A comparison of techniques for evaluating hydrologic model uncertainty

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Hydrologic models are composed of several components which are all parameter dependent. In the general setting, parameter values are selected based on regionalization of observed rainfall-runoff events, or upon calibration at local stream gauge data when available. Based on these data, a selected parameter set is then used for the hydrologic model. However, seldom are hydrologic model outputs examined as to the total variations in output due to the independent but coupled variations in parameter input values. In this paper, three of the more common techniques for evaluating model output distributions are compared as applied to a selected hydrologic model; i.e., an exhaustion techniques, Monte Carlo simulation method, and the more recently advanced Rosenblueth technique. It is concluded that, for the hydrologic model considered, the Monte Carlo technique provides more accuracy in comparison to Rosenblueth technique (for the same computational effort), but is less accurate than Exhaustion.

I. INTRODUCTION

Almost all policy statements regarding the design of flood control systems involve the use of a hydrologic mathematical model to estimate design flows for subsequent channel sizing and floodplain delineation. Typically, these models are hybridizations of other 'standardized' models which have been calibrated to regions dissimilar to the region where the model is intended for use. Or if the model has been regionalized by calibration to several local catchments, a prescribed set of rules are decided upon (e.g., a 'hydrology manual') as to how to select model parameter values for use in the subsequent estimation of runoff quantities. And finally, if the model has been calibrated to catchment rainfall-runoff data when available, an 'optimized' set of parameters are concluded which minimize, in some norm sense, the errors between model-produced and stream gauge measured runoff hydrographs.

In all of the above settings, the hydrologic model of the catchment is based upon a set of parameters which produce a single model output for decision making purposes. Oftentimes these model output values are compared to other models' single-output values and decisions are concluded as to which model is 'best'.

Because all hydrologic models are gross simiplifications of the several hydrologic processes, and due to the sparse data (if any) available to compute the model parameters, each of the model parameters and submodels (e.g., loss functions, routing methods, etc.) have an associated distribution-frequency which relates parameter values to the probability of exceedence (or frequency of occurrence), or relates the probability of which submodel (e.g., which loss function to use, which flow routing submodel to use, etc.) best represents the actual ongoing hydrologic/hydraulic process. The use of a single set of parameter values or choices of submodel algorithms to be used in the global model is but a single vector in an infinity of possible vectors.

Consequently, the policy decisions need to include the uncertainty of the hydrologic model in the selection of a level for flood protection. Generally, policy statements reflect the control of a peak flow rate or Q in the design of flood control facilities. The policy may incorporate the accommodation of a time distribution of runoff volume by means of a design storm runoff hydrograph (for example), but ultimately a Q is developed for channel sizing, for the delineation of a floodplain, and for the planning of urbanization. In this paper, only the uncertainty in model estimated Q (i.e., peak flow rate) values is considered in order to simplify the presentation in the comparison of the uncertainty evaluation techniques.

The policy statement, then, is reflected by the calculation of a specific Qm based upon using a single input vector (which represents parameter estimates, submodels to use, land use assumptions, etc.) into the model. Should another input vector be used, another Q estimate would result where typically $Q \neq Qm$. The question then becomes: what is the distribution of possible Q values as all possible input vectors are considered according to their respective probabilities of occurrence? To address this issue, a frequency distribution of the range of model estimated Q needs to be computed. But the enormity of this task has precluded a precise evaluation for most problems. For example, in a simple four parameter unit hydrograph model based

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upon a timing parameter (lag), a unit hydrograph (S-graph equivalent), and a two parameter loss function (Fm= maximum loss rate or ϕ-index technique, $\bar{Y}$= constant percentage loss rate fraction) where the parameter values can be presumably represented by a simple distribution-frequency histogram of 9-, 5-, 6- and 5-values respectively, 1350 test runs must be made with the model to exhaust all possible input vector definitions. Should the catchment model be discretized into, say, 4 subareas linked together by a single reach two-parameter channel routing submodel where both of the routing parameter distribution-frequency histograms have 5 values, then 8.3×10^{13} model runs are required. It is apparent that an alternative approach is needed in order to determine a model output Q distribution-frequency relationship.

The main objective of this paper is to present the results in the development of a distribution-frequency relationship for a hydrologic model output, the peak Q, using three techniques: (i) Exhaustion; that is, using every possible vector to evaluate the model output (Q) values; (ii) Monte Carlo simulation method[80]; that is, randomly choosing input vectors to evolve a Q distribution-frequency; and (iii) The Rosenblueth technique[82] which utilizes an approximation to estimate the statistics of the Q, i.e., the mean and variance of Q. Details of the considered hydrologic model are presented in Section II. An examination of the 'model' response function to parameter uncertainty is contained in Section III. Specifics of the Monte Carlo simulation and Rosenblueth techniques are contained in Section IV. Section IV also presents the computational results from the three considered approaches for comparison purposes.

II. HYDROLOGIC MODEL

Model selection

Of the over 100 models available, a design storm/unit hydrograph model (i.e., 'model') is selected for this particular application. Some of the reasons are as follows: (1) the design storm approach – the multiple discrete event and continuous simulation categories of models have not been clearly established to provide better predictions of flood flow frequency estimates for evaluating the impact of urbanization and for design of flood control systems than a calibrated design storm model[3,4,57,67]; (2) the unit hydrograph method – it has not been shown that alternate approaches (i.e., the kinematic wave modelling technique) provide a significantly better representation of watershed hydrologic response than a model based on unit hydrographs (locally calibrated or regionally calibrated) that represent free-draining catchments[1,2,4,6,8,23,26,30,31,49,50,52,55,63,66,69,70,73,74,77]; (3) model usage – this class of 'model' has been used extensively nationwide and has proved generally acceptable and reliable[3,10,15,19,28,37,39,41,42,43,53]; (4) parameter calibration – the 'model' used in this application is based upon a minimal number of parameters, giving higher accuracy in calibration of model parameters to rainfall-runoff data, and the design storm to local flood flow frequency tendencies[3,4,12,18,20,22,23,29,32,36,38,44,45,46,58,62,66,70,79]; (5) calibration effort – the 'model' does not require large data or time requirements for calibration[3,4,12,18,20,29,36,38,66,70]; (6) application effort – the 'model' does not require excessive computation for application[3,66,70]; (7) acceptability – the 'model' uses algorithms that are accepted in engineering practice[3,37,66]; (8) model flexibility for planning – data handling and computational submodels can be coupled to the 'model' (e.g., channel and basin routing) resulting in a highly flexible modelling capability[3,66]; (9) model certainty evaluation – the certainty of modelling results can be readily evaluated as a distribution of possible outcomes over the probabilistics distribution of parameter values[34,59,60,66,75,76].

Runoff hydrograph model parameters

The design storm unit hydrograph model ('model') is based upon several parameters; namely, two loss rate parameters (a ϕ-index coupled with a fixed percentage), an S-graph, catchment lag, storm pattern (shape, location of peak rainfalls, duration), depth area (or depth-area-duration) adjustment, and the return frequency of rainfall.

Loss function

The loss function, $f(t)$, used in the 'model' is defined by

$$f(t)=\begin{cases}\bar{Y}I(t), \text{ for } \bar{Y}I(t)<Fm\\ Fm, \text{ otherwise}\end{cases} \tag{1}$$

where $\bar{Y}$ is the low loss fraction and Fm is a maximum loss rate defined by

$$Fm=\sum a_{pj}F_{pj} \tag{2}$$

where a_{pj} is the actual pervious area fraction with a corresponding maximum loss rate of Fp_j; the infiltration rate for impervious area is set at zero; and $I(t)$ is the design storm rainfall intensity at storm time t.

The use of a constant percentage loss rate fraction $\bar{Y}$ in equation (1) is reported in Scully and Bender[57], Williams *et al.*[77], and Schilling and Fuchs[56]. The use of a phi index (ϕ-index) method in effective rainfall calculations is also well-known.

