Kinematic Wave Routing and Computational Error

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Abstract: The standard kinematic wave (KW) method used in many models of open channel flow routing of runoff hydrographs in watershed models is examined as to the significance of the computational errors due to numerical-diffusion and the selection of computational effort. It is shown that a wide range of modeling results are possible from a KW model depending on the choice of computational reach length and time-step size used in the KW approximation. In comparison, the simple convex hydrologic routing method demonstrates only a small fraction of the variation in results demonstrated by the KW model. It is recommended that use of the KW method for channel routing in watershed models be reconsidered.

Introduction

Models of watershed runoff typically include a submodel for approximating the effects of unsteady flow in open channels (i.e., channel routing) for routing a runoff hydrograph through a channel reach. The various methods used to approximate the unsteady flow routing process can be grouped primarily into two categories: hydraulic routing methods which approximate the governing flow equations of continuity, energy, or momentum; or hydrologic routing methods which represent the effects of translation and channel storage on the inflow runoff hydrograph. By far, the most popular hydraulic method used in watershed models is the kinematic wave approach. One of the most popular hydrologic channel routing models is the convex method.

In this paper, the standard kinematic wave routing method is compared to the standard convex routing method such as described and employed in the HEC-1 kinematic wave (KW) program (HEC 1979) and the SCS Engineering Handbook (1972), respectively. Several watershed models use the KW method for channel routing such as used in the HEC-1 KW program and, therefore, the results of this study apply to KW programs in general. The focus of this paper is not toward the accuracy of either routing method in the approximation of flow routing effects but rather the computational errors that are associated to either method. It is shown that except for those conditions where there is no attenuation or subsidence of the runoff hydrograph peak flow rate due to channel storage effects and where the inflow hydrograph includes a mild rising and falling limb, the KW model exhibits significant computational error and numerical-diffusion effects which depend on the user-specification of the KW modeling reach

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length an[illegible] ie-step sizes. In comparison, however, the simple convex hydrologic [illegible]uting method shows only a small fraction of the irregularities associated with the KW modeling results.

As a result of the identified inconsistencies, use of a KW model to approximate channel routing effects may be questionable for both hydrologic design studies where there is no model calibration, and for watershed model calibration studies where the errors in the KW channel routing models is accounted for in the watershed model by modifying the runoff hydrograph subarea parameters (e.g., modifying the overland flow-plane roughness factors).

Background

Use of the KW channel routing technique is popular among many of the watershed models developed during the last decade. [To avoid confusion, the KW routing method is defined to be the technique described in the HEC TD-10 (1979)]. However, the literature contains several examples of KW channel routing performance that indicate that this procedure may be of limited value in comparison to other methods. For example, Akan and Yen (1981) show that their comparisons of KW routing results to the diffusion and fully dynamic computational solutions indicate that the KW peak flow-rate estimates and hydrograph timing differ significantly from the other comparable modeling results. Similar results were obtained by Katapodes and Schamber (1983) that demonstrate the significant errors developed from the KW models where the standard KW model is "corrected" for dynamic routing effects. Weinmann and Laurenson (1979) demonstrate the significant errors developed from the standard KW approach.

The source of the KW routing errors can be grouped into two categories: (1) Errors in the KW model fundamental assumptions; and (2) computational errors from the finite-difference numerical solution of the KW approach. Typically, both errors are "seen" together, and comparisons are reported in the literature that do not isolate the two sources of error. For example, Doyle et al. (1983) write that, "It has been shown repeatedly in flow-routing applications that the kinematic wave approximation always predicts a steeper wave with less dispersion and attenuation than may actually occur." Generally speaking, however, the KW does not attenuate the peak flow rate, i.e., modeled attenuation of the hydrograph peak flow rate is under most circumstances a result of the computational errors in approximating the KW flow equations. Ponce et al. (1978) write, " . . . the kinematic model, by definition, does not allow for subsidence." In consideration of solving the KW flow equations by using the method of characteristics, Strelkoff (HEC 1980) writes that " . . . kinematic waves can attenuate under certain conditions. Such attenuation is enhanced by overflow into flood plains, but can occur when kinematic shocks (as distinguished from bores) are formed in the channel at the intersection of the characteristics."

