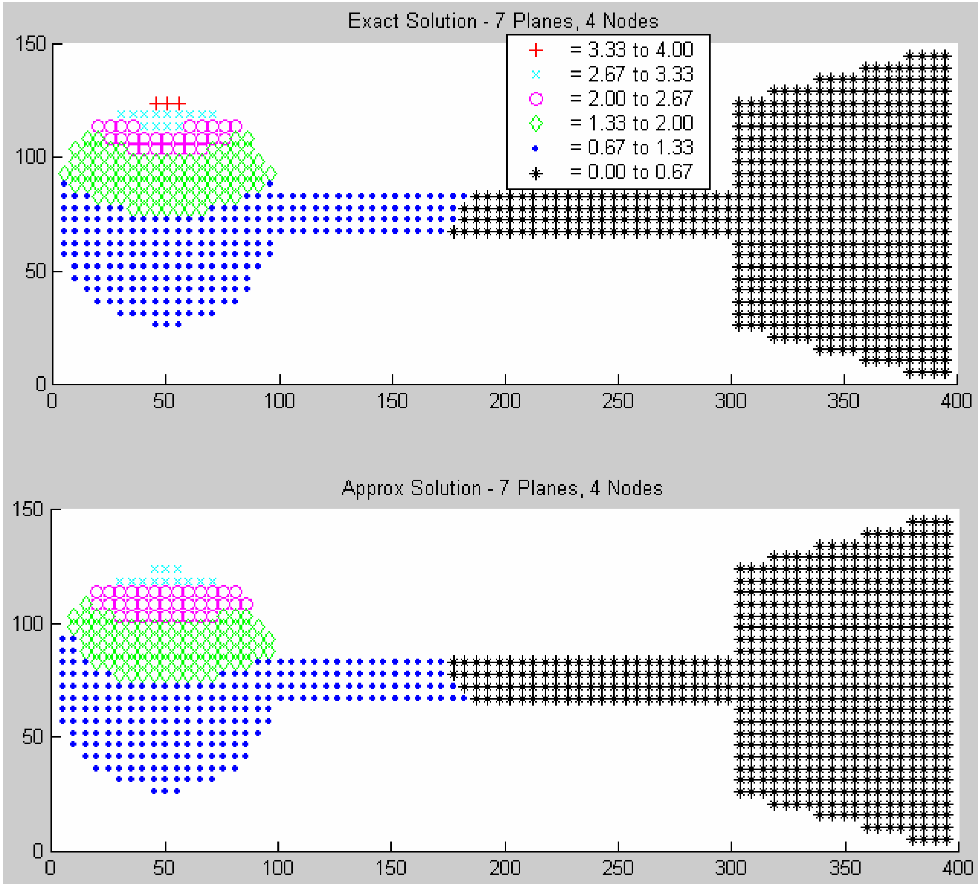

Exact Solution - 5 Planes, 8 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 5 Planes, 8 Nodes

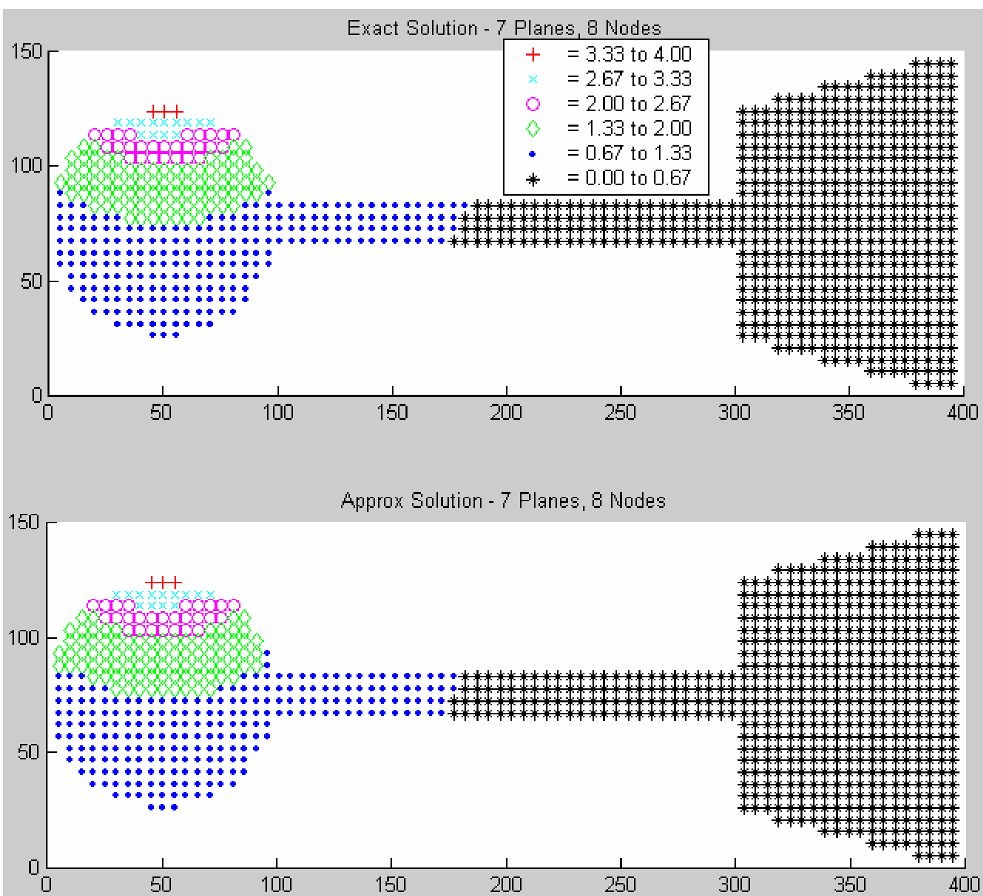

Exact Solution - 5 Planes, 12 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
0 50 100 150
0 50 100 150 200 250 300 350 400
Approx Solution - 5 Planes, 12 Nodes
0 50 100 150
0 50 100 150 200 250 300 350 400

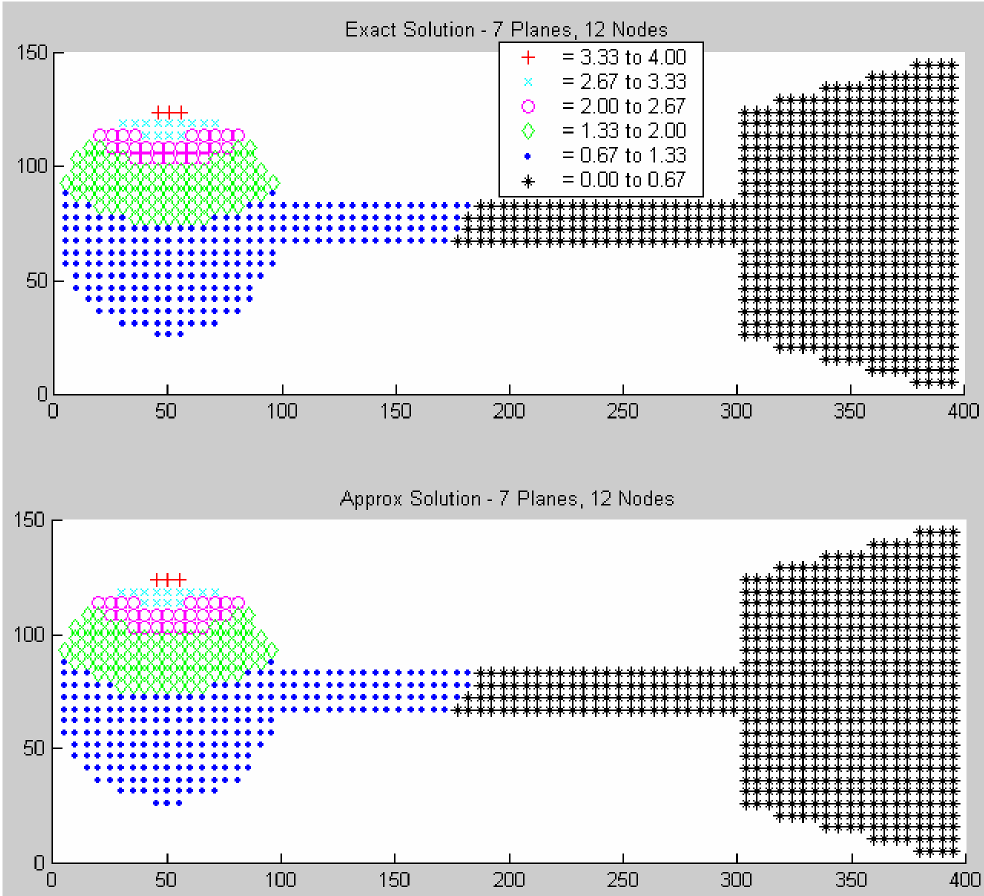

Exact Solution - 5 Planes, 16 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
0 50 100 150
0 50 100 150 200 250 300 350 400
Approx Solution - 5 Planes, 16 Nodes
0 50 100 150
0 50 100 150 200 250 300 350 400

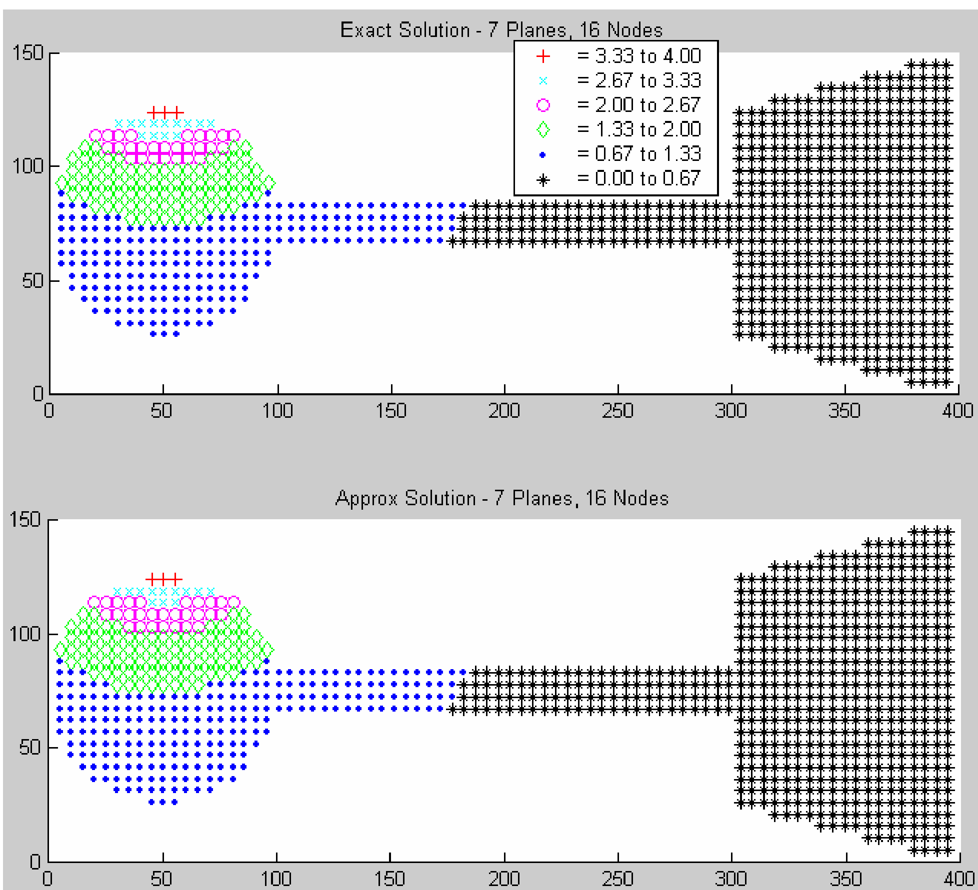

Exact Solution - 5 Planes, 20 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
0 50 100 150
0 50 100 150 200 250 300 350 400
Approx Solution - 5 Planes, 20 Nodes
0 50 100 150
0 50 100 150 200 250 300 350 400

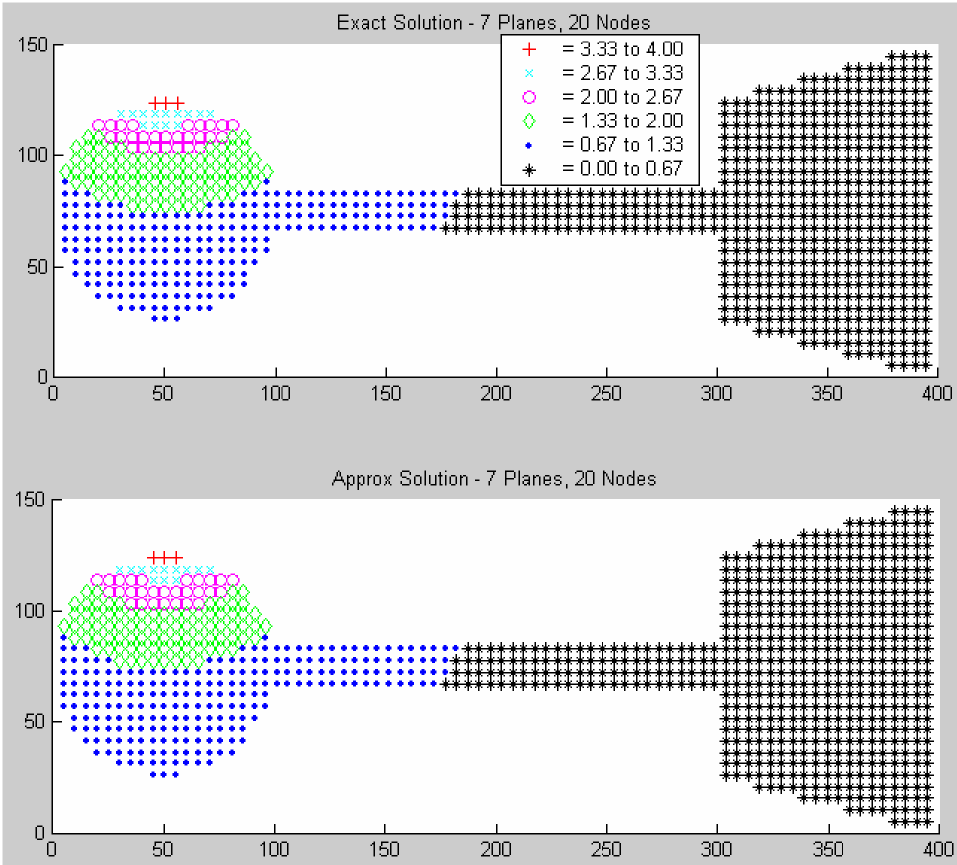

Exact Solution - 5 Planes, 24 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
0 50 100 150
0 50 100 150 200 250 300 350 400
Approx Solution - 5 Planes, 24 Nodes
0 50 100 150
0 50 100 150 200 250 300 350 400