The low loss rate fraction is estimated from the SCS loss rate equation (US Dep. of Agric., 1972) by

$$\bar{Y}=1-Y \tag{3}$$

where Y is the catchment yield computed by

$$Y=\sum a_jY_j \tag{4}$$

In equation (4), Y_j is the yield corresponding to the catchment area fraction a_j and is estimated using the SCS curve number (CN) by

$$Y_j=\frac{(P_{24}-Ia)^2}{(P_{24}-Ia+S)P_{24}} \tag{5}$$

where P_{24} = the 24-hour T-year precipitation depth; Ia is the initial abstraction of $Ia=0.2S$; and $S=(1000/\text{CN})-10$.

From the above relationships, the low loss fraction, $\bar{Y}$, acts as a fixed loss rate percentage, whereas Fm serves as an upper bound to the possible values of $f(t)=\bar{Y}I(t)$.

Values for Fm are based on the actual pervious area cover percentage (a_p) and a maximum loss rate for the pervious area, F_p. Values for F_p are developed from rainfall-runoff calibration studies of several significant storm events for several watersheds within the region under study. Further discussions regarding the estimation of parameter values are contained in a subsequent section.

A distinct advantage afforded by the loss function of equation (1) over loss functions such as Green-Ampt or Horton is that the effect of the location of the peak rainfall intensities in the design storm pattern on the model peak flow rate (Q) becomes negligible. That is, front-loaded, middle-loaded, and rear-loaded storm patterns all result in nearly equal peak flow estimates. Consequently, the shape (but not magnitude) of the design storm pattern is essentially eliminated from the list of parameters to be calibrated in the runoff hydrograph 'model' (although the time distribution of runoff volumes are affected by the location of the peak rainfalls in the storm pattern which is a consideration in detention basin design).

S-graph

The *S*-graph representation of the unit hydrograph (e.g., McCuen and Bondelid[41], Chow and Kulandalswamy[10], Mays and Coles[39]) can be used to develop unit hydrographs corresponding to various watershed lag estimates. The *S*-graph was developed by rainfall-runoff calibration studies of several storms for several watersheds. By averaging the *S*-graphs for each watershed studied, a representative *S*-graph is developed for each watershed. By comparing the representative *S*-graphs, regional *S*-graphs were derived to represent the average of watershed-averaged *S*-graphs.

Lag

Fundamental to any hydrologic model is a catchment timing parameter. For the 'model', watershed lag is defined as the time from the beginning of effective rainfall to that time corresponding to 50-percent of the *S*-graph ultimate discharge. To estimate catchment lag, it is assumed that lag is related to the catchment time of concentration (Tc) as calculated by a sum of normal depth flow calculated travel times; i.e., a mixed velocity method (e.g., Beard and Chang[3], McCuen *et al.*[42]). To correlate lag to Tc estimates, lag values measured from watershed calibrated *S*-graphs were plotted against Tc estimates. A least-squares best fit line gives the estimator

$$\text{lag} = 0.80Tc \tag{6}$$

Design storm pattern

A 24-hour duration design storm composed of nested 5-minute unit intervals (with each principal duraction nested within the next longer duration) was adopted as part of the policy. The storm pattern provides equal return frequency rainfalls for any storm duration, i.e., the peak 5-minute, 30-minute, 1-hour, 3-hour, 6-hour, 12-hour, and 24-hour duration rainfalls are all of the selected T-year return frequency. Such a storm pattern construction is found in HEC Training Document No. 15 (1982) which uses a nested central-loaded design storm pattern.

Runoff hydrograph model

The 'model' produces a time distribution of runoff $Q(t)$ given by the standard convolution integral representation of

$$Q(t) = \int_0^t e(s)u(t-s)\,\mathrm{d}s \tag{7}$$

where $Q(t)$ is the catchment flow rate at the point of concentration; $e(s)$ is the effective rainfall intensity; and $u(t-s)$ is the unit hydrograph developed from the particular S-graph. In equation (7), $e(s)$ represents the time distribution of the 24-hour duration design storm pattern modified according to the loss function definition of equation (1).

In the use of equation (7) for a particular watershed, an estimate of catchment lag is used to construct a unit hydrograph $u(x)$. Then, based on the catchment area (depth-area adjustment) and loss rate characteristics, $e(s)$ is determined. Because the peak flow rate $Q = \max Q(t)$ shows a negligible variation due to a change in storm pattern shape (except for a severe front loaded, near-monotonically decreasing pattern or a rear loaded, near-monotonically increasing pattern); the model parameters that affect Q are loss rates ($\bar{Y}I(t)$ and F_p), S-graphs, lag estimates, depth-area adjustment curve set, and design storm rainfall return frequency. Note that Fm is not a calibration parameter as $Fm = a_pF_p$ where a_p is the actual measured pervious area fraction.

Parameter calibration

Considerable rainfall-runoff calibration data has been prepared by the Corps of Engineers (COE) for use in their flood control design and planning studies. The watershed information available includes rainfall-runoff calibration results for three or more significant storms for watershed, which is used to develop optimized estimates for the S-graph, lag, and loss rate at the peak rainfall intensities. Although the COE used a more rational Horton type loss function which decreases with time, only the loss rate that occurred during the peak storm rainfalls was used in the calibration effort reported herein.

Fig. 1. Location of drainage basins used in model parameter calibration effect

Table 1. Watershed characteristics

Watershed name	Watershed geometry: Area (mi²)	Length (mi)	Length of centroid (mi)	Slope (ft/mi)	Percent impervious (%)	Tc (hrs)	Calibration results: Storm date	Peak F_p (inch/hr)	Lag (hrs)	Basin factor
Alhambra Wash[1]	13.67	8.62	4.17	82.4	45	0.89	Feb. 78	0,59, 0.24	0.62	0.015
							Mar. 78	0.35, 0.29		
							Feb. 80	0.24		
Compton 2[1]	24.66	12.69	6.63	13.8	55	2.22	Feb. 78	0.36	0.94	0.015
							Mar. 78	0.29		
							Feb. 80	0.44		
Verdugo Wash[1]	26.8	10.98	5.49	316.9	20	–	Feb. 78	0.65	0.64	0.016
Limekiln[1]	10.3	7.77	3.41	295.7	25	–	Feb. 78	0.27	0.73	0.026
							Feb. 80	0.27		
San Jose[2]	83.4	23.00	8.5	60.0	18	–	Feb. 78	0.20	1.66	0.020
							Feb. 80	0.39		
Sepulveda[2]	152.0	19.0	9.0	143.0	24	–	Feb. 78	0.22, 0.21	1.12	0.017
							Mar. 78	0.32		
							Feb. 80	0.42		
Eaton Wash[1]	11.02[4] (57%)	8.14	3.41	90.9	40	1.05	–	–	–	0.015[7]
Rubio Wash[1]	12.20[5] (3%)	9.47	5.11	125.7	40	0.68	–	–	–	0.015[7]
Arcadia Wash[1]	7.70[6] (14%)	5.87	3.03	156.7	45	0.60	–	–	–	0.015[8]
Compton 1[1]	15.08	9.47	3.79	14.3	55	1.92	–	–	–	0.015[8]
Dominguez[1]	37.30	11.36	4.92	7.9	60	2.08	–	–	–	0.015[8]
Santa Ana Delhi[3]	17.6	8.71	4.17	16.0	40	1.73	–	–	–	0.053[9]
										0.040[1]
Westminster[3]	6.7	5.65	1.39	13	40	–	–	–	–	0.079[9]
										0.040[1]
El Modena-Irvine[3]	11.9	6.34	2.69	52	40	0.78	–	–	–	0.028[9]
Garden Grove-Wintersberg[1]	20.8	11.74	4.73	10.6	64	1.98	–	–	–	–
San Diego Creek[1]	36.8	14.2	8.52	95.0	20	1.39	–	–	–	–

Notes 1: Watershed Geometry based on review of quadrangle maps and LACFCD strom drain maps.
2: Watershed Geometry based on COE LACDA Study.
3: Watershed Geometry based on COE Reconstitution Study for Santa Ana Delhi and Westminster Channels (June 1983).
4: Area reduced 57% due to several debris basins and Eaton Wash Dam reservoir, and groundwater recharge ponds.
5: Area reduced 3% due to debris basin.
6: Area reduced 14% due to several debris basins.
7: 0.013 basin factor reported by COE (subarea characteristics, June 1984).
8: 0.015 basin factor assumed due to similar watershed values of 0.015.
9: Average basin factor computed from reconstitution studies.
10: COE recommended basin factor for flood flows.