Therefore, attenuation of the hydrograph peak flow rate when using the KW technique is essentially the result of computational errors including numerical-diffusion and not due to the application of the KW flow equations. This paper focuses on the magnitude and significance of these

computational errors as produced by the well-known HEC-1 KW [illegible]ram. In this way, the second category of errors associated with the K[illegible] [illegible]ethod are evaluated. The first category of KW errors (i.e., the appropriateness of the use of the KW flow equations) is essentially addressed by the statement in Li et al. (1975) regarding the limitations of the kinematic wave approximation: "local and convective accelerations must be negligible, and the water surface slope is nearly equal to the channel bed slope."

Study Procedure

The reported difficulties in the referenced KW model were investigated during the course of a study to evaluate the accuracy of hydraulic and hydrologic channel routing models. During the course of that study, the significance of the KW computational errors were evaluated and then separately studied to identify the implications, if any, in the use of a KW channel routing model in a hydrologic model setting.

Several test cases were considered involving various rectangular channel reach lengths, slopes, friction coefficients, and base widths. In all cases, a runoff hydrograph shape typical of those anticipated for flood control studies was used. Use of a more peaked runoff hydrograph worsened the computational errors identified for the set of test cases reported in this paper.

Each test case involved a total channel length of 25,000 ft (7,620 m). Throughout the length, all channel properties are held constant. The inflow hydrograph was then routed through the channel using various (constant) channel segment (Δx) and time-step (Δt) sizes in the KW model. The convex method was then applied to the same problem conditions using identical channel segment sizes used for the KW model test, but with a constant time-step size of five min.

In the following are presented the set of test results involving the rectangular channel of 40-ft (12-m) base width, a bottom slope of 0.0010, and a Manning's friction factor of $n = 0.050$. In this test, the largest magnitude of computational error was noted for the set of tests considered in our study.

Typically, depending on hydrograph shape, the slower the flow velocities, the more significant the computational errors. However, for steep or peaked hydrographs, the errors were of the significance reported herein. It is repeated that the errors reported herein are due to computational errors, e.g., numerical-diffusion, and not due to the model's underlying assumptions as to hydraulics of the flow. It is also noted that although the HEC-1 KW model is used for KW modeling purposes, other similar KW models will also exhibit the properties described herein. The HEC-1 KW model is used for KW routing demonstration purposes only, and because this particular KW model is well known and is one of the most frequently used KW model programs.

Case Study Results

In HEC-1, the program selects Δx on the basis of Δt, or Δt is chosen on the basis of Δx. The routing reach is always divided into at least two segments, so that the maximum Δx is 1/2 the reach length. Because the finite-difference solution used in the kinematic wave routing equations

LE 1. 10,000-ft Channel Length KW Model Results

t (min) (1)	x (ft) (2)	Q_{peak} outflow (cfs) (3)	Time of peak (hr) (4)
1	2,000	769	2.13
2	3,333	705	2.17
3	5,000	650	2.15
4	5,000	658	2.13
5	5,000	665	2.17
6	5,000	677	2.10

introduces numerical diffusion into computational results, noticeable differences in routed hydrographs occur as Δt is varied in the reach. Table 1 gives the results of routing a hydrograph (see the inflow hydrograph shown in Fig. 2) through a 10,000-ft (3,048-m) long channel reach using various values of Δt.

From Table 1, as Δt gets smaller, the Δx value used decreases such as to satisfy the well-known Courant condition. As $\Delta t \rightarrow 0$, $\Delta x \rightarrow 0$ and Q_{peak} (outflow) = Q_{peak} (inflow) = 940 cfs (26.2 m^3/s) (see Fig. 5) where outflow and inflow indicate the corresponding runoff hydrograph values. Thus, the KW model results vary between 677–940 cfs (19.2–26.2 m^3/s) based on the selection of the model's computational effort to be used.

Fig. 1 contains KW model results for channel lengths of L = 5,000 and 10,000 ft (1,528 and 3,048 m) for two modeling attempts each. For L = 10,000 ft (3,048 m), it is seen that depending on whether Δx = 2,500 or 5,000 ft (762 or 1,724 m), Q_{peak} (outflow) is 840 cfs or 680 cfs (23.8 or 18.3 m^3/s), respectively. Again, a smaller Δx would result in a higher Q_{peak} (outflow) until the 940-cfs Q_{peak} (inflow) value is reached.