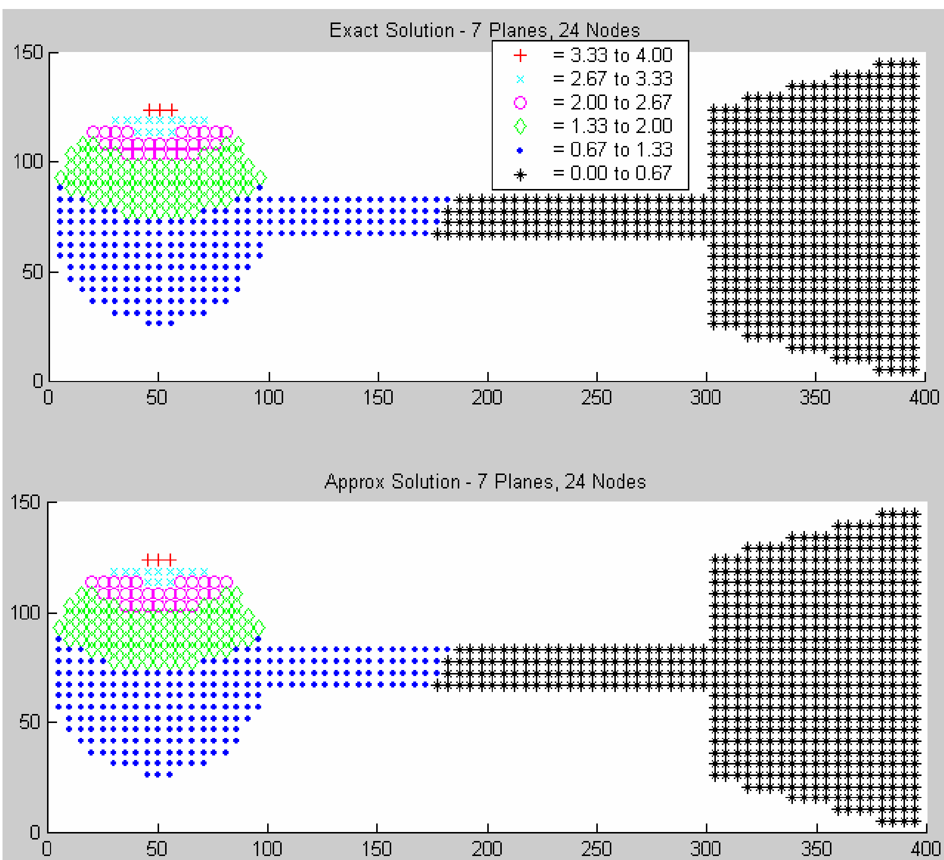

Exact Solution - 7 Planes, 4 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 7 Planes, 4 Nodes
0
50
100
150
200
250
300
350
400

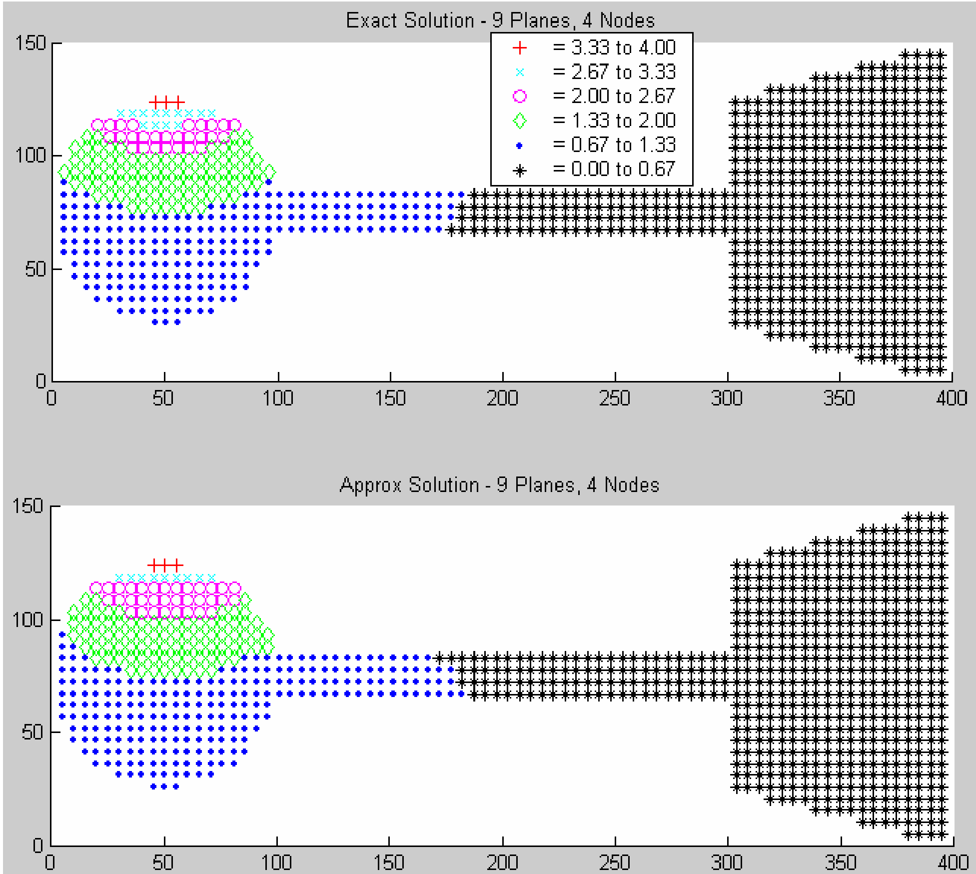

Exact Solution - 7 Planes, 8 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 7 Planes, 8 Nodes
0
50
100
150
200
250
300
350
400

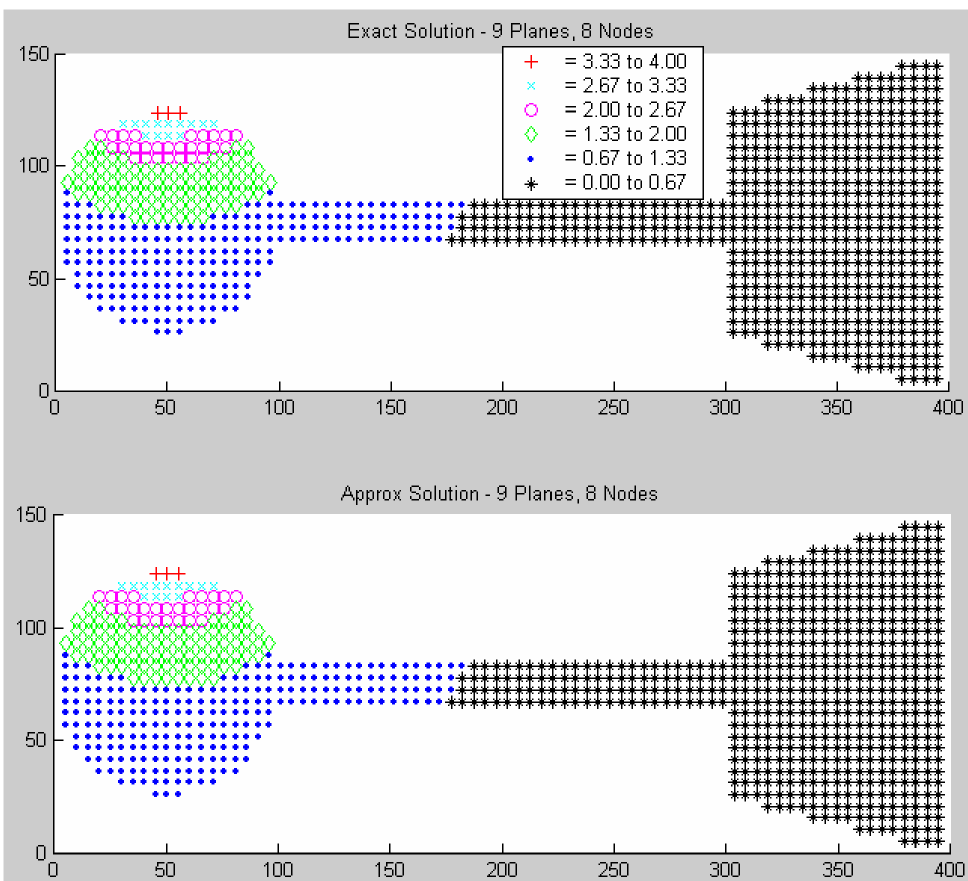

Exact Solution - 7 Planes, 12 Nodes
+ = 3.33 to 4.00
× = 2.67 to 3.33
○ = 2.00 to 2.67
◇ = 1.33 to 2.00
• = 0.67 to 1.33
* = 0.00 to 0.67
Approx Solution - 7 Planes, 12 Nodes


Exact Solution - 7 Planes, 16 Nodes
+ = 3.33 to 4.00
× = 2.67 to 3.33
○ = 2.00 to 2.67
◇ = 1.33 to 2.00
• = 0.67 to 1.33
* = 0.00 to 0.67
Approx Solution - 7 Planes, 16 Nodes


Exact Solution - 7 Planes, 20 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
0 50 100 150
0 50 100 150 200 250 300 350 400
Approx Solution - 7 Planes, 20 Nodes
0 50 100 150
0 50 100 150 200 250 300 350 400


Exact Solution - 7 Planes, 24 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
0 50 100 150
0 50 100 150 200 250 300 350 400
Approx Solution - 7 Planes, 24 Nodes
0 50 100 150
0 50 100 150 200 250 300 350 400


Exact Solution - 9 Planes, 4 Nodes
+ = 3.33 to 4.00
× = 2.67 to 3.33
○ = 2.00 to 2.67
◇ = 1.33 to 2.00
• = 0.67 to 1.33
✱ = 0.00 to 0.67
150
100
50
0
0 50 100 150 200 250 300 350 400
Approx Solution - 9 Planes, 4 Nodes
150
100
50
0
0 50 100 150 200 250 300 350 400