A total of 12 watersheds were considered in detail in this study. Seven of the watersheds are located in Los Angeles County while the other five catchments are in Orange County (Fig. 1). Several other local watersheds were also considered in light of previous COE studies that resulted in additional estimates of loss rates, *S*-graphs, and lag values. Table 1 provides an itemization of data obtained from the COE studies, and watershed data assumed for catchments considered hydrologically similar to the COE study catchments.

Peak loss rate, E_p

From Table 1, several peak rainfall loss rates are tabulated which include, when appropriate, two loss rates for double-peak storms. The range of values for all F_p estimates lie between 0.30 and 0.65 inch/hour with the highest value occurring in Verdugo Wash which has substantial open space in foothill areas. Except for Verdugo Wash, $0.20 \leqslant F_p \leqslant 0.60$ which is a variation in values of the order noted for Alhambra Wash alone. Fig. 2 shows a histogram of F_p values for the several watersheds. It is evident from the figure that 88 percent of F_p values are between 0.20 and 0.45 inch/hour, with 77 percent of the values falling between 0.20 and 0.40 inch/hour. Consequently, a regional mean value of F_p equal to 0.30 inch/hour is proposed; this value contains nearly 80 percent of the F_p values, for all watersheds, for all storms, within 0.10 inch/hour.

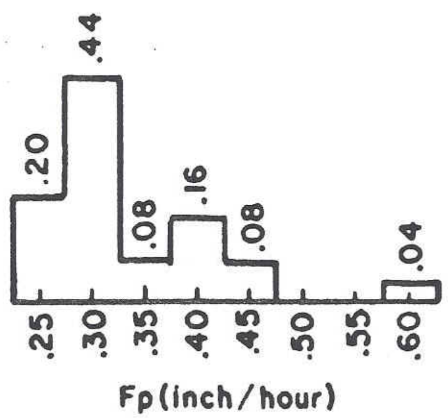

Fig. 2. Distribution-frequency of pervious area loss function, F_p

S-graph

Each of the watersheds listed in Table 1 has *S*-graphs developed for each of the storms where peak loss rate values were developed. For example, Fig. 3 shows the several *S*-graphs developed for Alhambra Wash. By averaging the several *S*-graph ordinates (developed from rainfall-runoff data), an average *S*-graph was obtained. By combining the several watershed average *S*-graphs (Fig. 4) into a single plot, an average of averaged *S*-graphs is obtained. This regionalized *S*-graph (Urban *S*-graph in Fig. 4) can be proposed as a regionalized *S*-graph for the several watersheds.

In order to quantify the effects of variations in the *S*-graph due to variations in storms and in watersheds (i.e., for ungauged watersheds not included in the calibration data set), the scaling of Fig. 5 was used where the variable 'X' signifies the average value of an arbitrary *S*-graph as a linear combination of the steepest and flattest *S*-graphs obtained. That is, all the *S*-graphs (all storms, all catchments) lie between the Feb. 1978 storm Alhambra *S*-graph ($X=1$) and the San Jose *S*-graph ($X=0$). To approximate a particular *S*-graph of the sample set,

$$S(X)=XS_1+(1-X)S_2 \tag{8}$$

where $S(X)$ is the *S*-graph as a function of X, and S_1 and S_2 are the Alhambra (Feb. 1978 storm) and San Jose *S*-graphs, respectively. Fig. 6 shows the population distribution of X where each watershed is weighted equally in the total distribution (i.e., each watershed is represented by an equal number of X entries). Table 2 lists the X values obtained from the Fig. 5 scalings of each catchment *S*-graph. In the table, an 'upper' and 'lower' X-value that corresponds to the X coordinate at 80 percent and 20 percent of ultimate discharge values, respectively, is listed. An average of the upper and lower X values is used in the population distribution of Fig. 6.

Catchment lag

In Table 2, the Urban *S*-graph, which represents a regionalized *S-graph* for urbanized watersheds in valley type topography, has an associated X value of 0.85. It is noted, however, that when the Urban *S*-graph is compared to the standard SCS *S*-graph, a striking similarity is seen (Fig. 7). Because the new Urban *S*-graph is a near duplicate of the SCS *S*-graph, it was assumed that catchment lag (COE definition) is related to the catchment time of concentration, *Tc*, as is generally assumed in the SCS approached.

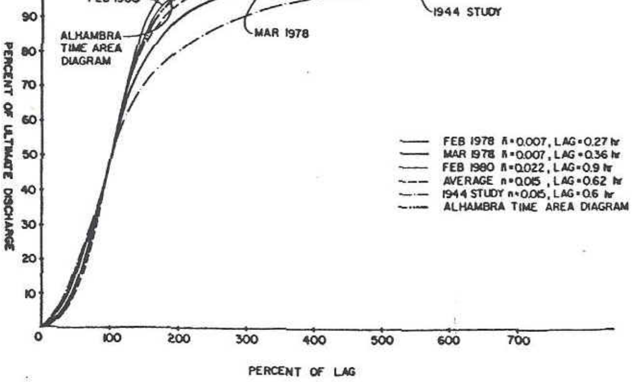

Fig. 3. Alhambra Wash best-fit S-*graphs*

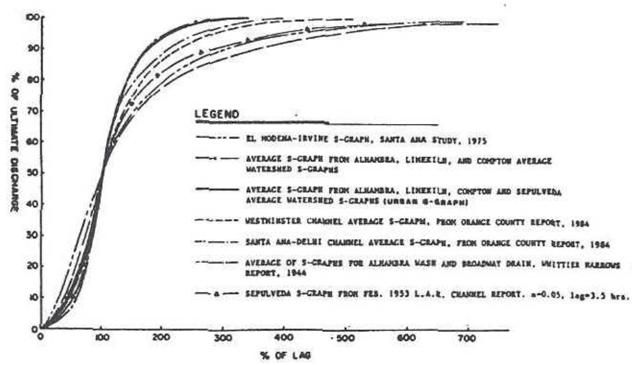

Fig. 4. Catchment averaged S-*graphs*

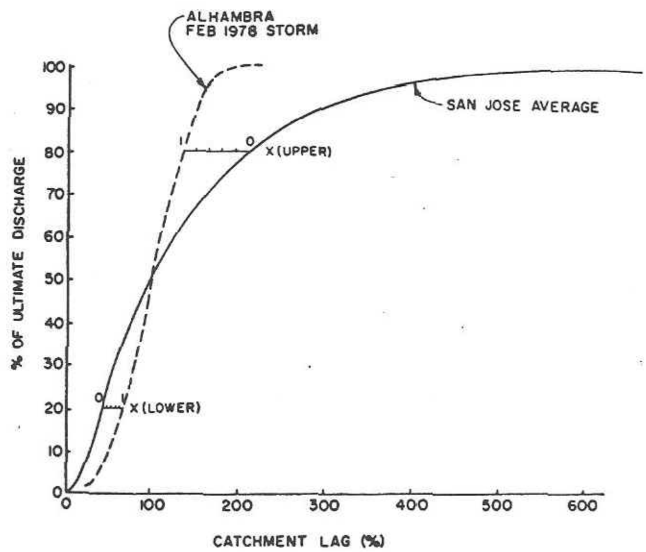

Fig. 5. S-graph scaling for S(X)