Fig. 2 shows the KW model outflow hydrographs for various channel lengths L from L = 0 ft (0 m) (i.e., the inflow hydrograph) to L = 25,000 ft (7,620 m). In all cases, Δx = 2,500 ft (762 m) and Δt = 6 min. Again, the Q_{peak} (outflow) values of Fig. 2 would raise (or lower) should a smaller (or larger) Δx value be specified in the KW model. This is demonstrated by using a Δx = 500 ft (172 m) and Δt = 2 min such as shown in Fig. 3. Comparing Figs. 2 and 3, it is seen that using more computational effort in the KW model [i.e., decreasing Δx from 2,500 to 500 ft (762 to 152 m)] increases the Q_{peak} (outflow) and also changes the hydrograph shape and time-to-peak.

Fig. 4 summarizes the KW modeling results for the total channel length of 25,000 ft (7,620 m). From the figure it is seen that depending on whether Δx = 2,500 ft or 8,333 ft (762 or 2,540 m), Q_{peak} (outflow) = 640 or 400 cfs (48.1 or 11.3 m^3/s), respectively. Recalling the Fig. 3 value for L = 25,000 ft (7,620 m) using Δx = 500 ft (152 m), Q_{peak} (outflow) = 800 cfs (22.7 m^3/s). Again, use of still smaller Δx would increase Q_{peak} (outflow) to the 940 cfs Q_{peak} (inflow) value.

Should the HEC-1 KW model user input Δt, the results of the L = 25,000 ft (7,620 m) case study vary according to Fig. 5. Again, as $\Delta t \rightarrow 0$, then $\Delta x \rightarrow 0$ and Q_{peak} (outflow) $\rightarrow$ Q_{peak} (inflow).

Fig. 6 shows the HEC-1 KW channel routing Q_{peak} (outflow) values for

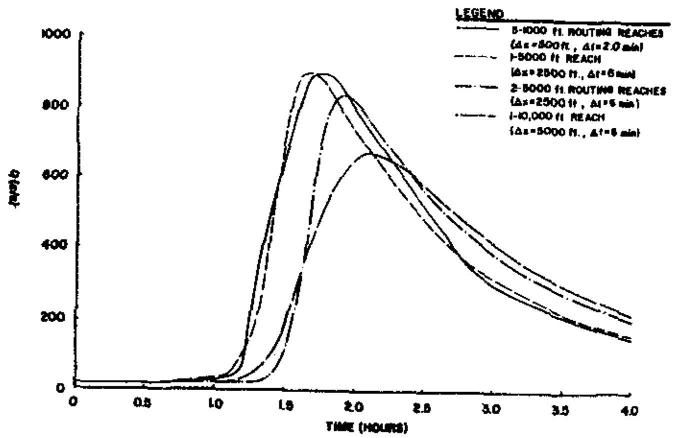

FIG. 1. KW Outflow Hydrographs for L = 5,000 ft and 10,000 ft (Inflow Hydrograph Shown in Fig. 2)

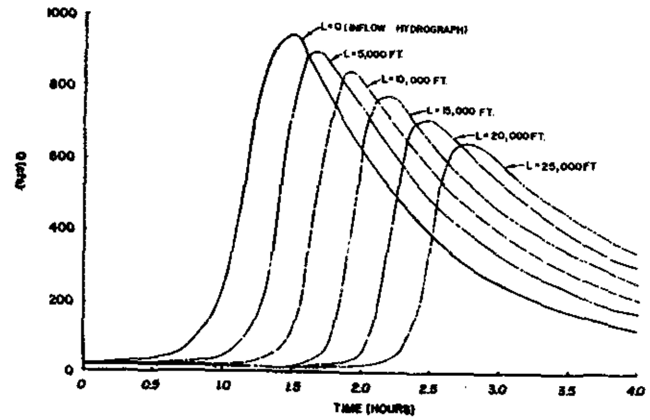

FIG. 2. Outflow Hydrographs for Δx = 2,500 ft, Δt = 6 min for Various Channel Lengths (L)

various L lengths and for an input Δt value of 6 min. Recalling that Q_{peak} (inflow) = 940 cfs (26.2 m^3/s), the shaded area shown in Fig. 6 is the KW Q_{peak} (outflow) values possible depending on the Δx value chosen.

The convex routing model was also used to approximate the unsteady flow problems attempted by the KW model. Typically, the convex model performed most "poorly" when the KW model did and, therefore, examination of the computational error for the same set of test problems described for the KW model is appropriate. Because the convex model demonstrated only a small fraction of the variation in results that the KW model demonstrated, the convex modeling results are shown in table form.