Exact Solution - 9 Planes, 8 Nodes
+ = 3.33 to 4.00
× = 2.67 to 3.33
○ = 2.00 to 2.67
◇ = 1.33 to 2.00
• = 0.67 to 1.33
✱ = 0.00 to 0.67
150
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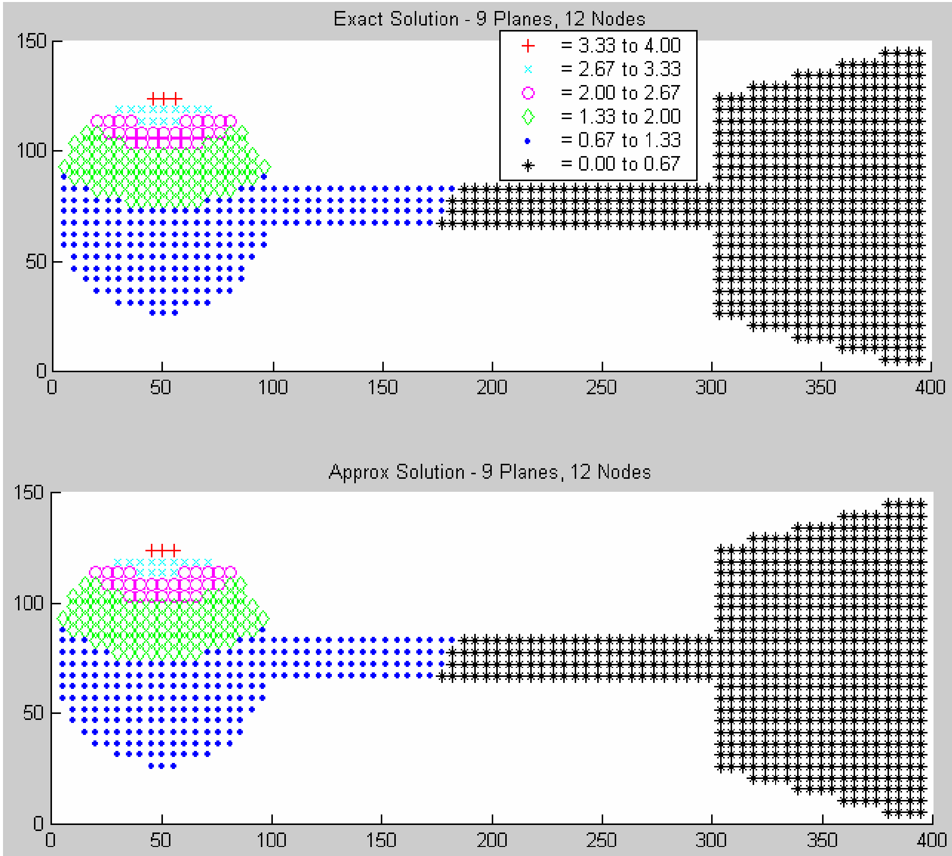
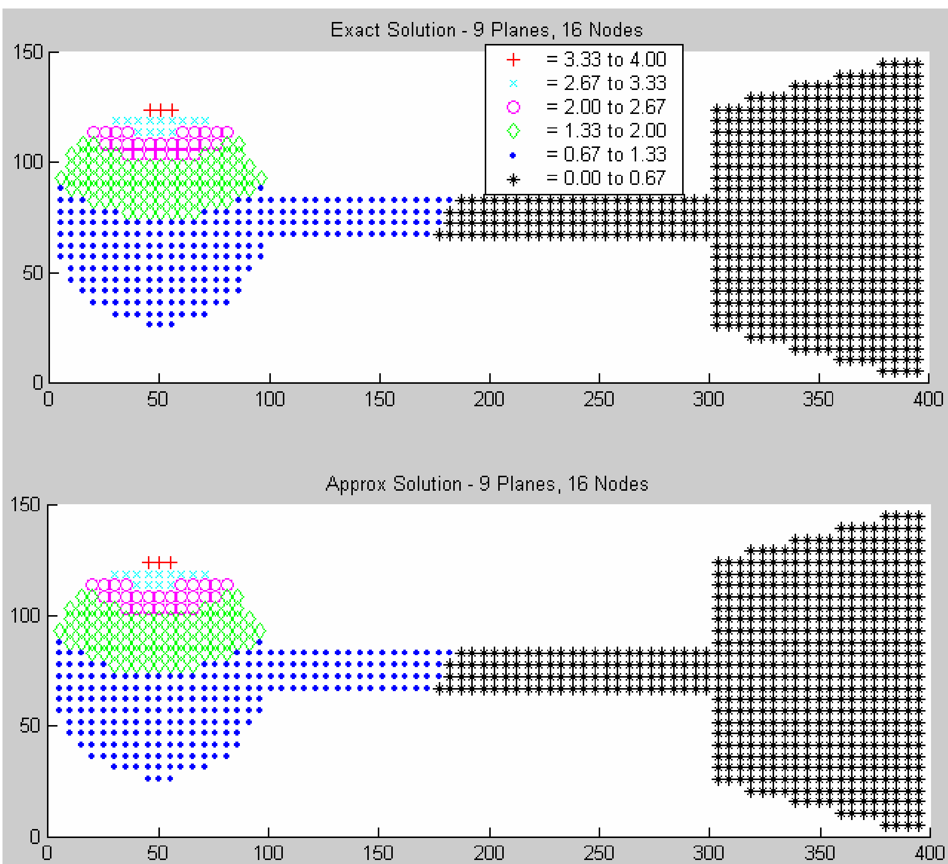
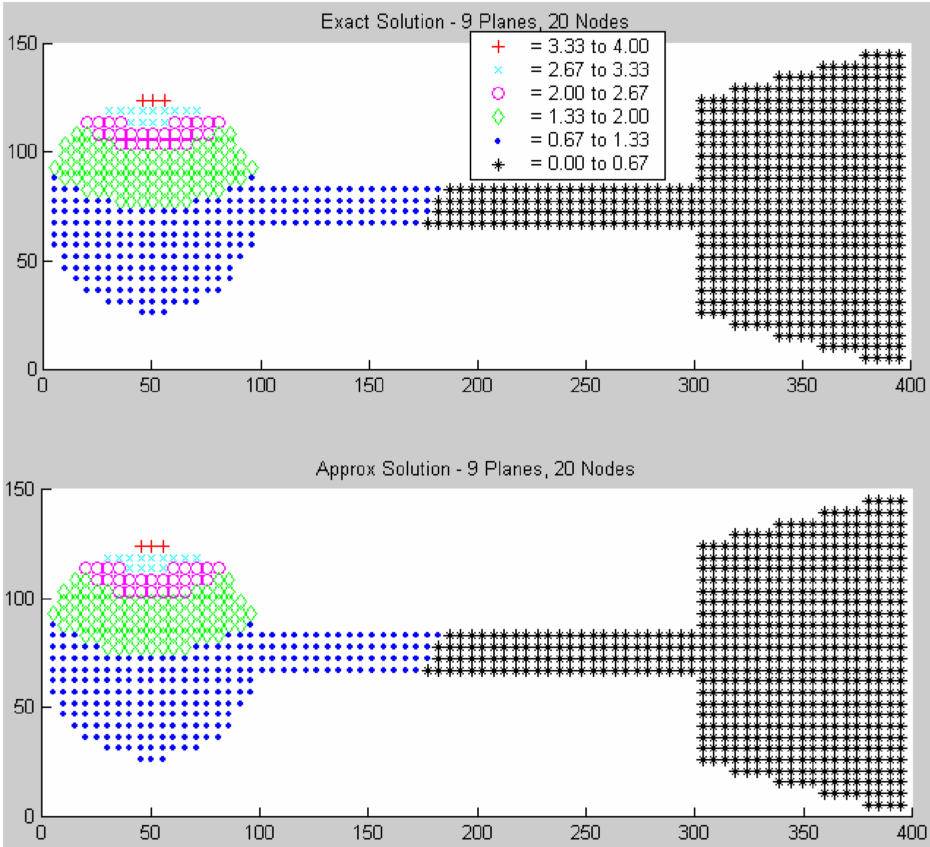
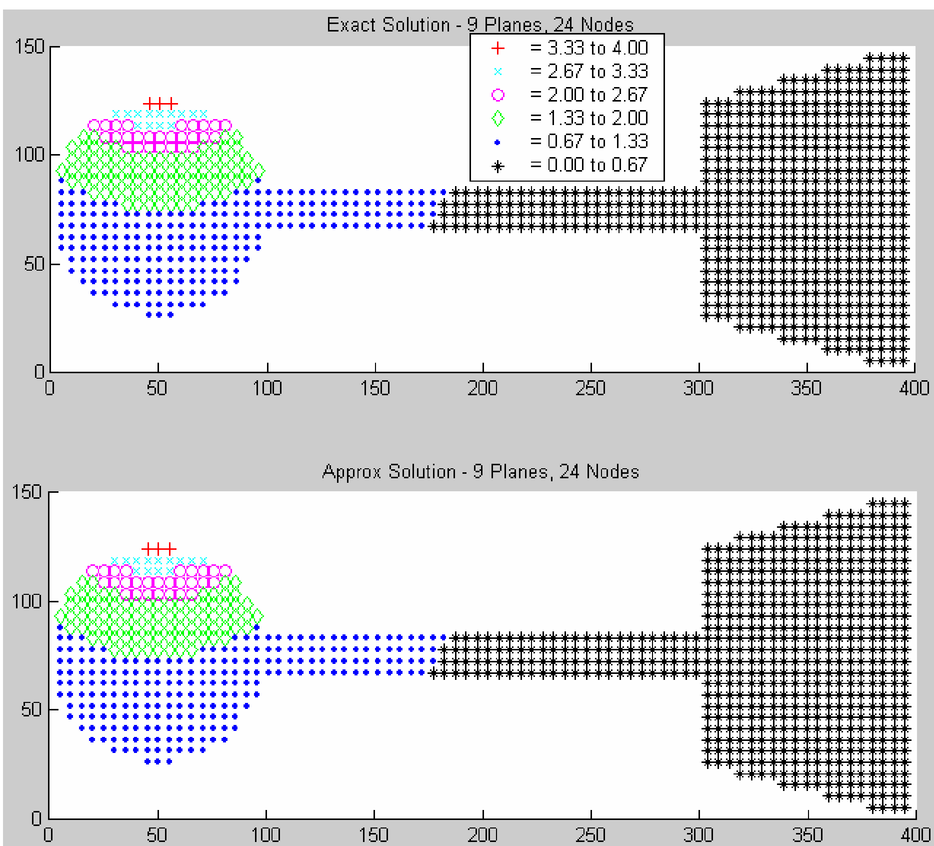
0 50 100 150 200 250 300 350 400
Approx Solution - 9 Planes, 8 Nodes
150
100
50
0
0 50 100 150 200 250 300 350 400


Exact Solution - 9 Planes, 12 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 9 Planes, 12 Nodes

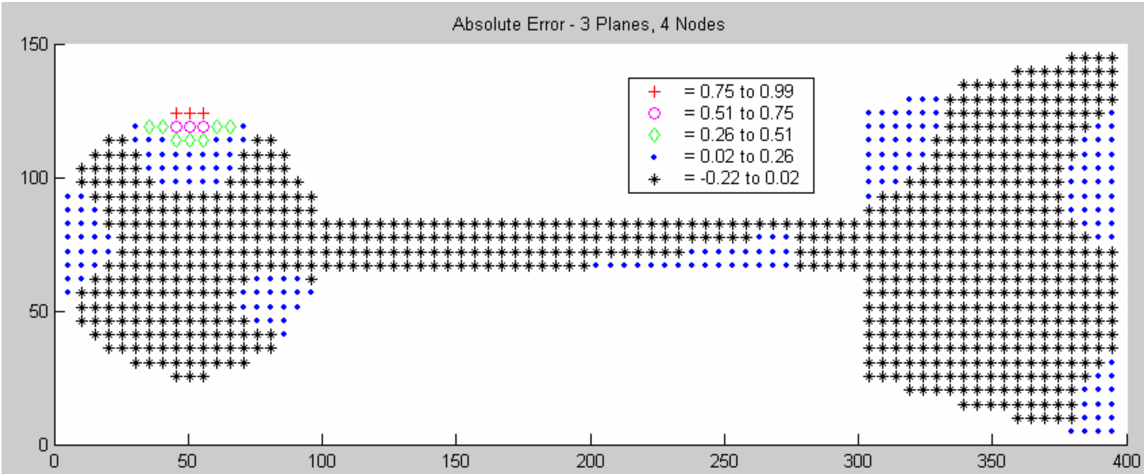

Exact Solution - 9 Planes, 16 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
Approx Solution - 9 Planes, 16 Nodes

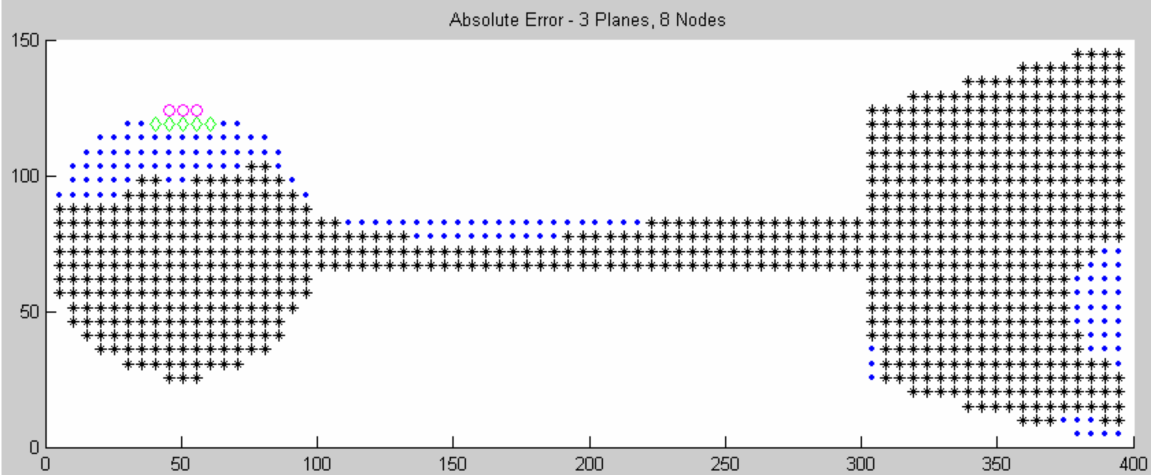

Exact Solution - 9 Planes, 20 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
150
100
50
0
0
50
100
150
200
250
300
350
400
Approx Solution - 9 Planes, 20 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

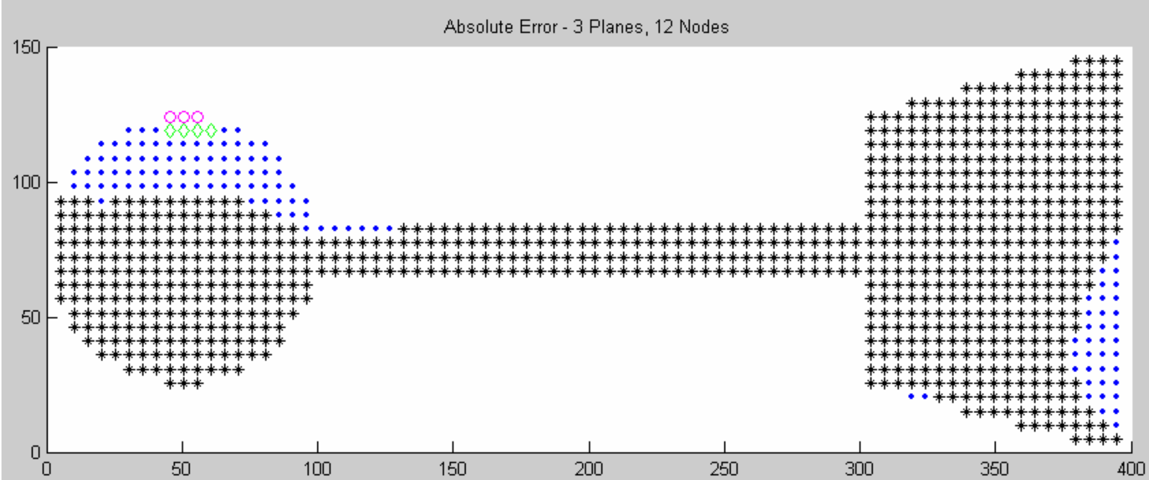

Exact Solution - 9 Planes, 24 Nodes
= 3.33 to 4.00
= 2.67 to 3.33
= 2.00 to 2.67
= 1.33 to 2.00
= 0.67 to 1.33
= 0.00 to 0.67
150
100
50
0
0
50
100
150
200
250
300
350
400
Approx Solution - 9 Planes, 24 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