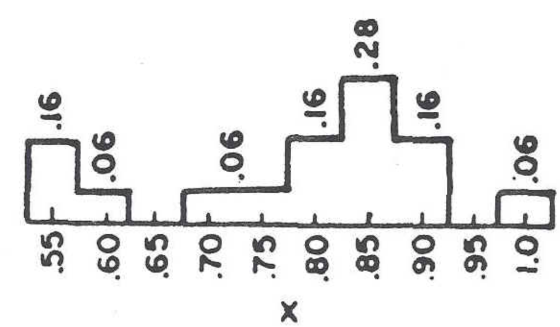

Fig. 6. Distribution-frequency for S-*graph 'X' parameter*

Catchment *Tc* values are estimated by subdividing the watershed into subareas with the initial subarea less than 10 acres and a flowlength of less than 1000 feet. Using a Kirpich formula, an initial subarea *Tc* is estimated, and a Q is calculated. By subsequent routing downstream of the peak flowrate (Q) through the various conveyances (using normal depth flow velocities) and adding successive estimated subarea contributions, a catchment *Tc* is estimated as the sum of travel times analogous to a mixed velocity method.

In this study, lag values are developed directly from available COE calibration data, or by using 'basin factors' calibrated from neighboring catchments (see

Table 2. Catchment S-graph X-values

Watershed name	Storm	X (Upper)	X (Lower)	X (Avg.)
Alhambra	Feb. 78	1.00	1.00	1.00
	Feb. 80	0.95	0.60	0.78
	Mar. 78	0.70	0.80	0.75
Limekiln	Feb. 78	0.50	0.80	0.65
	Feb. 80	0.80	1.00	0.90 (2)
Supulveda	Avg.	0.90	0.80	0.85 (3)
Compton	Avg.	0.90	1.00	0.95 (3)
Westminster	Avg.	0.60	0.60	0.60 (3)
Santa Ana Delhi	Avg.	0.80	1.00	0.90 (3)
Urban	Avg.	0.90	0.80	0.85

Note:
In Table 2, the numbers in parenthesis indicate the weighting of the average X value. That is, due to only the average S-graph (previously derived by the COE) being available, it is weighted to be equally represented in the sample set with respect to the other catchments.

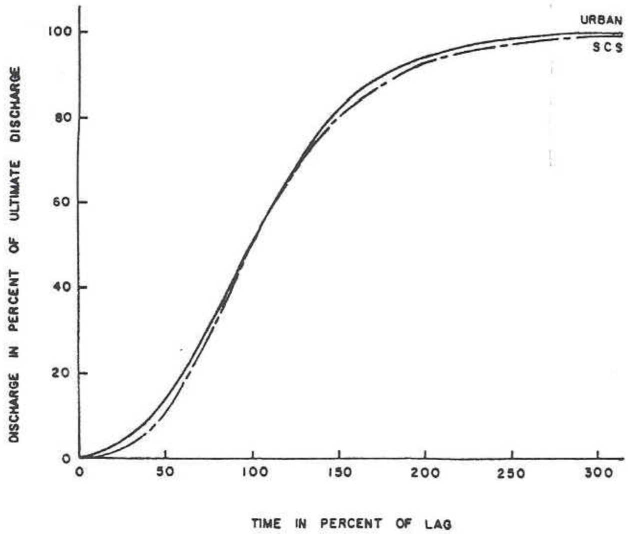

Fig. 7. Comparison of the standard SCS and the regionalized S-graphs

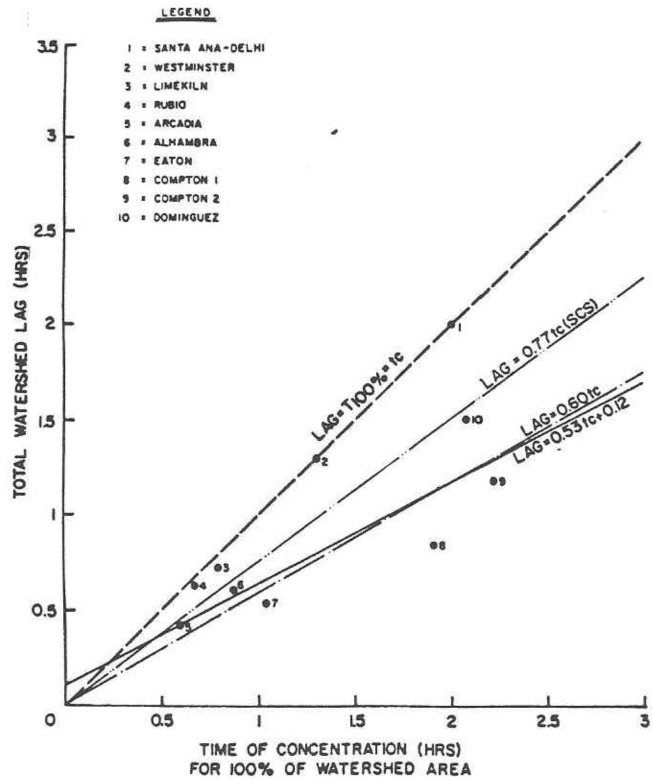

Fig. 8. Relationship between catchment lag and Tc *values for local region*

Fig. 1). The COE standard lag formula is:

$$\text{lag (hours)} = 24\bar{n}\left(\frac{LL_{ca}}{s^{0.5}}\right)^{0.38} \tag{9}$$

where L is the watershed length in miles; L_{ca} is the length to the centroid along the watercourse in miles; s is the slope in ft/mile; and $\bar{n}$ is the basin factor.

Because Eaton Wash, Rubio Wash, Arcadia Wash and Alhambra Wash are all contiguous (see Fig. 1), have similar shape, slopes, development patterns, and drainage systems, the basin factor of $\bar{n}=0.015$ developed for Alhambra Wash was also used for the other three neighboring watersheds. Then the lag was estimated using equation (9).

Compton Creek has two stream gauges, and the $\bar{n}=0.015$ developed for Compton 2 was also used for the Compton 1 gauge. The Dominguez catchment, which is contiguous to Compton Creek, is also assumed to have a lag calculated from equation (9) using $\bar{n}=0.015$.

The Santa Ana-Delhi and Westminster catchment systems of Orange County have lag values developed from prior COE calibration studies. Fig. 8 provides a summary of the local lag *versus* Tc data. A least-squares best fit results in

$$\text{lag} = 0.72Tc \tag{10}$$

McCuen *et al.*[42] provide additional measured lag values and mixed velocity Tc estimates which, when lag is modified according to the COE definition, can be plotted with the local data such as shown in Fig. 9. A least-squares best fit results in:

$$\text{lag} = 0.80Tc \tag{11}$$

In comparison, McCuen[41] gives standard SCS relationships between lag, Tc, and time-to-peak which, when modified to the COE lag definition, results in:

$$\text{lag} = 0.77Tc \tag{12}$$

Adopting a lag of $0.80Tc$ as the estimator, the distribution of (lag/$0.8Tc$) values with respect to equation (11) is shown in Fig. 10.

