Computational Error: How Low Can We Go?

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Overview

Over the course of this research, we looked at novel ways of positioning the nodes for solving boundary value problems via the Method of Fundamental Solutions (MFS) and the Complex Variable Boundary Element Method (CVBEM)

Background

The MFS and CVBEM techniques are two ways of using Dirichlet boundaries to approximate solutions to boundary value problems by using collocation. While a technique exists to position nodes and collocation points in preparation for the final solving step, we wanted to explore how we could improve on the existing method, which is quite slow.

The figures shown below are the two benchmark problems we used to analyze the success of our Node Positioning Algorithm (NPA). In Figure 1, we see a simulation of ideal fluid flow around a right-angle boundary. In Figure 2, we see a simulation of ideal fluid flow around a right-angle boundary, then proceeding to flow over a hump. Imagine these views as side-view cross-sections of a pipe.

Figure 1: Right-Angle Bend

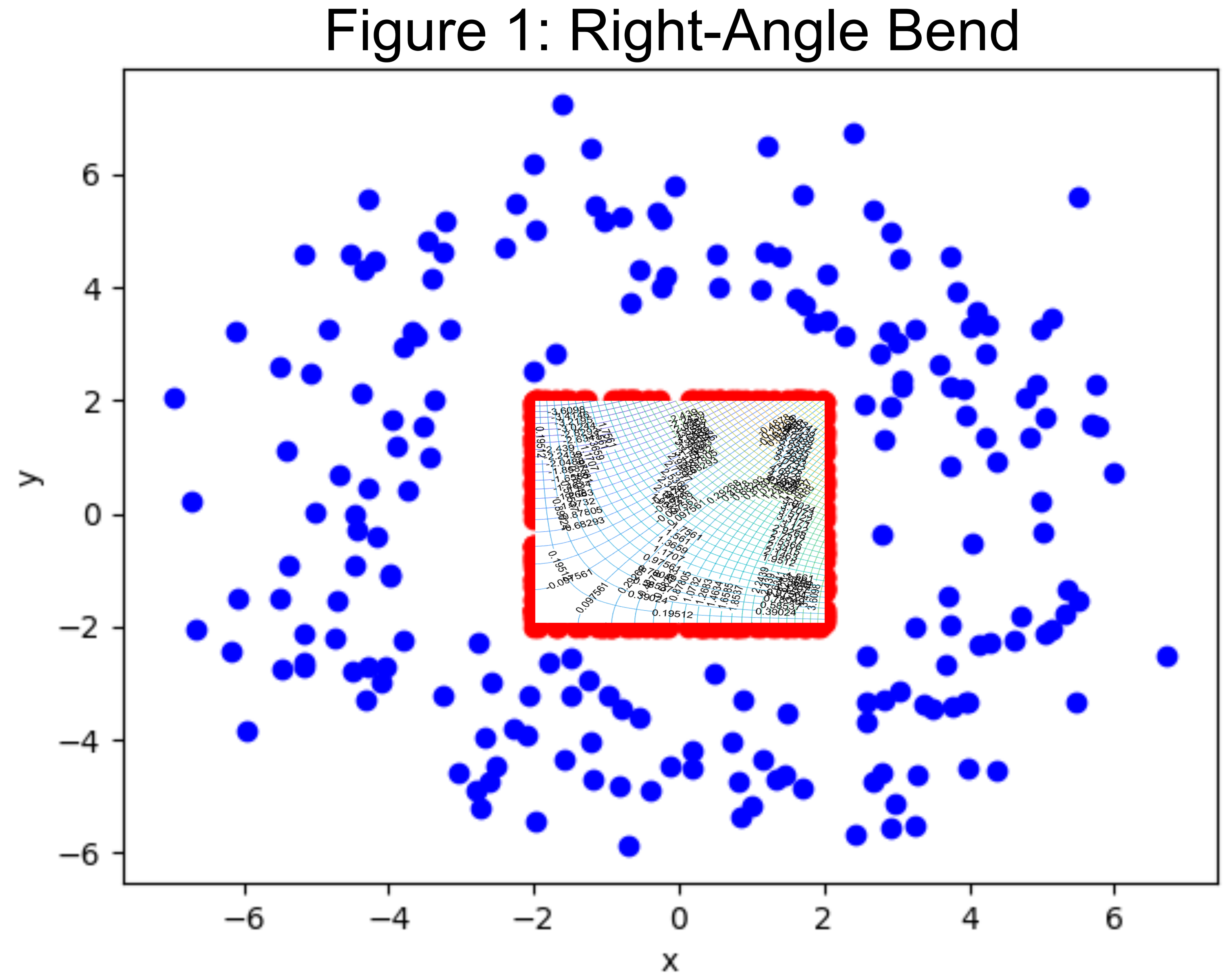
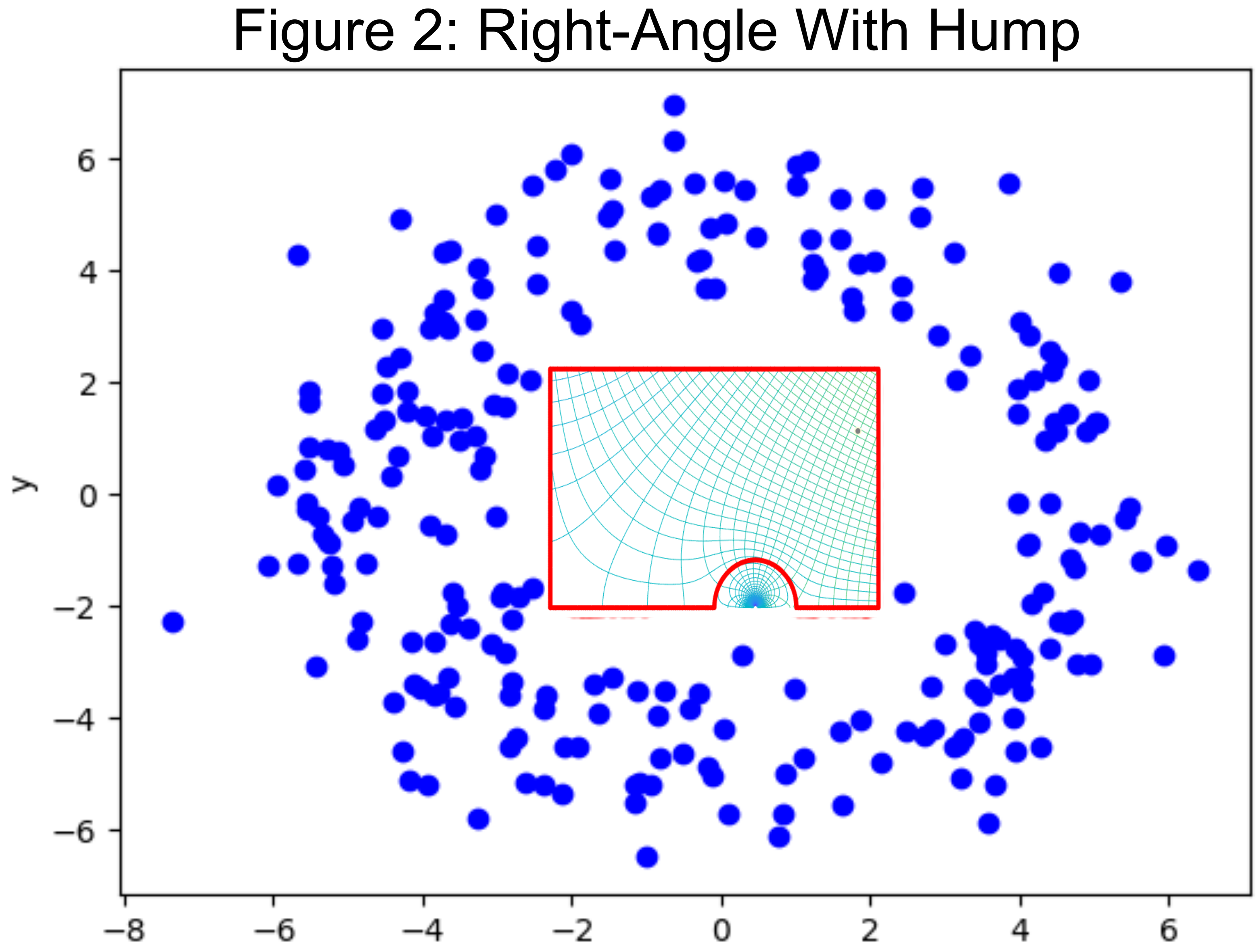