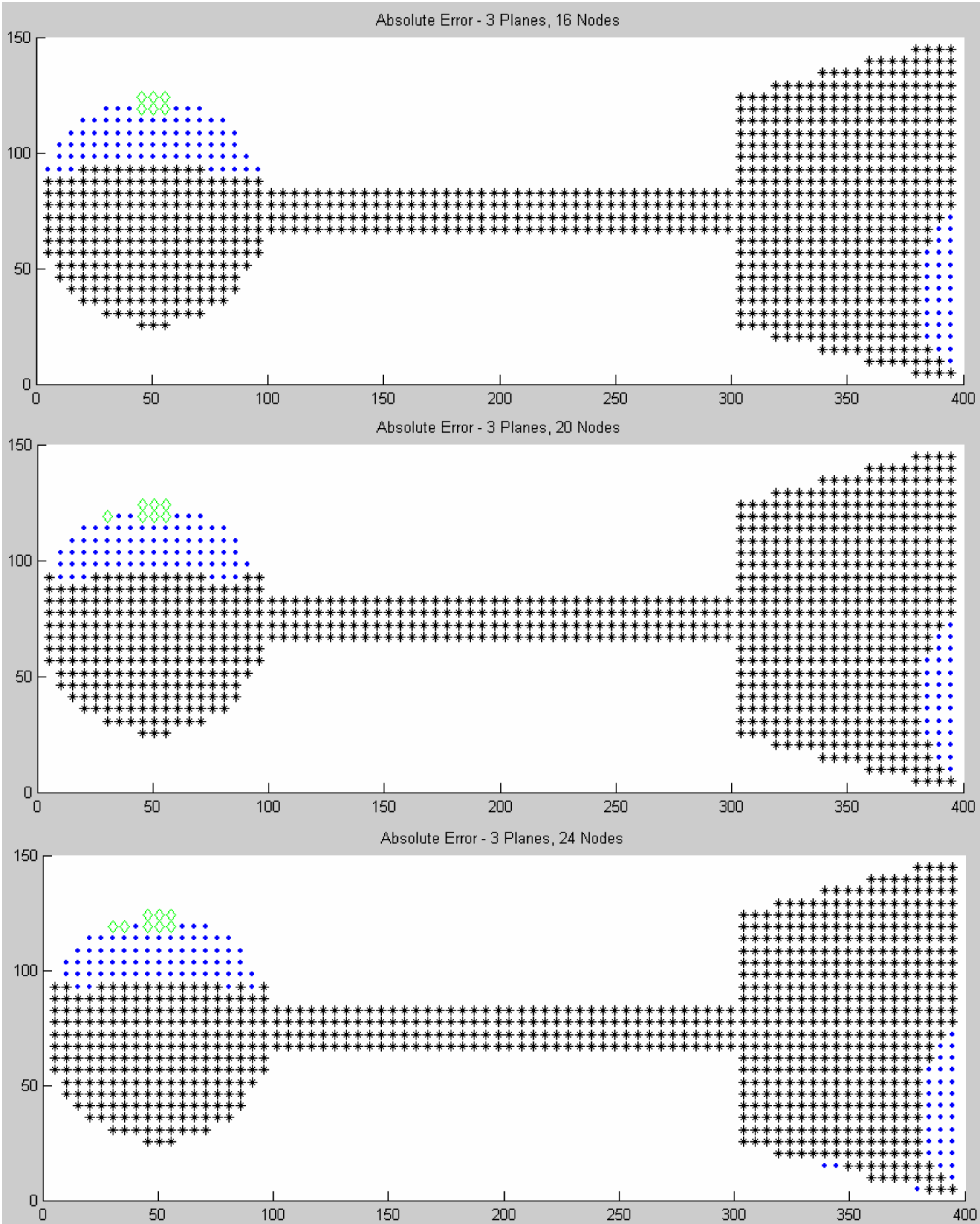
<u>Absolute Errors</u>

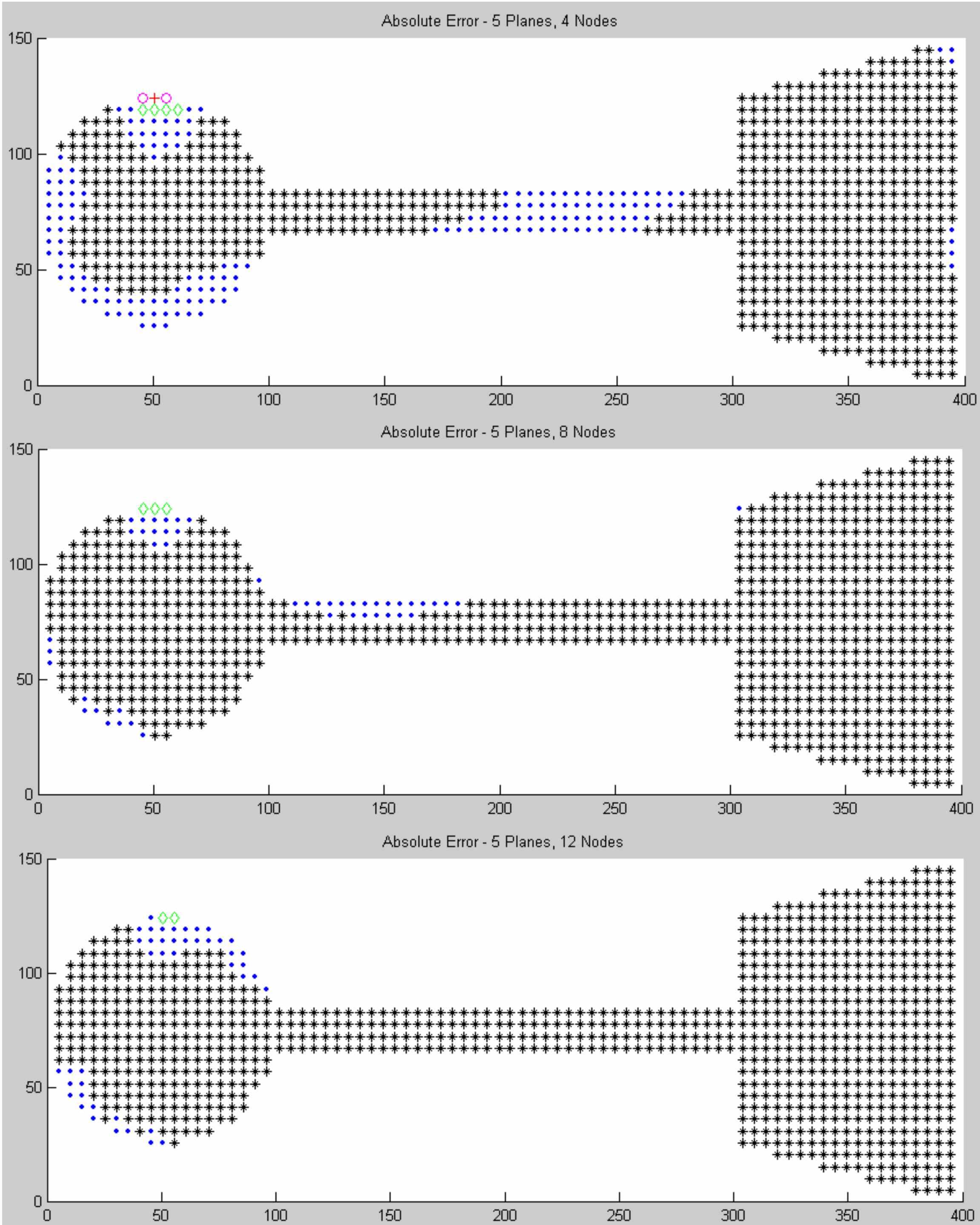
These graphs on pages 15-22 show the absolute errors for the various planes and nodes. From these graphs we see that the absolute errors are reduced as the number of planes and nodes increase.