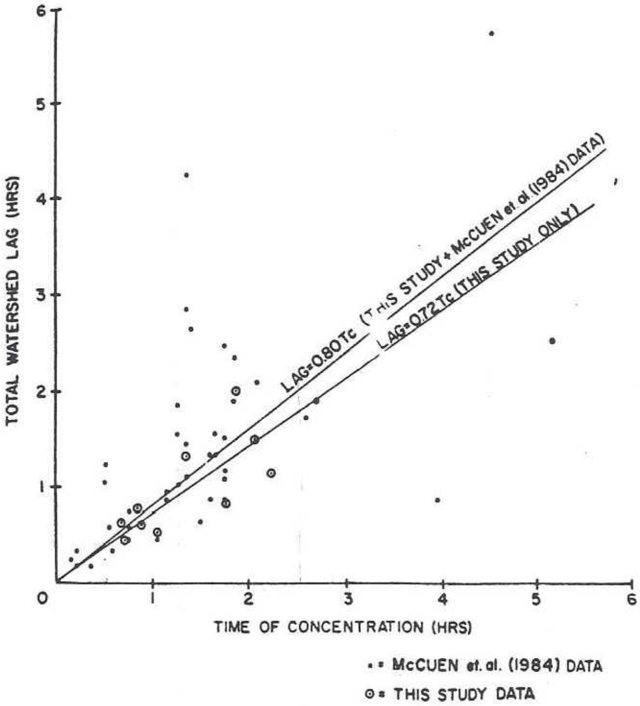

Fig. 9. Relationship between measured catchment lag and computed Tc

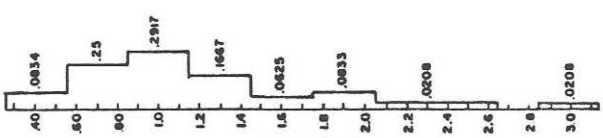

*Fig. 10. Distribution-frequency of lag/(0.8*Tc)

III. PARAMETER UNCERTAINTY AND MODEL RESPONSE

Each of the 'model' parameters (lag, F_p, and S-graph) are assumed to have the probability distribution functions (pdf) shown in a discrete histogram form in Figs 2, 6, and 10 for F_p, $S(X)$, and lag$=0.8Tc$, respectively. For example, if the 'model' is applied at a gauged site, say Alhambra Wash, then the variability in the S-graph is not given by Fig. 6 for $S(X)$, where $0.60 \leqslant 1$, but for $0.75 \leqslant X \leqslant 1$ (see Table 2). Similarly, the estimate for lag is much more certain for Alambra Wash than shown in Fig. 10. Consequently, the uncertainty in the 'model' output for a gauged site will typically show a significantly smaller range in possible outcomes than if the total range of parameter values of Figs 6 and 10 are assumed (as is done for the ungauged sites, or sites where an inadequate length of data exist for a constant level of watershed development).

To evaluate the 'model' uncertainty, a simulation that exhausts all combinations of parameter values shown in the several pdf distributions was prepared (i.e., the exhaustion approach). Because the lag/Tc plot is a function of Tc, several Tc values were assumed and lag values varied freely according to Fig. 10. The resulting Q/Qm distribution is shown in Fig. 11 for the case of Tc equal to 1 hour and a watershed area of 1 square mile (hence, depth-area adjustments are not involved). In the figure, Q is a possible 'model' peak flow rate outcome, and Qm is the peak flow rate obtained from the 'model' policy of assuming lag equal to $0.8Tc$, F_p equal to 0.30 inch/hour, and X equal to 0.85 (Urban S-graph). For different model parameters, the Q/Qm plots were all very close to Fig. 11 as a function of Tc. Therefore, Fig. 11 is taken to represent the overall Q/Qm distribution, which also reflects the inherent uncertainty in design Q values which is typically used in a flood control policy statement.

An important question arises as to whether or not the certainty of outcomes from the calibrated model can be reduced (i.e., the model made more certain) by introducing additional parameters. It is not clear in the current literature whether such a claim has validity. However, some pointed remarks can be taken from Klemes and Bulu[34] who evaluate the 'limited confidence in confidence limits derived by operational stochastic hydrologic models'. They note that advocates of modelling 'sidestep the real problem of modelling – the problem of how well a model is likely to reflect the future events – and divert the user to a more tractable, though less useful, problem of how best to construct a model that will reproduce the past events'. In this fashion, 'by the time the prospective modeller has dug himself out of the heaps of technicalities, he either will have forgotten what the true purpose of modelling is or will have invested so much effort into the modelling game that he would prefer to avoid questions about its relevance'. Of special interest is their conclusion that 'Confidence bands derived by more sophisticated models are likely to be wider than those derived by simple models'. That is, 'the quality of the model increases with its simplicitly'.

In a reply by Nash and Sutcliffe[45] to comments by Fleming[16], the simple model structure used by Nash and Sutcliffe[45] is defined as to modelling completeness in comparison to the Stanford Watershed Model variant, HSP. Nash and Sutcliffe write that '... We believe that a simple model structure is not only desirable in itself but is essential if the parameter values of the component parts are to be determined reliably through an optimization procedure.... One must remember that the data always constitute a limited sample and the optimized values are 'statistics' derived from this sample and therefore subject to sampling variance. The more complex the model structure the greater is the difficulty in obtaining optimum parameter values with low sampling variance. This difficulty becomes particularly acute... when two or more model components are similar in their operation...'.

Should a comparison be made between a simple model, such as described herein, and an 'advanced' model, the results would not be conclusive. As Nash and Sutcliffe write, '...The more complex model would almost certainly provide a better fit, as a linear regression analysis on a large number of variables will almost invariably provide a better fit than one of those independent variables whose significance has been established'. Hence, the hydrologist must be careful to evaluate modelling results obtained from a verification test rather than obtained from a calibration data set. Nash and Sutcliffe also note the dominating importance of errors in rainfall and effective rainfall estimates in complex models such as HSP: 'We wonder, however, how the parameters expressing spatial variation of rainfall or infiltration capacity, can be optimized at all, let alone with stability or significance, in the typical case where the short-term rainfall data are based on a single recording rain gauge...'.

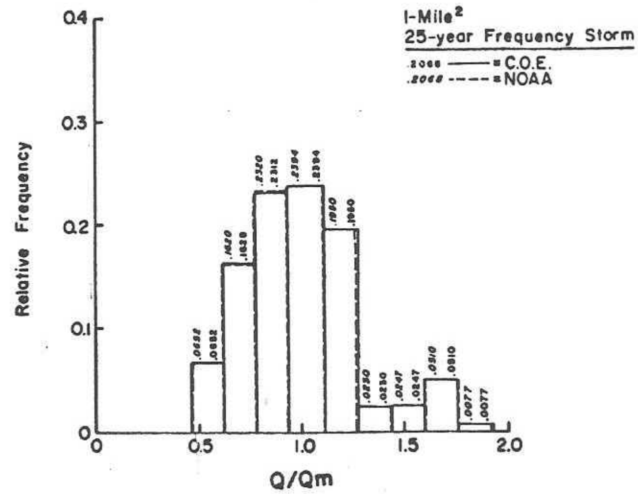

Fig. 11. Q/Qm *Distribution for* Tc = *1-hour, Area* = *1-mi.*², *using the exhaustion method*

IV. STATISTICAL MODELS FOR EVALUATING HYDROLOGIC MODEL UNCERTAINTY

Many hydrology models involve complex systems of submodels whose performance fluctuates because of variations in their parameters. From a knowledge of the probable parameter distributions and an understanding of the model structure, the model output distribution due to parameter uncertainty can be evaluated.

Several statistical models are available for studying the relationship between the model output distribution and parameter uncertainty. In a statistical model, the parameters of a hydrology model are considered as random variables sampled from respective value distributions usually developed from regionalized data analysis. For parameters with discrete probability density functions (pdf) an exhaustion model can be used to calculate the mean and standard deviation of the model output. However in the general case, the number of trials needed to exhaust the parameter vector field is so large, that this technique cannot be used. If continuous or discrete probability density functions of each parameter are known, then the Monte Carlo simulation method can be used to develop estimates of the mean and variance of model output. The moments generation method is another technique which can be applied to a simple performance-parameters relationship model with known mean and standard deviation for each parameter. If the coefficient of variation (which is defined as the ratio between standard deviation and mean) for each parameter is small, then a simplified two-point estimate method[82] may be considered.