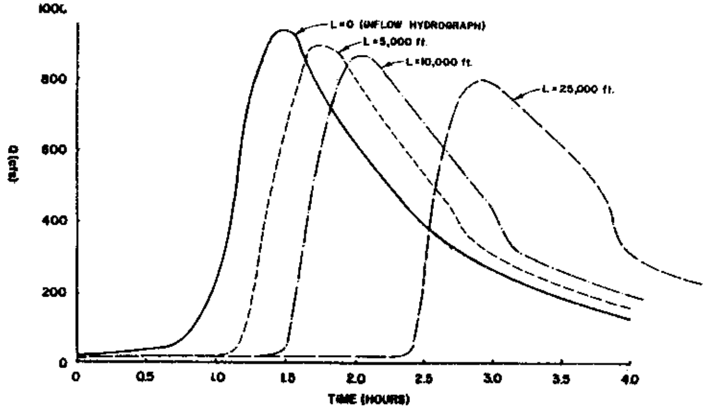

FIG. 3. Using Δx = 500 ft and Δt = 2 min in KW Model Test of Fig. 2

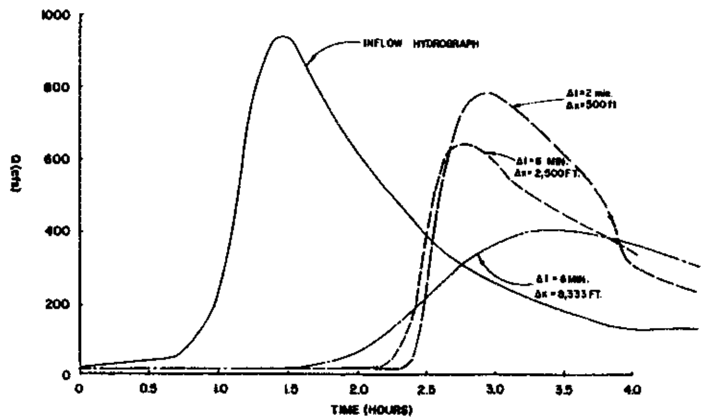

FIG. 4. KW Results for L = 25,000 ft

In Table 2 are contained the Q_{peak} (outflow) values from use of the convex model for the inflow hydrograph of Fig. 2 and for various values of L. Three cases are considered for Δx values; namely, $Q1$ values indicate three reaches composed of two 10,000-ft (3,048-m) lengths and one 5,000-ft (1,524-m) length; $Q2$ values indicate five 5,000-ft (1,524-m) lengths; and $Q3$ values indicate twenty-five 1,000-ft (305-m) lengths. For all tests, a Δt of 5 min was used. Also included in Table 2 is an additional convex test case for a different set of channel conditions that results in considerably higher channel-flow velocities. It is readily seen that after 25,000 ft (7,620 m), the convex routing method involves computational errors due to the selection of Δx values of the order of 5%.

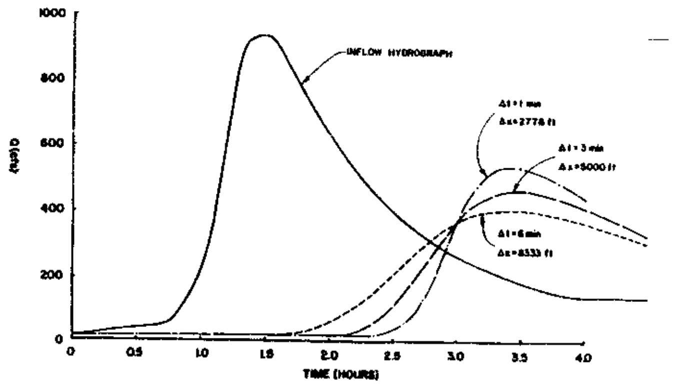

FIG. 5. Effect of Δt Input in HEC-1 KW Model (L = 25,000 ft) (as Selected by HEC-1 Program)

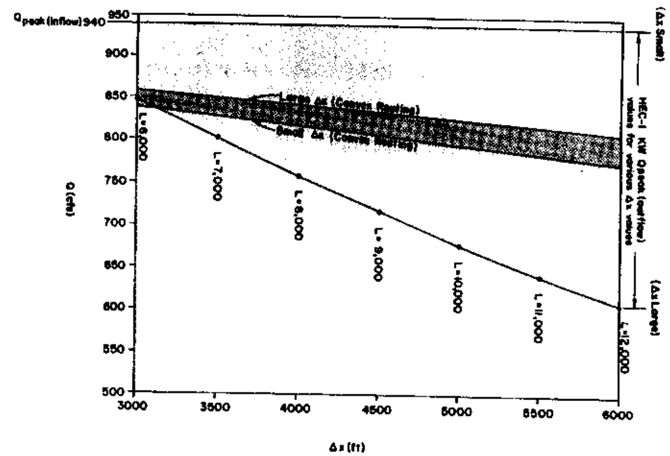

FIG. 6. Variation in KW and Convex Method Modeling Results of Q_{peak} (Outflow) for Various L Values from 6,000–12,000 ft