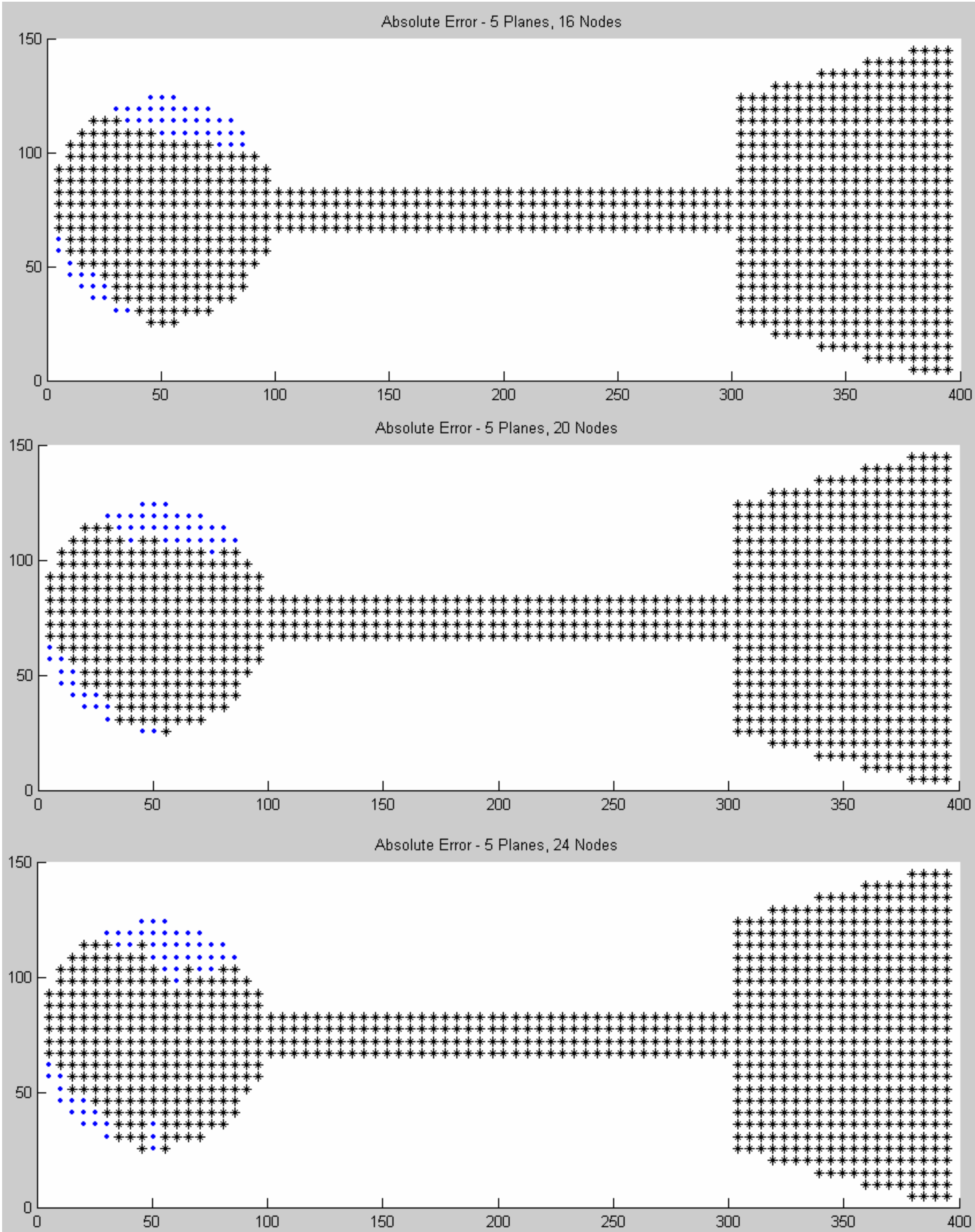


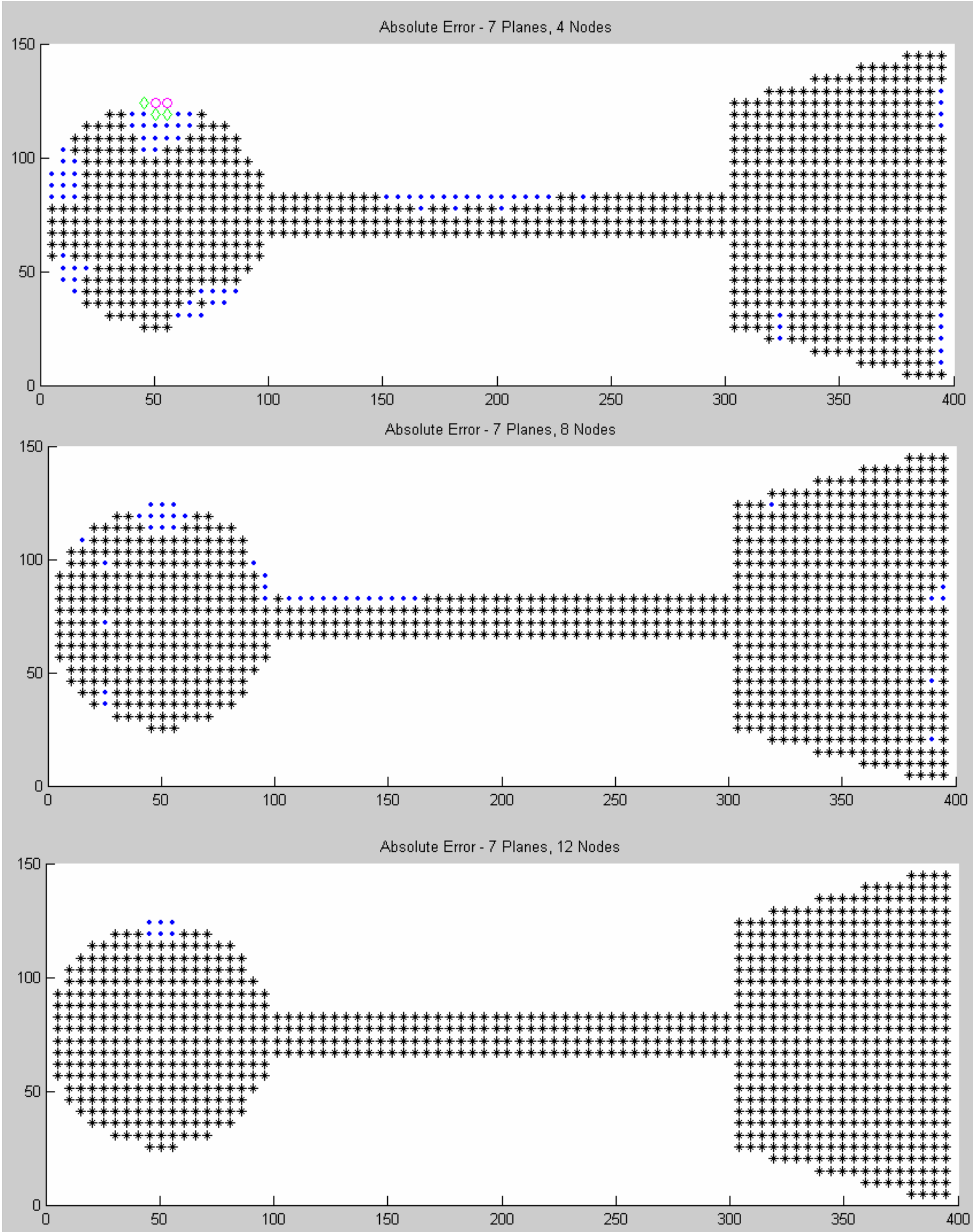


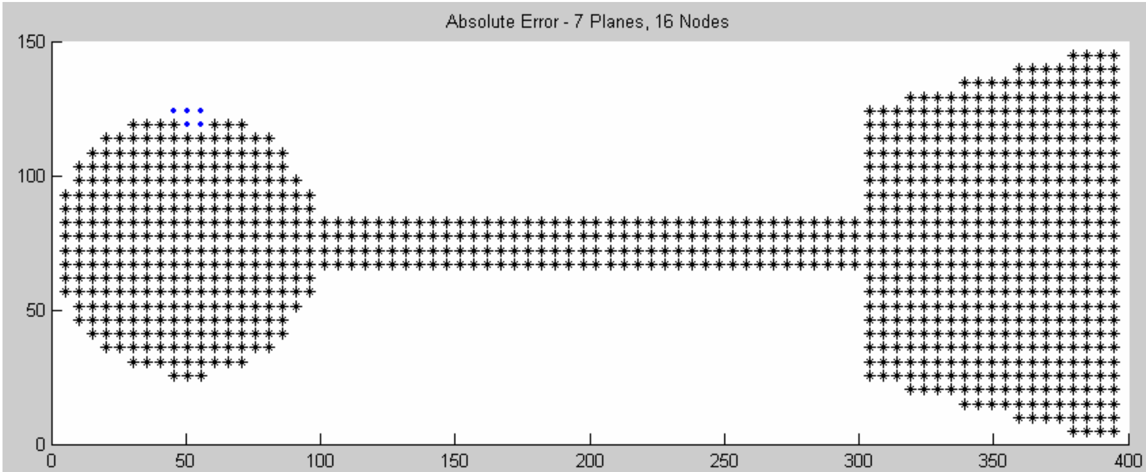


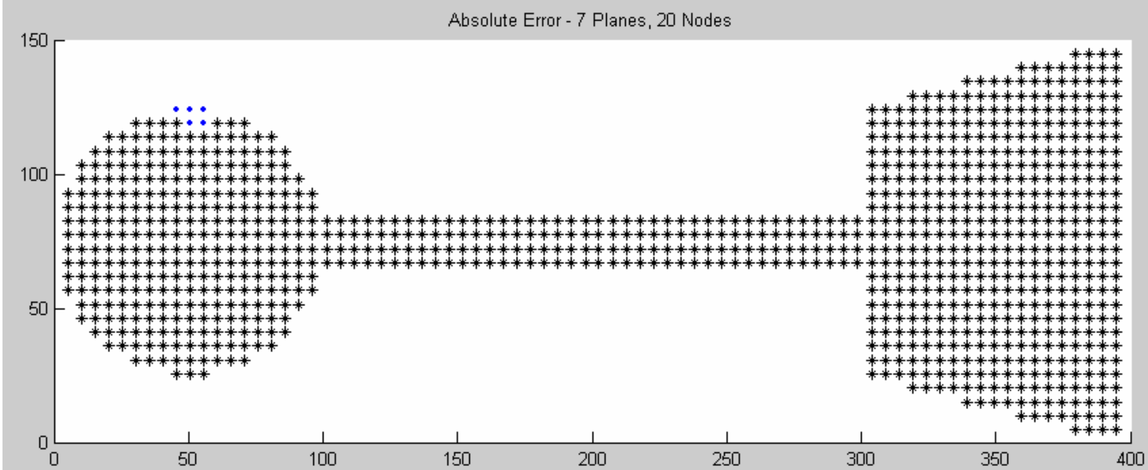

Absolute Error - 3 Planes, 16 Nodes
Absolute Error - 3 Planes, 20 Nodes
Absolute Error - 3 Planes, 24 Nodes

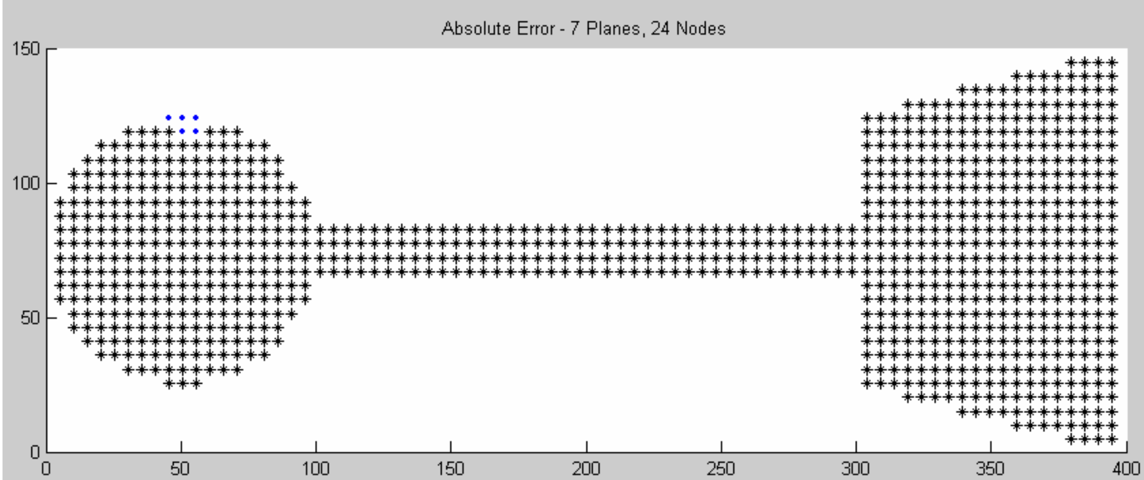

Absolute Error - 5 Planes, 4 Nodes
Absolute Error - 5 Planes, 8 Nodes
Absolute Error - 5 Planes, 12 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

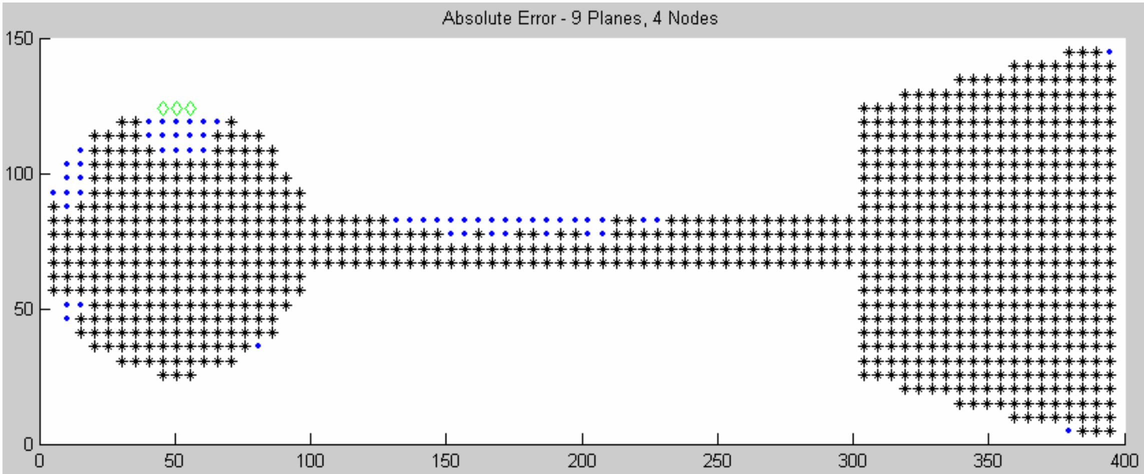

Absolute Error - 5 Planes, 16 Nodes
Absolute Error - 5 Planes, 20 Nodes
Absolute Error - 5 Planes, 24 Nodes
0
50
100
150
200
250
300
350
400

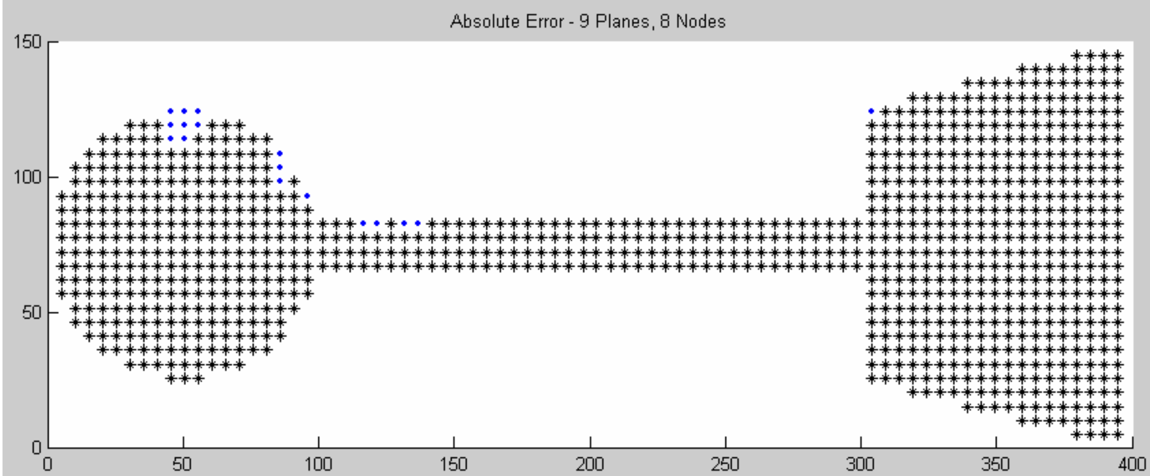

Absolute Error - 7 Planes, 4 Nodes
Absolute Error - 7 Planes, 8 Nodes
Absolute Error - 7 Planes, 12 Nodes
0
50
100
150
200
250
300
350
400

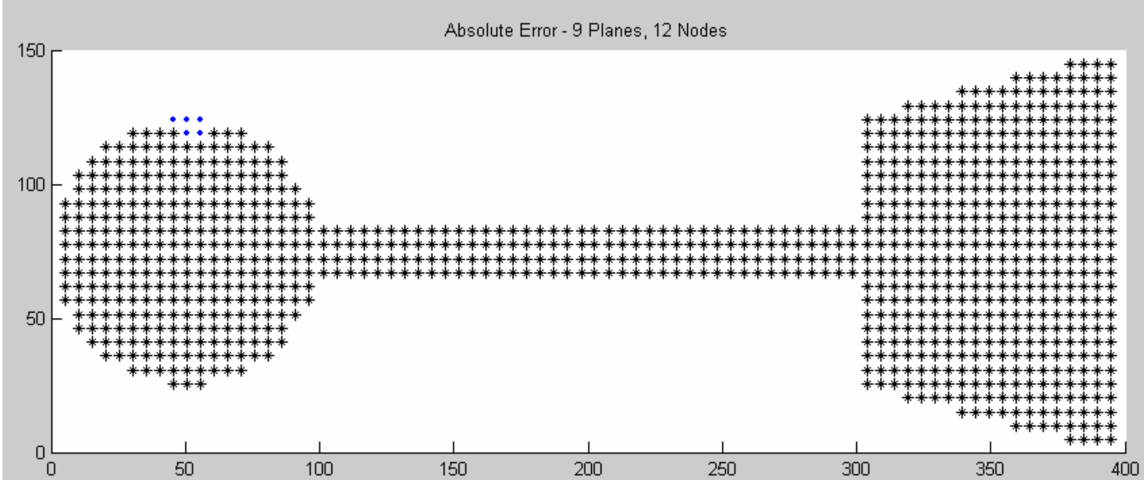

Absolute Error - 7 Planes, 16 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