Exhaustion model

Simple discrete probability density functions can be derived using histograms for each random variable. For demonstration purposes, consider a hydrology model output Q based upon three parameters; i.e., $Q = F(X_1, X_2, X_3)$, where the X_j are represented by three different discrete probability density functions (Fig. 12).

The mean of the subject model output, Q, is

$$\bar{Q} = \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} P_{1i} P_{2j} P_{3k} F(X_{1i}, P_{2j}, P_{3k}) \qquad (13)$$

and the standard deviation is

$$s = \left(\sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} P_{1i} P_{2j} P_{3k} [F(X_{1i}, X_{2j}, X_{3k}) - \bar{Q}]^2 \right)^{1/2} \qquad (14)$$

where P_{1i} is the probability weighting assigned to X_1 at outcome i. Equations (13) and (14) can be extended directly for any finite number of parameters and number of parameters-histogram values.

Monte Carlo simulation method

If probability density functions for each of the parameters can be obtained or estimated, the Monte Carlo simulation method can be used. Because the Monte Carlo simulation method involves randomly selected input vectors, the computed model output statistics (e.g., mean, variance) are themselves random variables and the estimates are also subject to statistical fluctuations. Thus any estimate will be a random variable and will have an associated error band. The larger the number of trials in the simulation, it is hoped the more precise will be the estimates for the statistics.

Moments generation method

Generally, the density functions are not available for most of the parameters in a hydrology model. However, oftentimes the mean and standard deviation of each parameter can be estimated from the limited information.

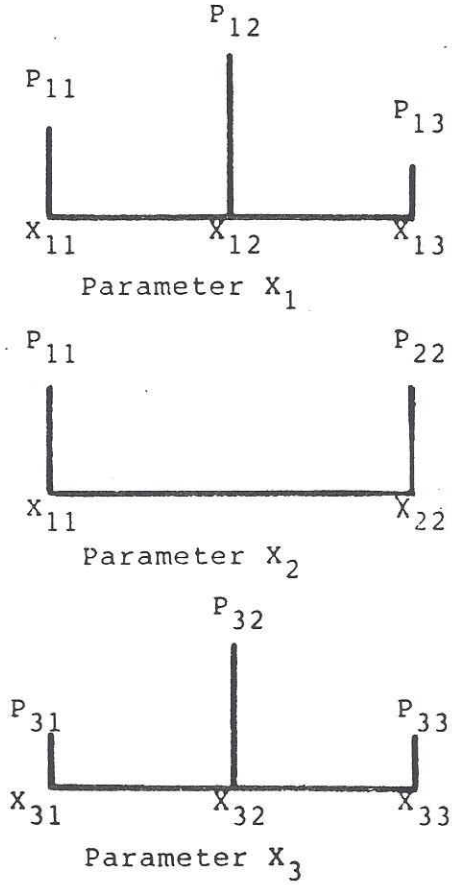

Fig. 12. Histogram representation of hydrologic model parameters

By using the previous example, the mean of the model output can be estimated from the moments generation method as

$$E[Q]=E[F(\bar{X}_1,\bar{X}_2,\bar{X}_3)] + 1/2\sum_{i=1}^{3}\frac{\partial^2 F(\bar{X}_1,\bar{X}_2,\bar{X}_3)}{\partial X_i^2}\mathrm{Var}(X_i) \quad (15)$$

and the variance of the system performance can be estimated as

$$\mathrm{Var}[Q]=\sum_{i=1}^{3}\left(\frac{\partial F(\bar{X}_1,\bar{X}_2,\bar{X}_3)}{\partial X_i}\right)^2\mathrm{Var}(X_i) \quad (16)$$

in which X_1, X_2, X_3 are the estimated means and $\mathrm{Var}(X_i)$ is the estimated variance for each parameter, respectively.

Rosenblueth method

If the first and second partial derivatives in the above equations are not available, then the two-point estimate method[82] can be considered. The estimated mean ($\bar{Q}$) and standard deviation(s) for the model output (Q) from the two-point estimate method are

$$\begin{aligned}\bar{Q}=\ &1/8F(\bar{X}_1+s_1,\bar{X}_2+s_2,\bar{X}_3+s_3)\\ &+1/8F(\bar{X}_1+s_1,\bar{X}_2+s_2,\bar{X}_3-s_3)\\ &+1/8F(\bar{X}_1+s_1,\bar{X}_2-s_2,\bar{X}_3+s_3)\\ &+1/8F(\bar{X}_1-s_1,\bar{X}_2+s_2,\bar{X}_3+s_3)\\ &+1/8F(\bar{X}_1-s_1,\bar{X}_2-s_2,\bar{X}_3+s_3)\\ &+1/8F(\bar{X}_1-s_1,\bar{X}_2+s_2,\bar{X}_3-s_3)\\ &+1/8F(\bar{X}_1+s_1,\bar{X}_2-s_2,\bar{X}_3-s_3)\\ &+1/8F(\bar{X}_1-s_1,\bar{X}_2-s_2,\bar{X}_3-s_3)\end{aligned} \quad (17)$$

