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Applications of a Two-Dimensional Diffusion Hydrodynamic Model

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ABSTRACT

The two-dimensional form of the diffusion equations for flow of water over large expanses of area has been used to model several important flooding/inundation scenarios including dam-break studies involving wide, expansive flows, flooding from flood control channels due to backwater conditions, and flows crossing watershed boundaries on alluvial fans. In this paper, several of these applications are reviewed and specific modeling approaches considered. The two-dimensional diffusion hydrodynamic model (DHM) is extended to include the one-dimensional channel constrictions such as occurs in bridgeways or culverts, and the diffusion model predicted backwater effects are immediately accommodated in the DHM by the one-dimensional diffusion routing model. Flow from the one-dimensional channel onto the two-dimensional surface, and return flow from the surface into the channel is handled by a simple interface model which preserves continuity of the flow volumes while forcing an identical water surface at flooding locations between the two-dimensional and one-dimensional models. Use of the DHM in general flood control studies is proposed due to advantages offered by the DHM approach over other commonly used flooding analysis type computer models.

INTRODUCTION

Each year, flood control projects and storm channel systems are constructed by Federal, State, County and City governmental agencies and also by private land developers which accumulatively cost in the billions of dollars. Additionally, floodplain insurance mapping, zoning, and insurance rates are continually being prepared or modified by the Federal Emergency Management Agency. Finally, the current state-of-the-art in flood system deficiency analysis often results in the costly reconstruction of existing flood control or protection measures which are based upon widely used analysis techniques which often are not adequate to represent the true hydraulic/hydrologic response of the flood control system to the standardized design storm protection level. The main drawbacks in the currently available analysis techniques lie in the inability of the current models to represent unsteady backwater effects in channels and overland flow, unsteady overflow of channel systems due to constrictions (e.g., culverts, bridges, etc.), unsteady flow of floodwater across watershed boundaries due to two-dimensional (horizontal plane) backwater, and ponding flow effects.

In this paper, we report on the current state of development of a Diffusion Hydrodynamic Model (DHM) which approximates all of the above hydraulic effects for channels, overland surfaces, and the interfacing of these two hydraulic systems to represent channel overflow and return flow. The overland flow effects are modeled by a two-dimensional unsteady flow hydraulic model based on the diffusion form of the governing flow equations. Similarly, channel flow is modeled using a one-dimensional unsteady flow hydraulic model based on the diffusion equation. The resulting models both approximate unsteady supercritical and subcritical flow (without the user predetermining hydraulic controls), backwater flooding effects, and escaping and returning flow from the two-dimensional overland flow model to the channel system.

The current simple version of the DHM has been successfully applied to a collection of one- and two-dimensional unsteady flows hydraulic problems including dam-break analyses and flood system deficiency studies. Consequently, the proposed DHM promises to result in a highly useful, accurate, and simple to use computer model which is of immediate help to practising flood control engineers (however, considerable topographic data may be needed depending on the area being modeled.)

Background

One approach to studying flood wave propagation is to simply estimate a maximum possible flow rate and route this flow as a steady state flow through the downstream reaches. This method is excessively conservative in that all effects due to the time variations in channel storage and routing are neglected.

A better approach is to rely on one-dimensional (1-D) full dynamic unsteady flow equations (e.g., St. Venant eqs.). Some sophisticated 1-D models include more terms and parameter to account for complexities in prototype reaches which the basic flow equations cannot adequately handle. However, the ultimate limit of the 1-D model can only be overcome by extending into the two-dimensional (2-D) realm. Several 2-D models employing full dynamic equations have been developed. Among them is one particularly aimed at flood flow analysis by Katopodes and Strelkoff[3]. Attendant with the increased power and capacity of 2-D fully dynamic models, are the greatly increased boundary, initial, geometry and other input data requirements they need for large amounts of computer memory and computational speed, as well as the increased computational time. Although it is often claimed that the extra computational cost and effort required for a more sophisticated model is negligible compared with the total modeling cost and effort in the 1-D realm, the parallel in the 2-D realm seems to be premature at present.

A coupled 1-D and 2-D diffusion hydrodynamic model (DHM) described in this paper appears to offer a simple and economic means for the estimation of flooding effects for diverging flood flows.