Figure 2: Right-Angle With Hump



Results

On our two benchmark problems, we found that we could achieve similarly accurate results to the previous method while drastically reducing the computational time necessary to compute these results. With approximately 250 nodes and collocation points, we were able to successfully reduce the error to approximately 10^{-16} for the right-angle problem, and 10^{-14} for the right-angle-with hump problem. Once we get to numbers these small, the floating-point arithmetic starts to break down, which introduces some error into the system.

Figure 3

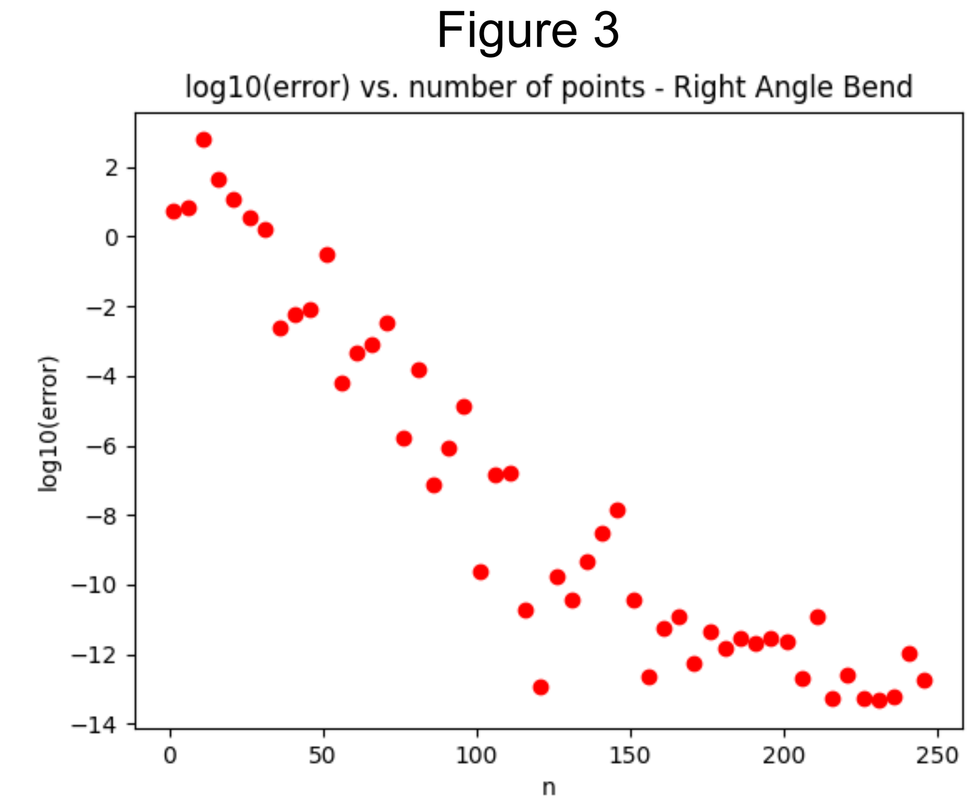


Figure 4

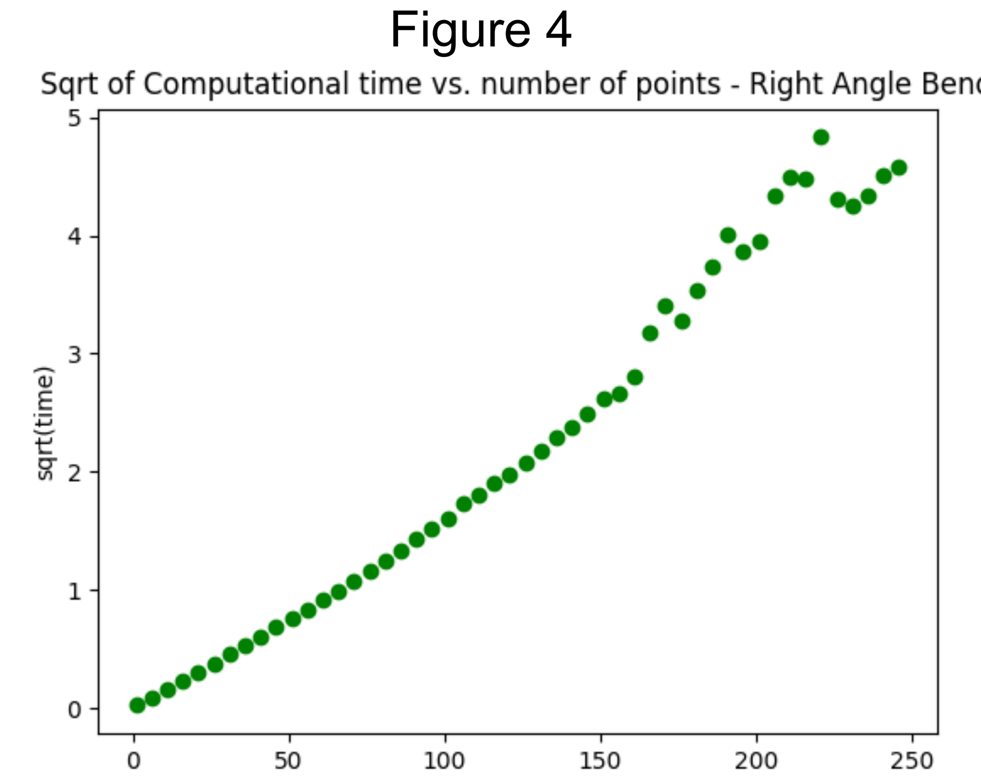


Figure 5

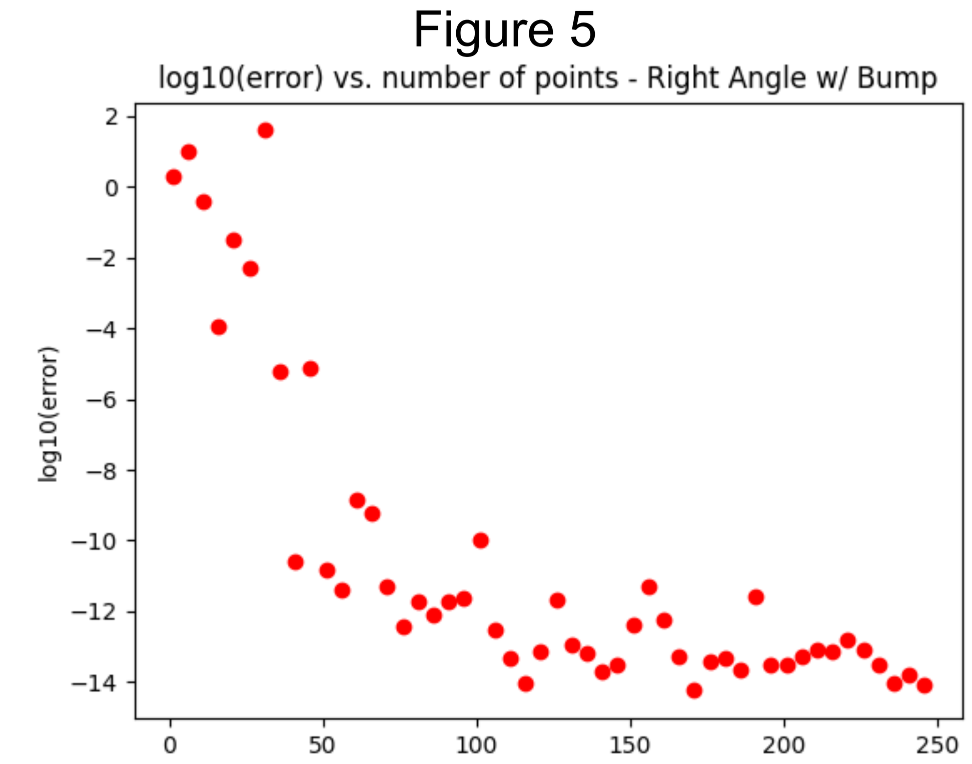


Figure 6