Fig. 7 shows the range of computational results from the HEC-1 KW model (where the program selects the computational parameters) and the convex routing method (for a constant time step of five min). The illustrated range of channel lengths vary from 0–25,000 ft (7,620 m). From the figure, the convex method shows a variation of 5%. In contrast, the KW model shows a variation of over 130% for L = 25,000 ft (7,620 m) depending on the Δx values selected.

Fig. 8 compares the KW-produced range of results and the convex routing results for the fast-flow problem of Table 2.

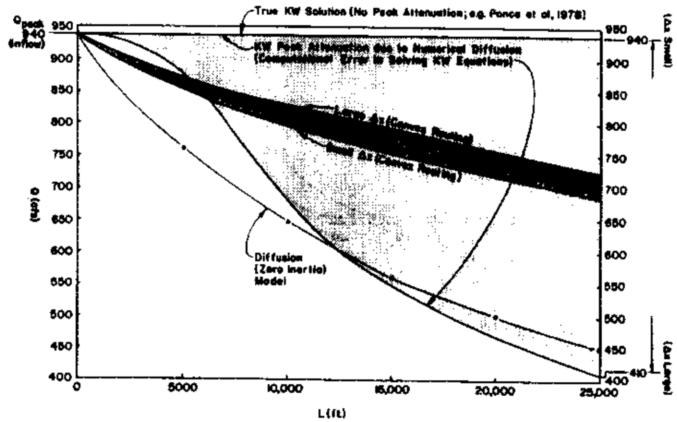

FIG. 7. Variation in KW and Convex Method Modeling Results of Q_{peak} (Outflow) for Various L Values from 0–25,000 ft; Slow-Flow Problem (Diffusion Model Results Shown by *)

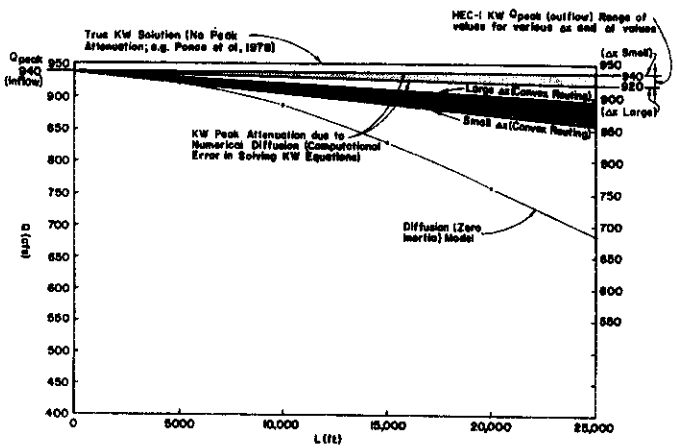

FIG. 8. Variation in KW and Convex Method Modeling Results of Q_{peak} (Outflow) for Various L Values from 0–25,000 ft; Fast-Flow Problem (Diffusion Method Results Shown by *)

TABLE 2. Convex Model and Diffusion Model Q_{peak} (Outflow) R .s

	SLOW-FLOW B = 40 ft; So = 0.001 ft; n = 0.050				FAST-FLOW B = 10 ft; So = 0.010 ft; n = 0.015			
	Convex				Convex			
L (ft) (1)	$Q1$ (2)	$Q2$ (3)	$Q3$ (4)	Diffusion (5)	$Q1$ (6)	$Q2$ (7)	$Q3$ (8)	Diffusion (9)
1,000			922				938	
2,000			903				935	
3,000			332				931	
4,000			869				926	
5,000		858	855	761		935	920	922
6,000			838				920	
7,000			828				918	
8,000			817				915	
9,000			804				910	
10,000	831	795	795	647	929	930	904	885
11,000			786				904	
12,000			775				903	
13,000			768				900	
14,000			760				895	
15,000		750	751	560		925	889	829
16,000			745				889	
17,000			737				888	
18,000			730				885	
19,000			725				881	
20,000	757	716	717	500	910	921	875	757
21,000			712				875	
22,000			706				874	
23,000			699				871	
24,000			695				867	
25,000	721	689	689	450	907	916	862	627