ONE-DIMENSIONAL MODEL FOR UNSTEADY FLOW

Generally, the 1-D flow approach used wherever there is no significant lateral variation in the flow. Land[4,5] examines four such unsteady flow models in their prediction of flooding levels and flood wave travel time, and compares the results against observed unsteady flow data. Ponce and Tsivoglou[6] examine the gradual failure of an earth embankment (caused by an overtopping flooding event) and present a detailed model of the total system: sediment transport, unsteady channel hydraulics, and earth embankment failure. Although many dam-break studies involve flood flow regimes which are truly two-dimensional (in the horizontal plane), the 2-D case has not received much attention in the literature. In addition to the aforementioned model of Katopodes and Strelhoff, which relies on the complete 2-D dynamic equations, Xanthopoulos and Koutitas[7] use the diffusion model to approximate a 2-D flow field. The model assumes that the flood plain flow regime is such that the inertia terms are negligible. In a 1-D model, Akan and Yen[1] also use the diffusion approach to model hydrograph confluences at channel junctions. In the latter study, comparisons of model results were made between the diffusion model, a complete dynamic wave model solving the total equation system, and the basic kinematic wave equation model. The comparisons between the diffusion model and the dynamic wave model were good for the study cases, showing only minor discrepancies.

MODEL ACCURACY IN PREDICTION OF FLOOD DEPTHS

In order to evaluate the accuracy of the proposed diffusion model in the prediction of flood depths, Land's model[4,5], referred in the preceding section, has been used for comparison purposes. This model solves 1-D full dynamic equations by an implicit finite scheme and is identified as the USGS dynamic model K-634. The study approach was to compare predicted flood depths for various channel clopes and inflow hydrographs using the above two models.

From the study of Hromadka, et al.[2] it is seen that the diffusion model provides estimates of flood depths that compare very well to the flood depths predicted from the K-634 model. Differences in predicted flood depths are less than three percent for the various channel slopes and peak flow rates considered.

In the following sections, the development of a two-dimensional diffusion hydrodynamic model (DHM) will be described. The model is based on a diffusion scheme in which gravity, friction, and pressure forces are assumed to dominate the flow equations. Earlier, Xanthopolous and Koutitas[7] employed such an approach in the prediction of dam-break flood plains in Greece. Good results were also obtained in their studies applying the 2-D model to flows that are essentially 1-D in nature. In the following, an integrated finite difference model is developed which (1) solves the two-dimensional topographic flood wave propagation, (2) the one-dimensional flood wave propagation for channel flow, and (3) the interface between the two models to accommodate flooding effects.

MATHEMATICAL DEVELOPMENT FOR TWO-DIMENSIONAL MODEL

The set of (fully dynamic) 2-D unsteady flow equations consist of one equation of continuity

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial H}{\partial t} = 0 \qquad (1)$$

and two equations of motion

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left[\frac{q_x^2}{h}\right] + \frac{\partial}{\partial y}\left[\frac{q_x q_y}{h}\right] + gh\left[S_{fx} + \frac{\partial H}{\partial x}\right] = 0 \qquad (2)$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial}{\partial y}\left[\frac{q_y^2}{h}\right] + \frac{\partial}{\partial h}\left[\frac{q_x q_y}{h}\right] + gh\left[S_{fy} + \frac{\partial H}{\partial y}\right] = 0 \tag{3}$$

in which q_x, q_y are flow rates per unit width in the x,y-directions; S_{fx}, S_{fy} represent friction slopes in x,y-directions; H, h, g stand for, respectively, water-surface elevation, flow depth, and gravitational acceleration; and x,y,t are spatial and temporal coordinates.

The above equation set is based on the assumptions of constant fluid density with zero sources or sinks in the flow field, hydrostatic pressure distributions, and relatively uniform bottom slopes.

The local and convective acceleration terms can be grouped together such that are rewritten as

$$m_z + \left[S_{fz} + \frac{\partial H}{\partial z}\right] = 0, \; z = x,y \tag{4}$$

where m_z represents the sum of the first three terms in Eqs. (2) and (3) divided by gh. Assuming the friction slope to be approximated by steady flow conditions, the Manning's formula in the U.S. customary units can be used to estimate q_z

$$q_z = \frac{1.486}{n} h^{5/3} S_{fz}^{1/2}, \; z = x,y \tag{5}$$

Equation 5 can be rewritten as

$$q_z = -K_z \frac{\partial H}{\partial z} - K_z m_z, \; z = x,y \tag{6}$$

where

$$K_z = \frac{1.486}{n} h^{5/3} \Big/ \left|\frac{\partial H}{\partial s} + m_s\right|^{1/2}, \; z = x,y \tag{7}$$

The symbol s indicates the flow direction which makes an angle $\theta = \tan^{-1}(q_y/q_x)$ with the +x-direction.