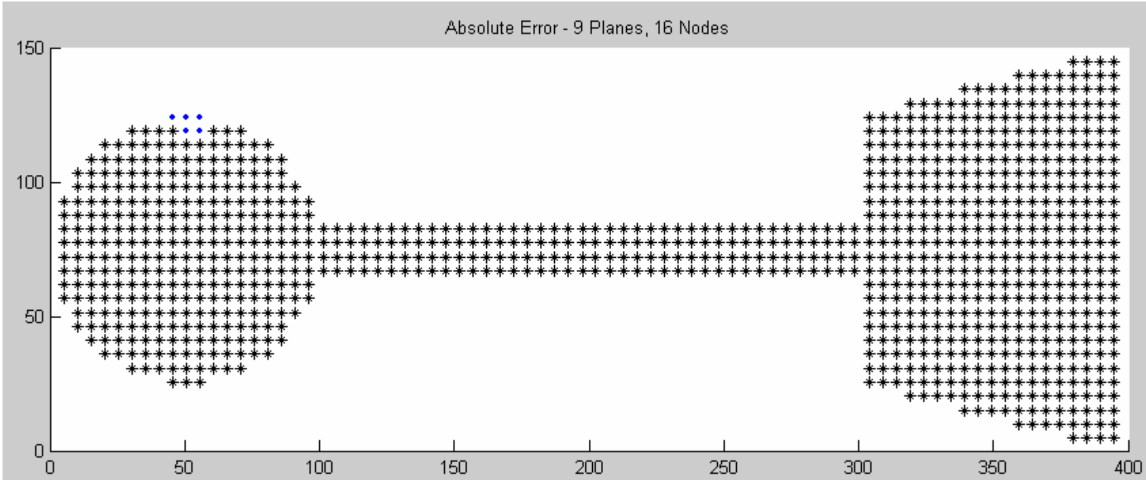

Absolute Error - 7 Planes, 20 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

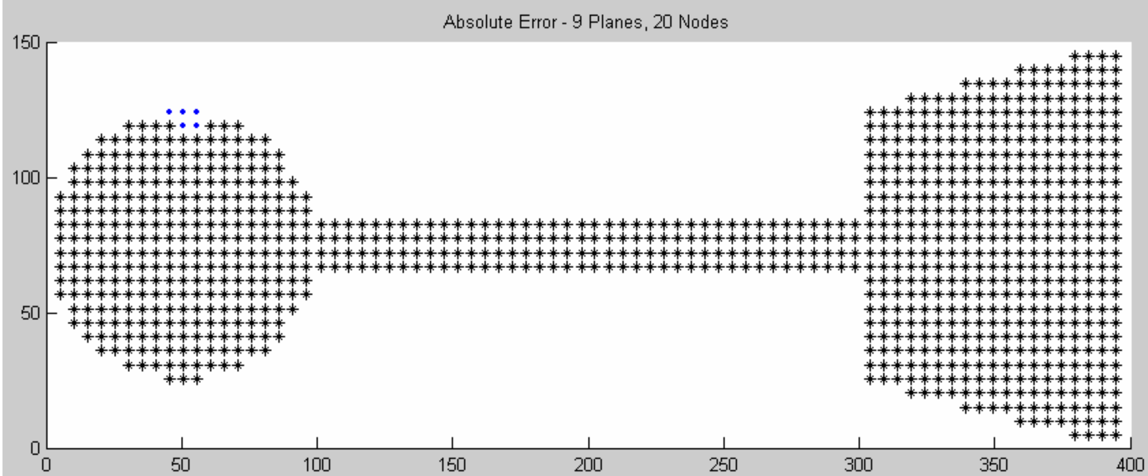

Absolute Error - 7 Planes, 24 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

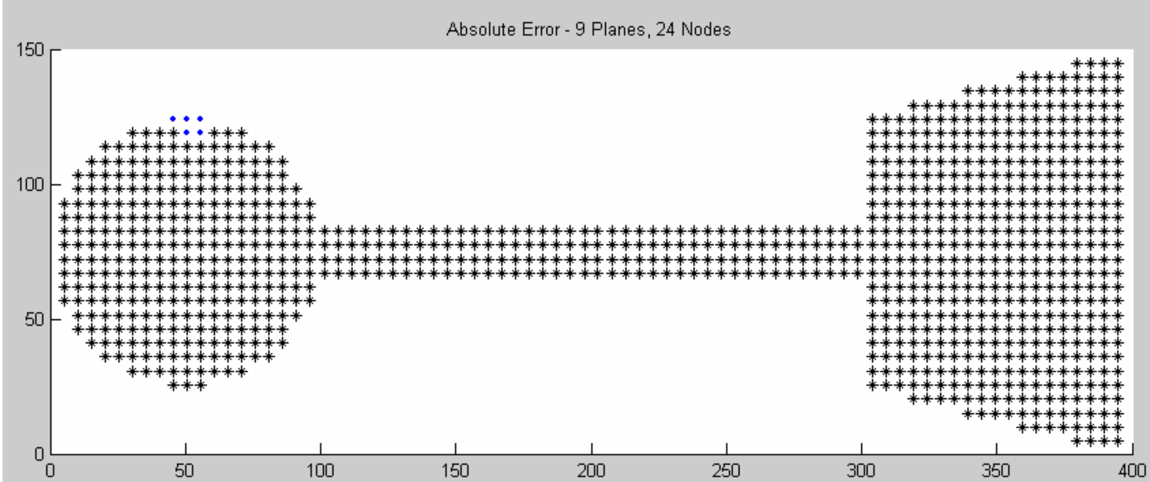

Absolute Error - 9 Planes, 4 Nodes
0
50
100
150
0
50
100
150
200
250
300
350
400


Absolute Error - 9 Planes, 8 Nodes
0
50
100
150
0
50
100
150
200
250
300
350
400


Absolute Error - 9 Planes, 12 Nodes
0
50
100
150
0
50
100
150
200
250
300
350
400

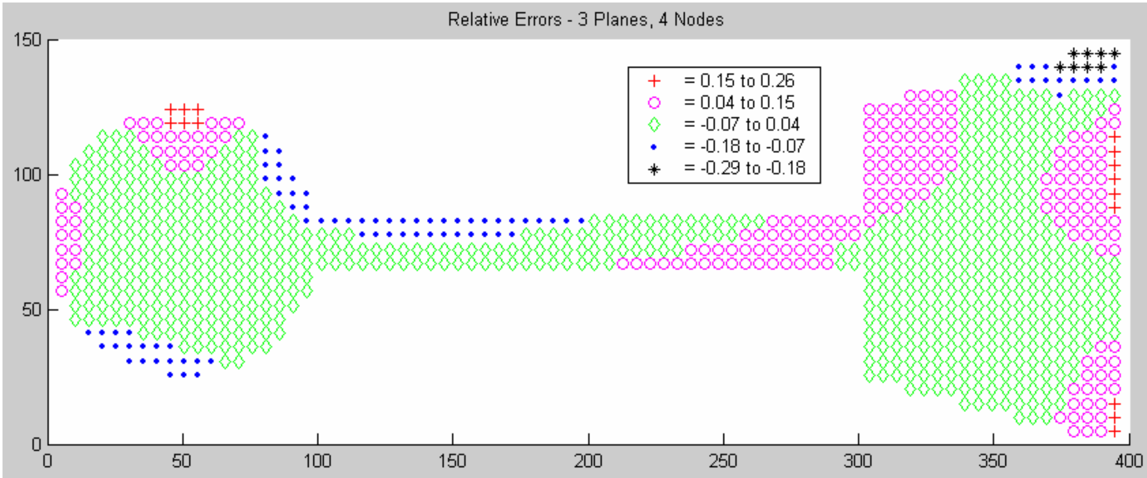

Absolute Error - 9 Planes, 16 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

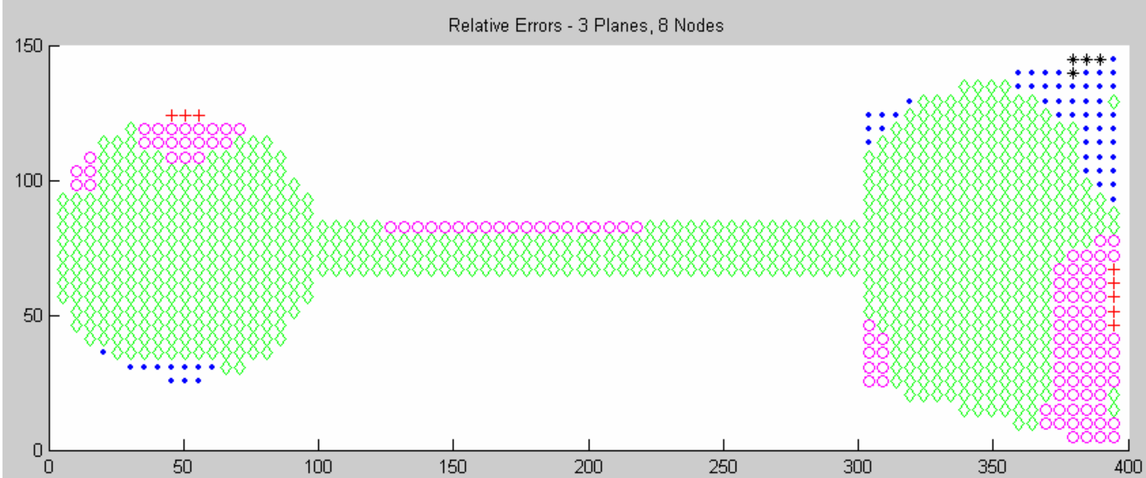

Absolute Error - 9 Planes, 20 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

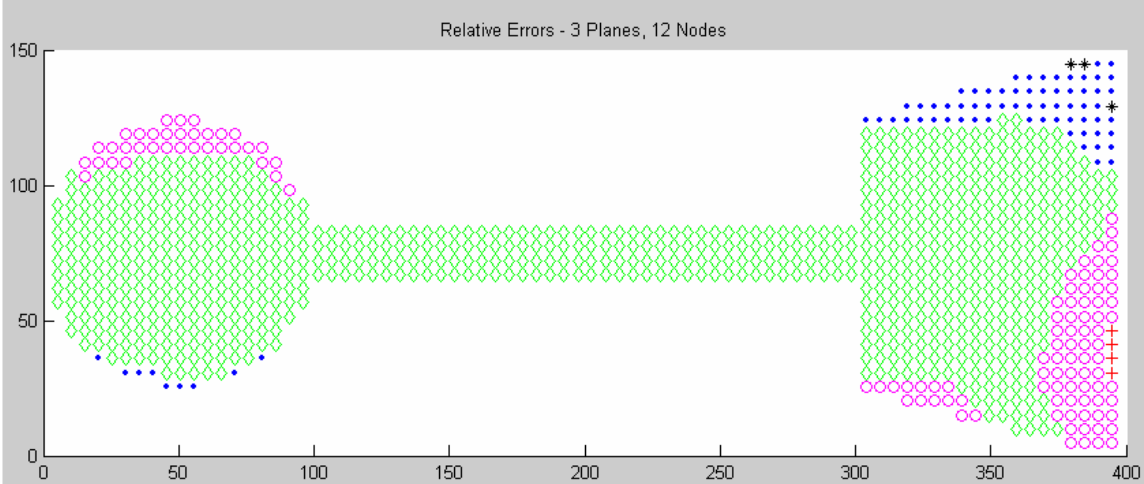

Absolute Error - 9 Planes, 24 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400