Comparison to Diffusion (Zero Inertia) Model

The next level of sophistication above the KW technique is the zero inertia or the diffusion routing method. Akan and Yen (1981), Tingsanchali and Manandhar (1985), Katopodes and Schamber (1983), Ponce et al. (1978), Weinmann and Laurenson (1979), Li et al. (1975), and Doyle et al. (1983), among others, have shown the significant improvement in computational accuracy using the diffusion analog in comparison to the KW technique.

Included in Table 2 are peak flow-rate values at 500-ft intervals obtained from a one-dimensional diffusion model of the test inflow hydrograph for both the considered slow-flow and fast-flow problems. The diffusion model results are also plotted on Figs. 7 and 8.

From Fig. 7, it appears that the lower curve of values associated to the KW approximation are close to the diffusion modeling results. However, it must be remembered these KW results are strictly due to the algorithmic errors (numerical-diffusion) in solving the KW equations. Had the KW equations been solved exactly, then the top line [i.e., a constant Q = 940 cfs (26.6 m^3/s)] would be the KW modeling results.

ANALYSIS OF RESULTS

From the preceding results it is seen that the arbitrary use of the KW method to model unsteady flow in open channels is subject to considerable scrutiny due to the potentially wide variation in results possible by the selection of Δx or Δt values. This "range of results" impacts the very credibility in using KW or channel routing hydrologic models. A possible remedy in using the standard KW approach (such as in HEC-1) may be to require that all users choose Δx values sufficiently small as to guarantee a good solution of the KW assumption, but in that case, Q_{peak} (outflow) = Q_{peak} (inflow) due the lack of subsidence of the peak flow-rate fundamental to the KW formulation. But many channel routing conditions do exhibit peak attenuation due to channel storage effects, and, therefore, use of the KW would contradict the fundamental channel routing characteristics. Possibly, KW should only be used when there is negligible peak attenuation in the channel. In that case, simple hydrograph translation would be a simpler method to use than KW.

The convex routing method, on the other hand, is simple to apply, does not demonstrate the computational deficiencies to the magnitude exhibited by the HEC-1 KW model, contains peak attenuation, and performs translation for high velocity flows.

Based on the observed computational errors of the KW channel routing method, the limitations fundamental to use of the KW method, and the computational effort needed to approach a true KW hydrograph routing approximation, we submit that use of the KW method for channel routing needs a re-evaluation for use in hydrologic models unless guidelines are developed to control the arbitrary use of KW in design studies.

It is not implied by this study that the simple convex routing technique should be used as the standard flow routing method, but rather that the uncertainty in the selection of KW discretization values for space and time needs further attention from the program developers. Even though the KW technique is conceptually more physically based than the convex method and can potentially achieve the "correct" routing effects, the typical general purpose computer program does not provide internal computational checks to optimize the time step and spatial increment sizes to achieve this "correct" solution. Indeed, the typical goal of most canned-program users is to simply achieve a successful run of the computer program. With the demonstrated range of results possible from a widely used program (i.e., HEC-1) based on KW, a policy statement regarding use of such programs should indicate supplemental procedures required to reach to the "correct" solution. Otherwise, such a policy statement may need to eliminate the use of routing techniques such as KW in favor of a crude convex approximation simply due to its reproducibility by the average practicing floor control engineer in industry and local government agencies involved with flood control planning.

CONCLUSIONS

The HEC-1 KW model is studied to evaluate the significance of computational errors due to the choice of the computational effort used to approximate the unsteady flow effects in channel routing. It is shown that the selection of Δx and Δt values may have a significant impact on the KW

modeling results, and that the simple convex hydrologic routing method demonstrates but a small fraction of the variations in results demonstrated by the KW model used. It is recommended that hydrologic models that use the standard KW method for channel routing be re-evaluated as to their credibility and reliability in their use in the typical flood control design setting of practicing engineers. Guidelines are needed in KW routing models in order to eliminate the possible range of values due to computation error, or KW channel routing programs need internal checks to select Δx and Δt such that an accurate solution of the KW equation is achieved. KW programs also need internal checks to notify program users when the KW flow equations may be inappropriate due to channel storage effects becoming significant.

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