Values of m are assumed negligible by several investigators[1,2,7], resulting in the simple diffusion model:

$$q_z = -K_z \frac{\partial H}{\partial z}, \; z = x,y \tag{8}$$

The proposed 2-D flood flow model is formulated by substituting Equation 8 into Equation 1,

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} = \frac{\partial H}{\partial t} \tag{9}$$

NUMERICAL MODEL FORMULATION (GRID ELEMENTS)

For uniform grid elements, the numerical modeling approach used is the integrated finite difference version of the nodal domain integration (NDI) method. For grid elements, the *NDI* nodal equation is based on the usual nodal system (see Fig. 3). Flow rates along the boundary Γ are estimated using a linear trial function assumption between nodal points.

For a square grid of width δ,

$$q\Big|_{\Gamma_E} = -\left\{ K_x\Big|_{\Gamma_E} \right\}\left(H_E - H_c \right) \Big/ \delta \tag{10}$$

where

$$K_x\Big|_{\Gamma_E} = \begin{cases} \left\{ \dfrac{1.486}{n} \bar{h}^{5/3} \right\}_{\Gamma_E} \Bigg/ \left| \dfrac{H_E - H_c}{\delta \cos \theta} \right|^{1/2} ; & \bar{h} > 0 \\ 0 ; \ \bar{h} \le 0 \text{ or } \left| H_E - H_c \right| < 10^{-3} \end{cases} \tag{11}$$

In Equation 11, h and n are both the average of the values at c and E, *i.e.* $h = (h_c + h_E)/2$ and $n = (n_c + n_E)/2$. Additionally, the denominator of K_x is checked such that K_x is set to zero if $|H_E - H_c|$ is less than a tolerance such as 10^{-3} ft.

The model advances in time by an explicit approach

$$\underset{\sim}{H}^{i+1} = \underset{\sim}{K}^{i} \underset{\sim}{H}^{i} \tag{12}$$

where the assumed input flood flows are added to the specified input nodes at each time step. After each time step, the conduction parameters of Eq. (11) are reevaluated, and the solution of Eq. (12) reinitiated. Using grid sizes with uniform lengths of one-half mile, time steps of size 3.6 sec were found satisfactory. Verification of the 2-D model is given in Hromadka[2], Hromadka et al.[2], Hromadka and Durbin[3], and Hromadka and Lai[4]. Hromadka and DeVries[5] demonstrate the use of the two-dimensional model in the analysis of dam-break flood plains.

MATHEMATICAL DEVELOPMENT FOR ONE-DIMENSIONAL MODEL

By eliminating a directional component in Eq. (9), a one-dimensional formulation is developed which provides a good *approximation of one-dimensional unsteady flow routing* cluding backwater effects and subcritical/supercritical f regimes. Hromadka[5] demonstrates the good results obtained from a one-dimensional version of DHM to model unsteady flow effects. Figure 1 shows the comparison between 1-D DHM results *and the USGS K-634 model results for various channel slopes*, and peak flow rates. The figure illustrates that good results are obtained from the DHM.

INTERFACE MODEL (FLOODING SOURCE/SINK TERM)

To model flood flows exiting from and returning to a one-dimensional channel, an interface model is needed to couple the 1-D DHM and 2-D (topography) DHM. Figure 2 illustrates the mass conservation scheme assumed to represent the source/sink *term of flows flooding/draining from the topographic model to* the channel model.

INCLUSION OF CHANNEL DEFICIENCY EFFECTS

The main cause of existing flood plain difficulties is the existence of channel constrictions due to bridges, undersized culverts, and other factors. By specifying a stage-discharge relationship at points within the channel system (or on the topography), constrictions are modeled efficiently in the DHM.

Preliminary Results

Herein we will describe our most current and advanced work. The 1-D and 2-D DHM formulations are coupled through the interface model of the flooding source/sink term (due to *channel overflows onto the topographic model*). Using *simple* grid elements, Figure 3 shows the integrated finite difference scheme for the mass balance. Figure 4 shows the problem definition involving 160 grid elements for the topographic model and 3 channel systems. Included in the middle channel system *is a junction. The channel systems* drain towards culverts (located at the bottom of the domain) which have a limited capacity (e.g., freeway crossings). Figure 5 shows the channel inflow hydrographs and pertinent topographic data to indicate catchment divides. Figures 6, 7, 8, 9, and *10 show the flood plain extent at various model time values.* Figure 11 shows the three channel outflow hydrographs which reflect the release of ponded waters due to culvert deficiencies. Figure 12 shows the hydrographs at grid points 1 and 2 as flow escapes to the left of the domain. (For this application, *zero flux is assumed on the right side of the domain*, with critical depth assumed for the left side). Figures 13 and 14 show the maximum flood depths calculated in both plan and profile views, respectively.