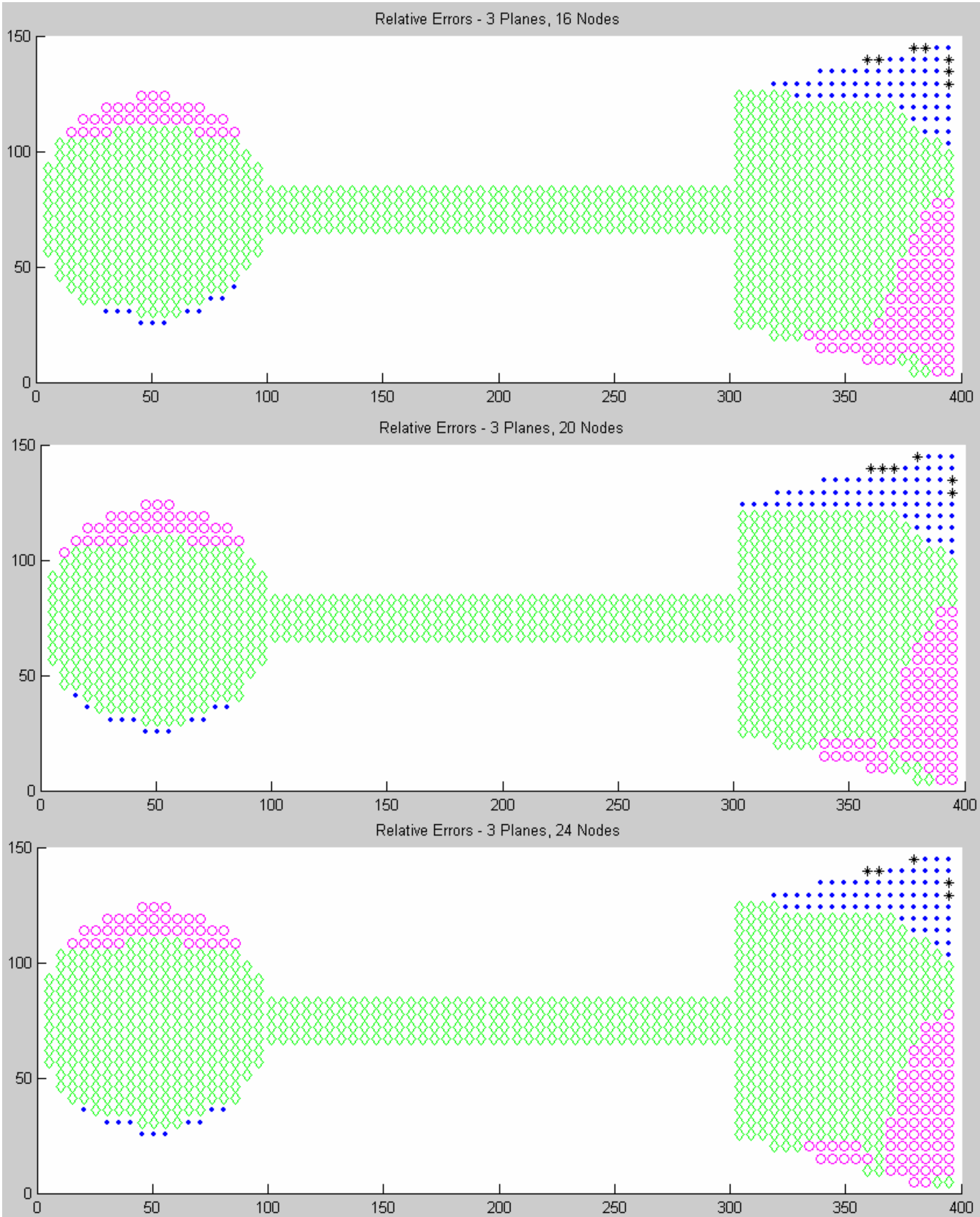
Relative Errors

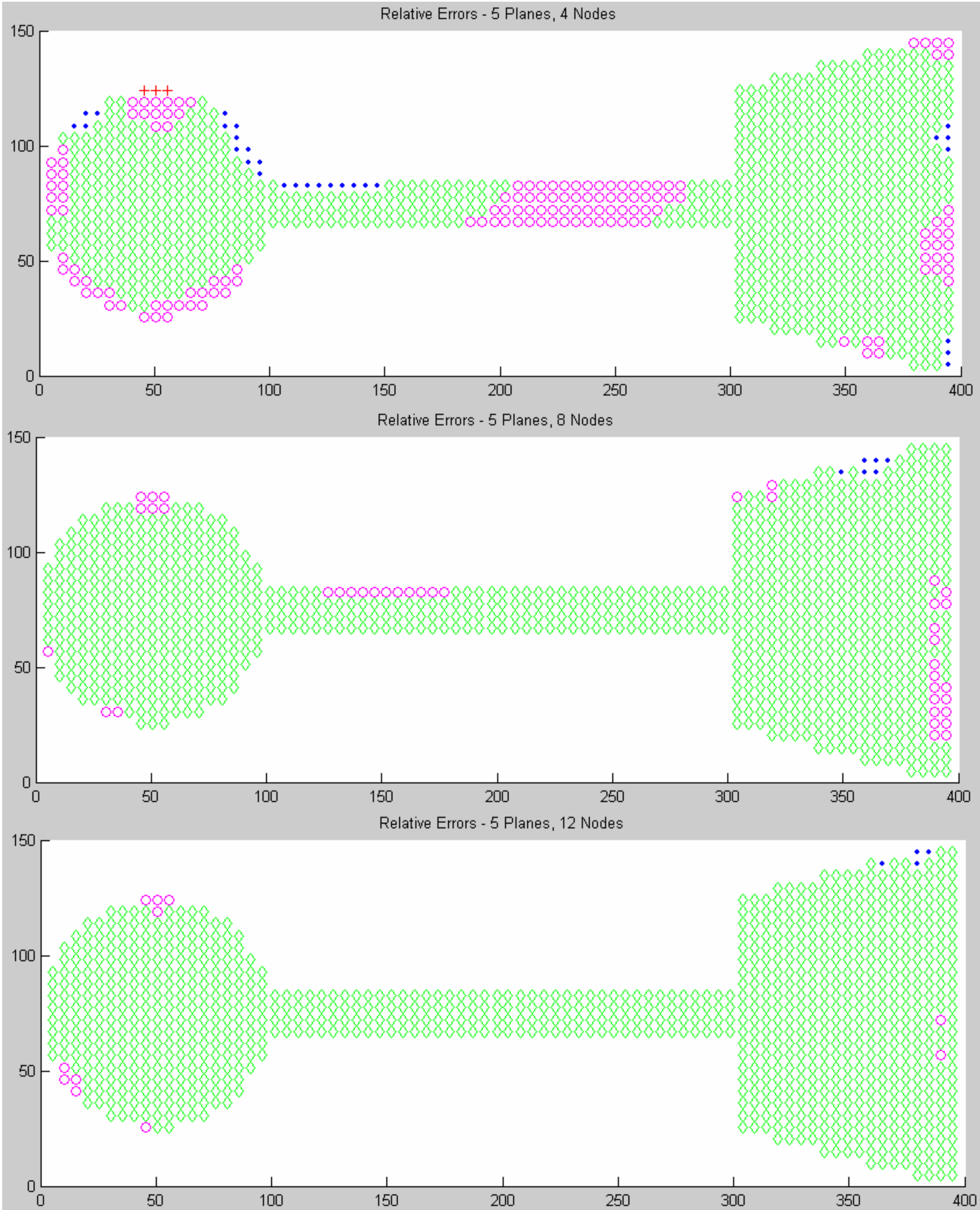
These graphs on pages 23-30 show the relative errors for the various planes and nodes. From these graphs we see that the relative errors are reduced as the number of planes and nodes increase.