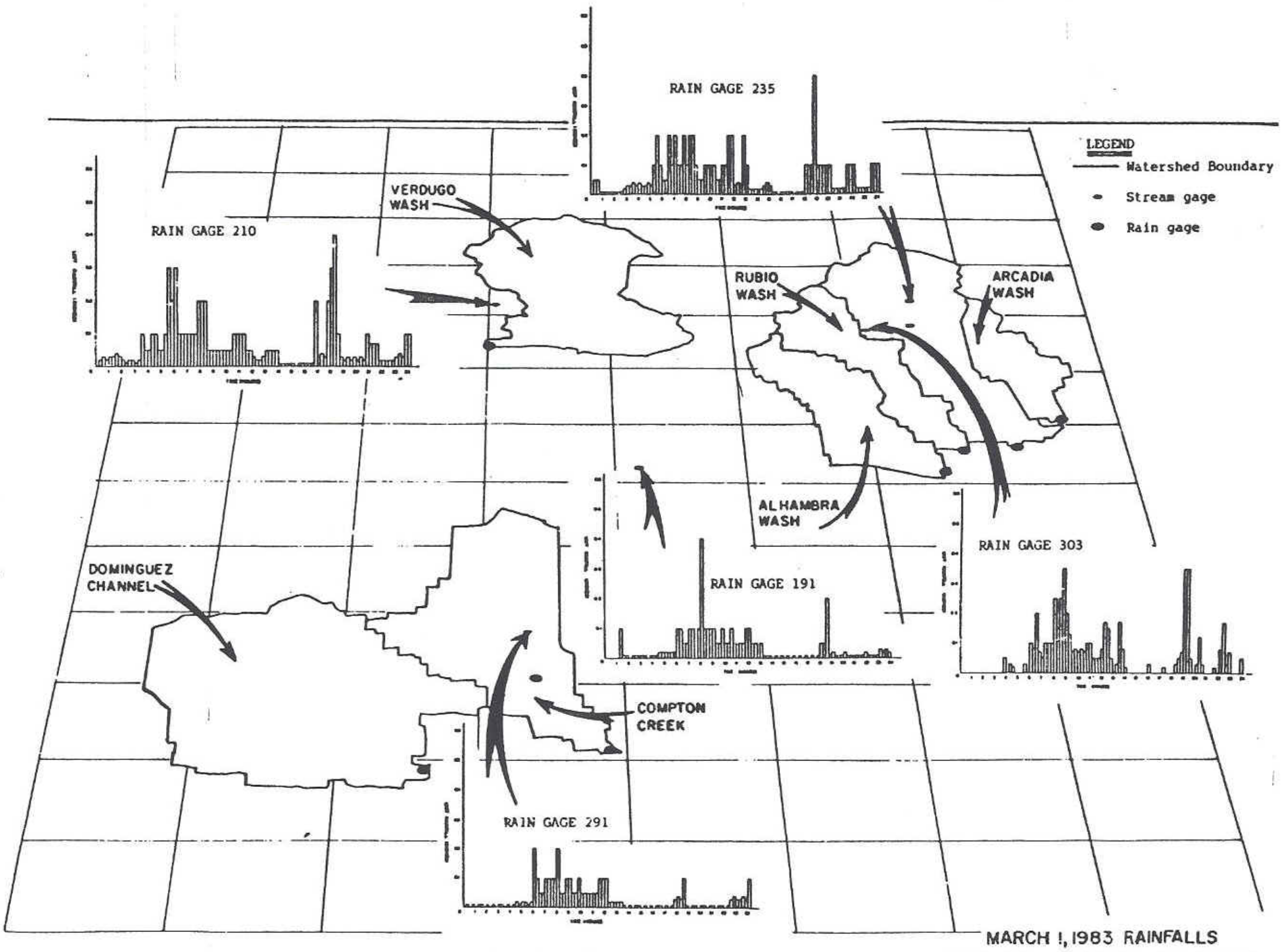

Fig. 13. March 1, 1983 recorded storm rainfall distribution

Table 3. Criteria and information needed for different statistic models

Statistical model	1st and 2nd Partial derivative	Probability density function: Continuous	Probability density function: Discrete	Iteration number	Mean and standard deviation
Exhaustion model	No	No	Yes	$Ni(pi)^{(1)}$	No
Moment generation model	Yes	No	No	No	Yes
Rosenblueth method	No	No	No	$2^{n\,(2)}$	Yes
Monte Carlo simulation method	No	Yes	No	$500\sim5000^{(3)}$	No

Notes: (1). *Ni(pi)* denotes the discrete points on the discrete distribution function for parameter *pi*.
(2). *n* denotes the number of parameters.
(3). Number of iterations based on this study's results.

and

$$s=\{1/8[F(\bar{X}_1+s_1,\bar{X}_2+s_2,\bar{X}_3+s_3)-\bar{Q}]^2$$
$$+1/8[F(\bar{X}_1+s_1,\bar{X}_2+s_2,\bar{X}_3-s_3)-\bar{Q}]^2$$
$$\vdots$$
$$+1/8[F(\bar{X}_1-s_1,\bar{X}_2-s_2,\bar{X}_3-s_3)-\bar{Q}]^2\}^{1/2} \quad (18)$$

in which s_1, s_2, s_3 are the standard deviations of X_1, X_2, X_3, respectively.

Discussion

The exhaustion model is suitable for models involving few parameters with sparse discrete pdf's. When the discrete pdf's become dense, or the number of parameters becomes large, resulting in an infeasible number of model outcome runs for the exhaustion technique, then the Monte Carlo simulation or Rosenblueth techniques become more attractive. In most engineering problems, only the means and variances of the parameters need to be estimated; in which case, the moments generation method is preferred when the first and second partial derivatives can be evaluated or approximated. If the model output/parameters relationship is linear, then either the moments generation method or the two-point estimate method is suitable. For a non-linear system with parameters that have small coefficients of variation, the results from the moment generation method and the two-point estimate method are usually satisfactory. The exhaustion model and the Monte Carlo simulation method are suitable for non-linear systems and/or for parameters having large coefficients of variation.

Since different criteria and information are needed for each statistical model (Table 3), the selection of a statistical model should be conducted with care, and attention must be given to development of a realistic description of the underlying physical situation to serve as input to a statistical model.

Comparison of model uncertainty evaluation techniques: peak flow estimates

The estimation of the peak flow rates, Q, is a basic problem in hydrology. An important source of uncertainty in this estimate is that caused by the uncertain estimation of model parameters. This uncertainty can have a significant effect on the flood design value, and its quantification is an important aspect of evaluating the risk involved in a chosen level of flood protection.

Table 4. Results from different statistical models in the study

Model	Iterations	Q (cfs) Mean	Standard deviation
Exhaustion (see Fig. 11)	378	683 (687)	190 (101)
Monte Carlo[1]	10	649 (658)	166 (102)
	25	688 (675)	199 (101)
	100	670 (676)	171 (94)
	500	683 (682)	181 (94)
	1000	681 (684)	185 (94)
	5000	683 (685)	187 (93)
Rosenblueth	8	728 (712)	404 (98)

Note: 1. Single sample results. All values for the mean and standard deviation for Q are estimates (see text).
2. Values in parentheses indicate results by using smaller coefficients of variation for the time of concentration and soil loss function.

For simplicity, the hydrologic model of Section II is assumed to be a function of only the three representative parameters of soil loss (phi index), watershed time of concentration and the unit hydrograph (S-graph form) variable, (X). Figs 2, 6, and 10 show the assumed histograms for these three random variables. In this application, the model output considered is the runoff hydrograph peak flow rate, or Q. Using the several parameter values from each histogram, an exhaustion study was performed as described in this sections and the result is depicted in Fig. 11. For the second analysis using Monte Carlo technique, a uniform [0, 1] distribution is used to represent each histogram shown in Figs 2, 6, and 10. The Monte Carlo simulation is used to develop random model outcomes by use of random input vectors containing randomly selected parameters values. Finally, the mean and standard deviation values for each histogram were used in the Rosenblueth method as the third model. Table 4 shows the results developed from a runoff hydrograph model of 1 square mile. Because only 3 parameters are used in this model setting, the Rosenblueth technique requires only 2^3 or 8 trials of the model, potentially affording a considerable cost savings over an exhaustion analysis.

As shown in Table 4, the Rosenblueth model gives the highest estimated mean value of the Q and also the largest standard deviation of the estimated Q. This probably reflects the high coefficients of variation of watershed time of concentration (and similarly, lag) and soil loss function (phi index) in the above study.

For further comparison of the Monte Carlo and Rosenblueth models, the pdf's of watershed lag time and soil loss function were modified to have small coefficients of variation. Table 4 also includes the results of the three statistical approaches for this second study. In this case, the Rosenblueth model shows acceptable results compared to the exhaustion model and the Monte Carlo method.

It is noteworthy that in the actual field case study where the coefficients of variation of two model parameters are 'large', the Rosenblueth technique performed poorly, even when compared to a Monte Carlo analysis with a small sample set. Of course, the Monte Carlo technique estimates are random variables themselves, but in this study it was found that for over 90% of the time, the Monte Carlo technique resulted in better estimates of the mean and standard deviation than the Rosenblueth method for the same number of trials (i.e., sample size of 8). And when the model parameters are modified to have a 'small' coefficient of variation, the Rosenblueth technique produced acceptable estimates but so did the Monte Carlo method for the same effort.

Because it is not clear beforehand whether a model uncertainty analysis based upon the Rosenblueth technique will result in adequate estimates of the mean and standard deviation, and because the Monte Carlo performed as good as or better than the Rosenblueth technique (as applied to this hydrologic model), the Monte Carlo technique may be preferable for use in other studies as well where information about the parameters' pdf's are known.

Comparison of model uncertainty evaluation techniques: rainfall-runoff analysis

Rainfall-runoff analysis may be used for generating estimates of the peak flow rates, Q, needed for the design of flood control channels, and estimates for a time distribution of runoff volume for the design of detention basins.

The hydrologic model (described in Section II) was applied to a severe storm condition which occurred on March 1, 1983 in southern California. This storm resulted in various rainfall intensities ranging between 10-year and 200-year return frequencies, causing severe flooding damage throughout the region. The variation in rainfalls is reflected by Fig. 13 which shows the rain gauge measured time-distributions in the Los Angeles area. As can be seen, the variation in rainfall is significant even though the storm was of a rare return frequency. Stream gauge data was also available at several catchments. Consequently, it is feasible to prepare a rainfall-runoff analysis using the previously described hydrologic model. It is noted that this storm was not included in the data set used to determine the parameter calibration of the hydrologic model. It is also noted that in this study effort, the focus is upon the modelling of a particular storm event rather than the development of a probabilistics design storm runoff hydrograph for flood control purposes. Hence, the nested design pattern (see Section II) used for flood control design purposes is replaced, in this application, by the actual measured storm pattern recorded at the available rain gauges.

The subject model is based upon 5-minute unit intervals for both rainfall and runoff. Assuming each 5-minute unit interval of runoff to be a random variable, an uncertainty analysis can be prepared for the entire runoff hydrograph as a collection of 5-minute unit interval random variables. Such an analysis was prepared for two catchments using the parameter value histograms of Figs 2, 6, and 10, for both the Rosenblueth and Monte Carlo techniques. For comparison purposes, additional studies were also prepared using the Monte Carlo approach, but with successively larger sample sizes. A comparison of results in the estimates of Alhambra Wash and Compton (2) Creek of the several peak flow rate statistics are contained in Table 5. Figs 14 and 15 show the modelling outcomes developed from the two uncertainty analysis techniques considered.

As with the previous application, the Monte Carlo technique provides in general, a 'better' estimate for the mean and standard deviation than provided by the Rosenblueth technique. Even with the same computational effort (8 trials), the Monte Carlo technique outperformed (for this study) the Rosenblueth technique.

V. CONCLUSION

Advances in hydrologic modelling techniques typically involves the incorporation of higher complexity into the hydrology model by use of hydraulic submodels. With

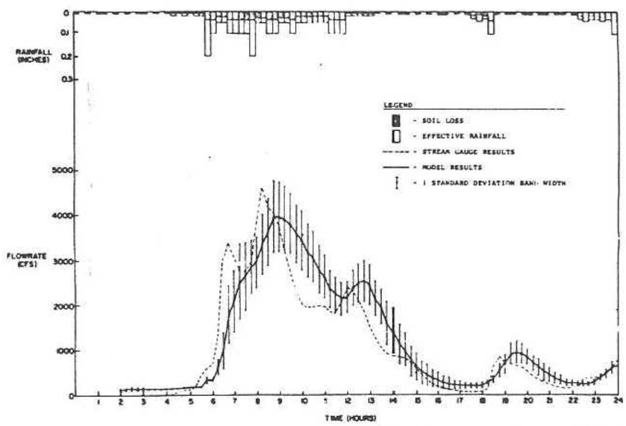

Fig. 14a. Rosenblueth analysis of the March 1, 1983 storm for the Compton Creek watershed

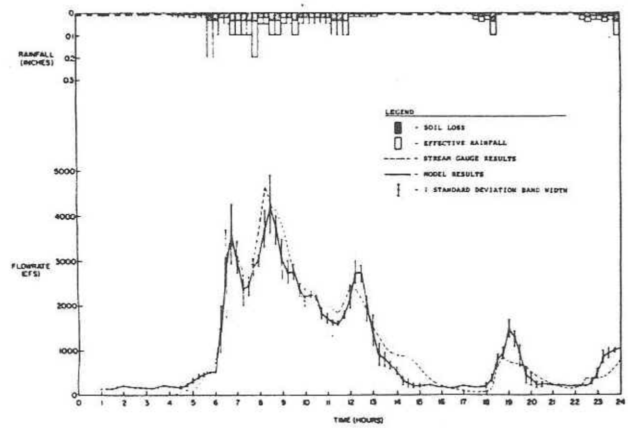

Fig. 14b. Monte Carlo analysis of the March 1, 1983 storm for the Compton Creek watershed

Table 5. Comparison of Rosenblueth and Monte Carlo techniques in modelling March 1, 1983 storm for two catchments

Watershed runoff hydrograph		Rosenblueth		Monte Carlo $n=8$		$n=50$		$n=100$		$n=500$	
	5-minute interval no.	$\bar{Q}$	s	$\bar{Q}$	s	$\bar{Q}$	s	$\bar{Q}$	s	$\bar{Q}$	s
Alhambra Wash	82	2511	491	2303	365	2155	362	2101	339	2086	336
	109	6486	858	5758	864	5444	798	5329	713	5305	739
	152	2618	530	2404	338	2285	315	2227	302	2211	293
	234	5602	1263	5218	1154	4740	1170	4600	1088	4535	1058
	271	1585	443	1521	343	1387	320	1342	307	1327	295
Compton (2) Creek	81	1828	647	3583	681	3279	740	3171	680	3145	656
	105	3984	784	4232	650	4036	513	3942	465	3919	436
	151	2570	451	2717	250	2637	195	2621	189	2616	177
	233	946	257	1507	184	1428	191	1417	182	1410	181
	291	762	139	1018	33	1004	60	1002	57	999	47

Notes: n = sample size.
$\bar{Q}$ = mean value.
s = standard deviation.

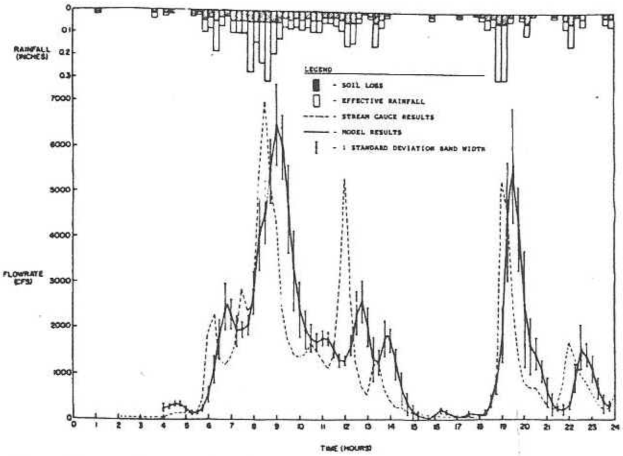

Fig. 15a. Rosenblueth analysis of the March 1, 1983 storm for the Alhambra Wash watershed

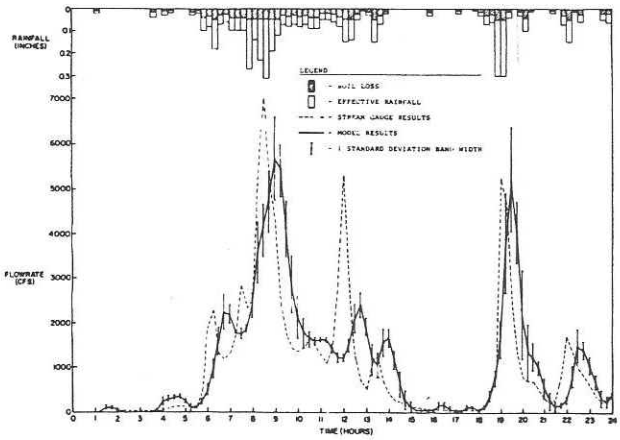

Fig. 15b. Monte Carlo analysis of the March 1, 1983 storm for the Alhambra Wash watershed

over 150 models reported in the open literature, it is still not clear whether the general level of success afforded by the many types of complex models provide a marked improvement over that achieved by the more commonly used and simpler models such as the unit hydrograph method. Such a review indicates that it is still not clear, in general, whether as modelling complexity increases, modelling accuracy increases.

It is important to consider the uncertainty in a hydrologic model, because the estimates of the peak flow rates, Q, and estimates of runoff volume will have significant effect on the flood design values, e.g., the design of flood control channels, and the design of detention basins.

In order to prepared such an analysis, a technique to develop model outcome statistics is needed. Due to computational effort limitations, an exhaustion study which considers the total universe of parameter inputs is usually precluded. Two alternatives to an exhaustion study is a Monte Carlo simulation and the more recently advanced Rosenblueth technique. Although the Rosenblueth technique potentially affords a significant savings in computational effort over that usually needed with a Monte Carlo analysis, it was found that in this application the Monte Carlo technique was superior in accuracy, for even the same computational effort.

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