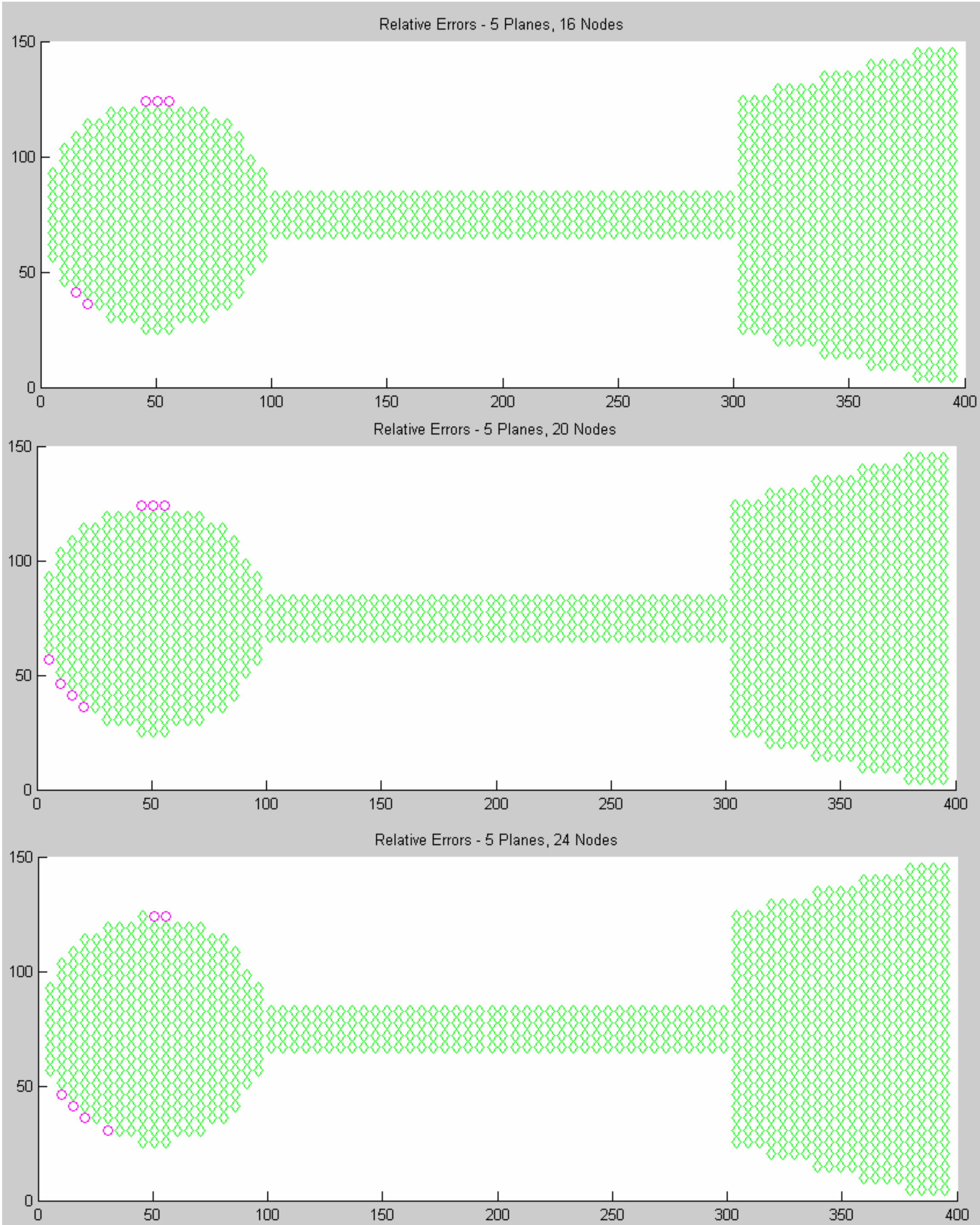


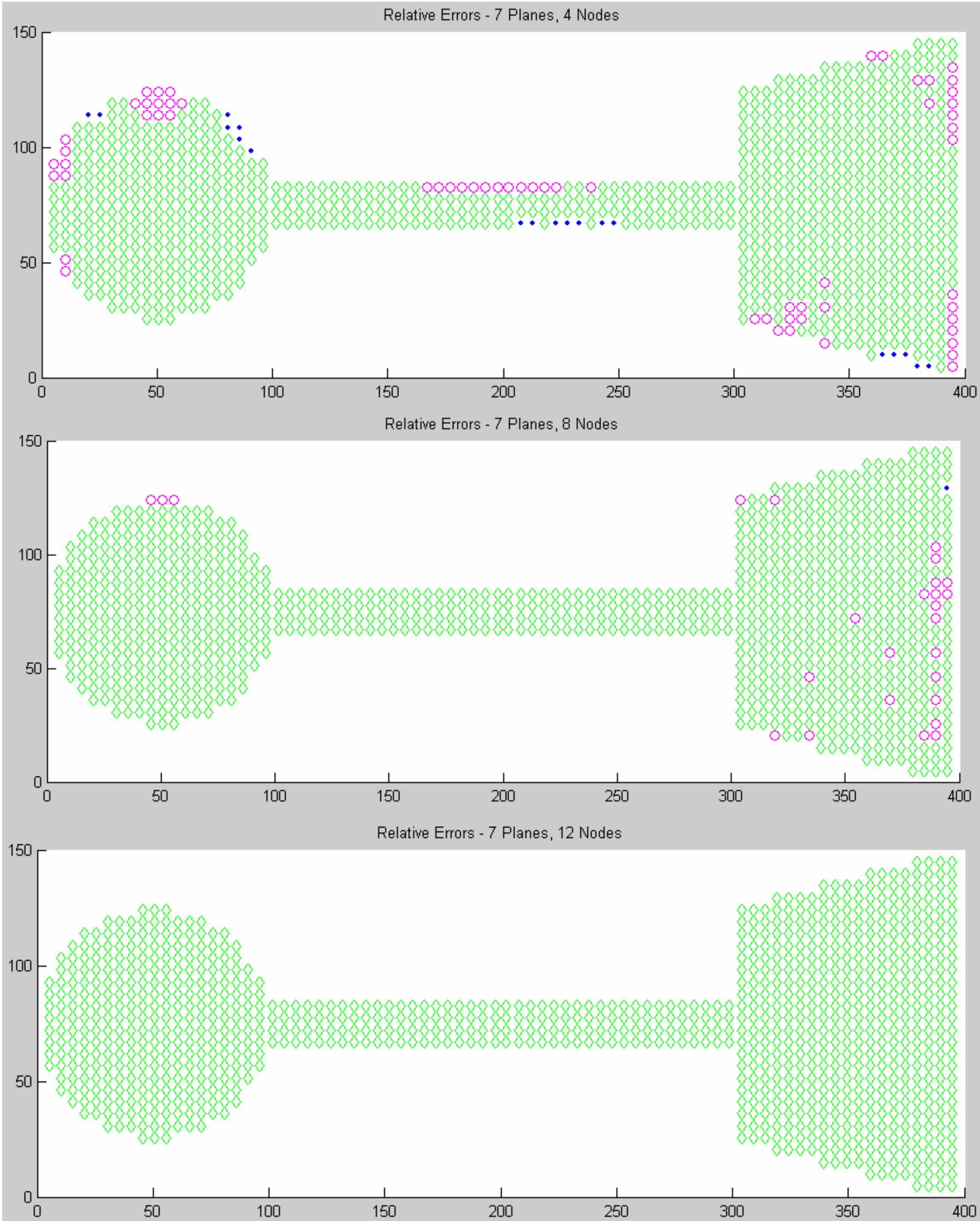


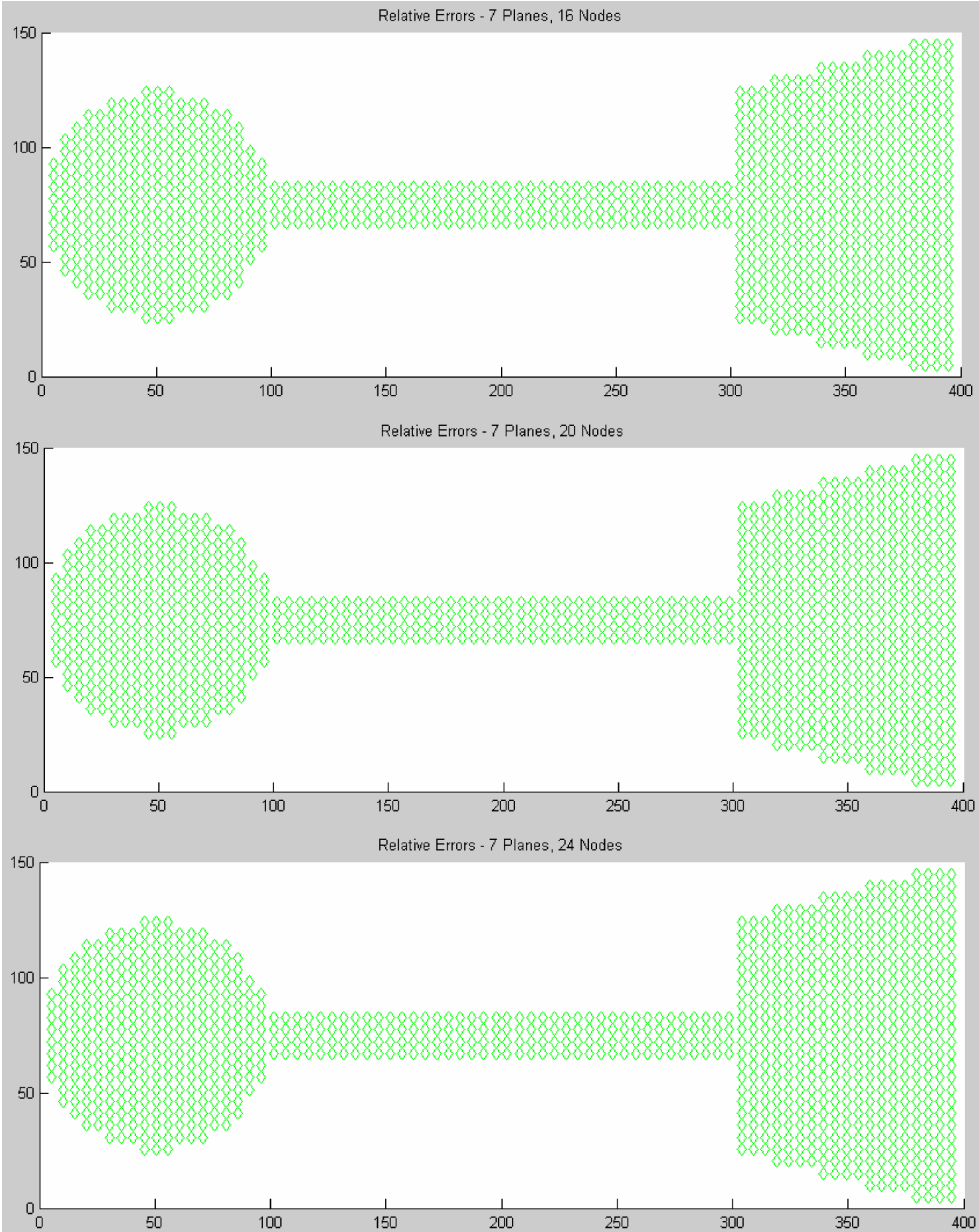


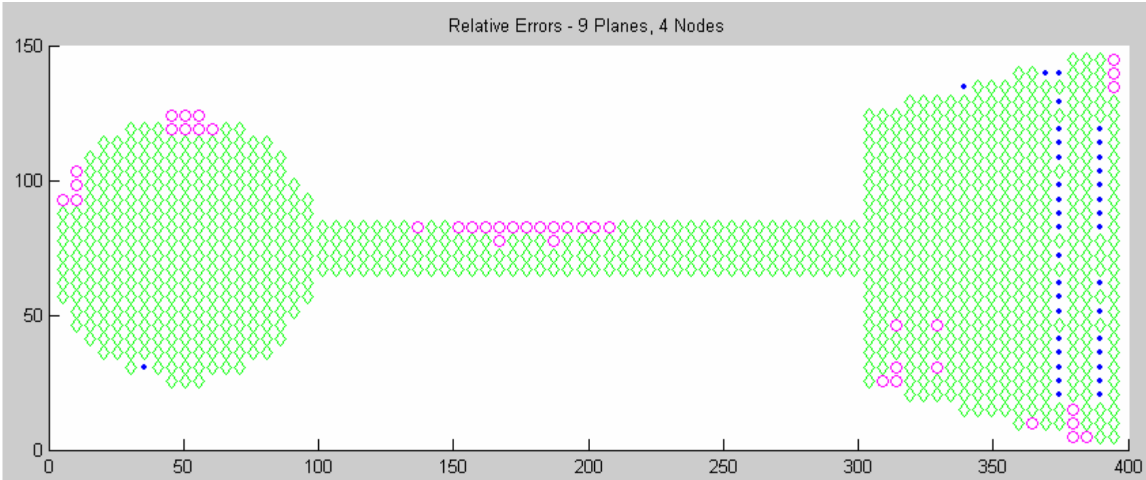

Relative Errors - 3 Planes, 16 Nodes
Relative Errors - 3 Planes, 20 Nodes
Relative Errors - 3 Planes, 24 Nodes
0
50
100
150
200
250
300
350
400

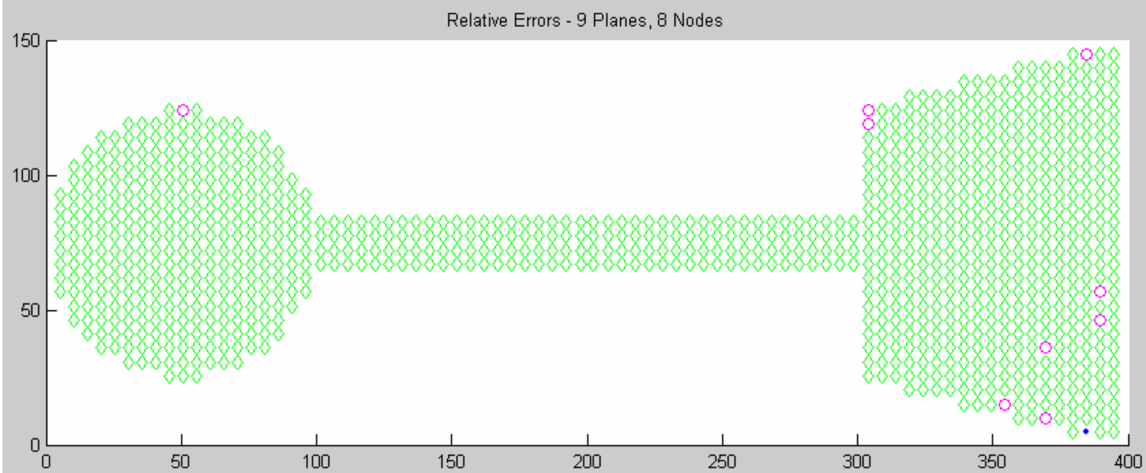

Relative Errors - 5 Planes, 4 Nodes
Relative Errors - 5 Planes, 8 Nodes
Relative Errors - 5 Planes, 12 Nodes
0
50
100
150
200
250
300
350
400

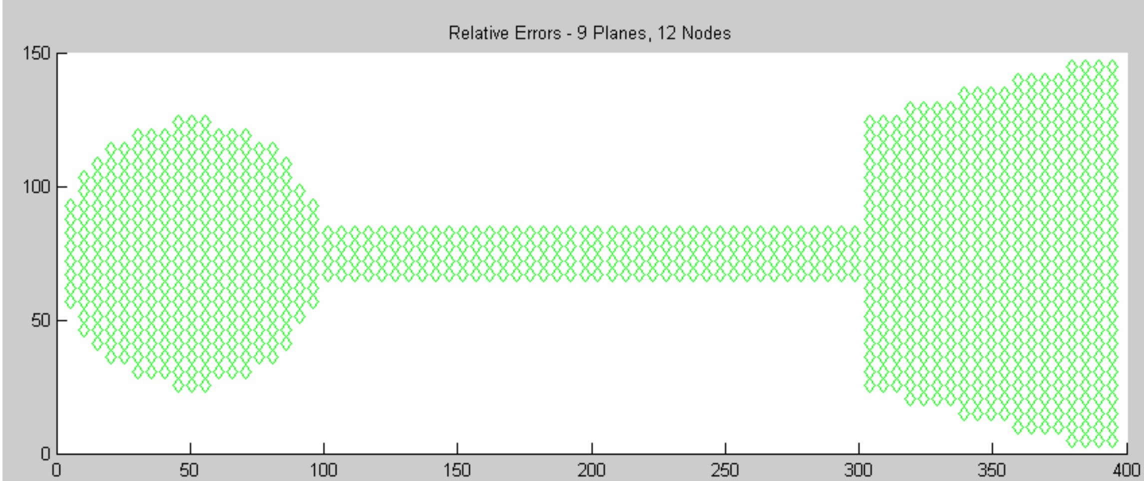

Relative Errors - 5 Planes, 16 Nodes
Relative Errors - 5 Planes, 20 Nodes
Relative Errors - 5 Planes, 24 Nodes
0
50
100
150
200
250
300
350
400

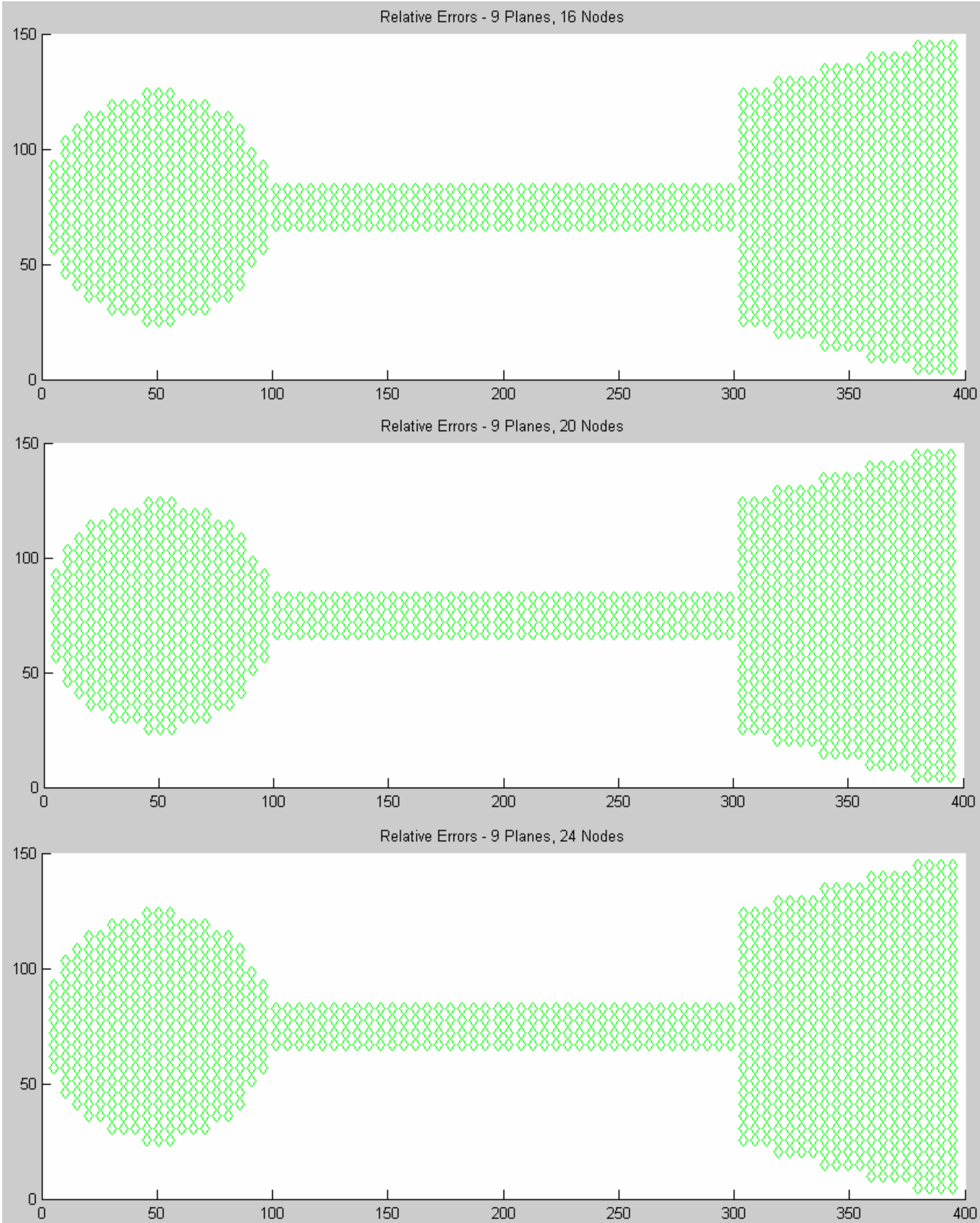

Relative Errors - 7 Planes, 4 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400
Relative Errors - 7 Planes, 8 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400
Relative Errors - 7 Planes, 12 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400


Relative Errors - 7 Planes, 16 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400
Relative Errors - 7 Planes, 20 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400
Relative Errors - 7 Planes, 24 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400


Relative Errors - 9 Planes, 4 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400


Relative Errors - 9 Planes, 8 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400


Relative Errors - 9 Planes, 12 Nodes
150
100
50
0
0
50
100
150
200
250
300
350
400


Relative Errors - 9 Planes, 16 Nodes
Relative Errors - 9 Planes, 20 Nodes
Relative Errors - 9 Planes, 24 Nodes
0 50 100 150
0 50 100 150 200 250 300 350 400