An examination of the model results indicate that the current simple version of the DHM already provides a considerable advance in floodplain determination over that currently available.

CONCLUSIONS

The DHM provides a novel tool for hydrologic and hydraulic engineers who are involved in floodplain management or flood control. By the continuing development of this new modeling approach, the analysis of flood control system deficiencies and development of methodology on how to best spend the available dollars is now feasible in both a qualitative and quantitative sense.

1. Akan, A. O. and Yen, B. C., "Diffusion-Wave Flood Routing in Channel Networks," ASCE Hyd. Div., HY6, 1981.

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7. Katopodes, N., and Strelkoff, T., "Computing Two-Dimensional Dam-Break Flood Waves," A.S.C.E., Journal of Hydraulics Division, HY9, 1978.

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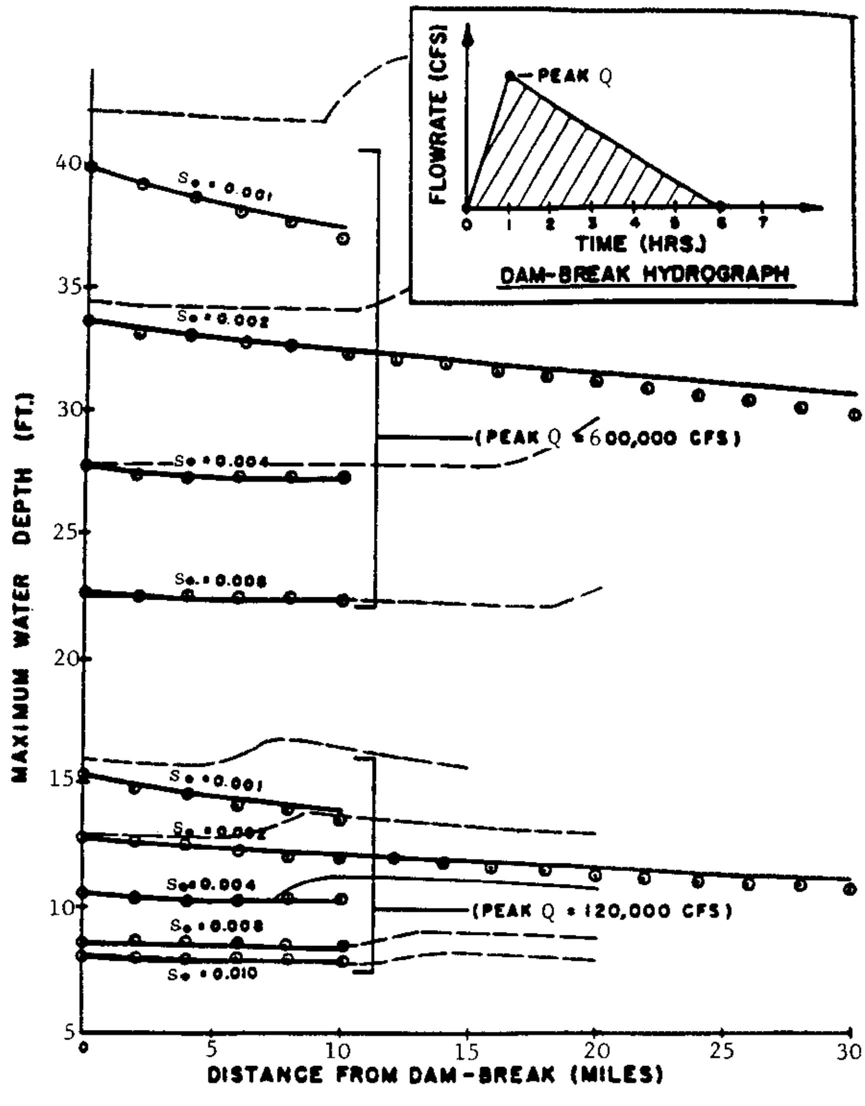

Figure 1. Diffusion model (⊙), kinematic routing (dashed line) and K-634 model results (solid line) for 1,000-foot width channel, Manning's n = 0.040, for various channel slopes, S_o.

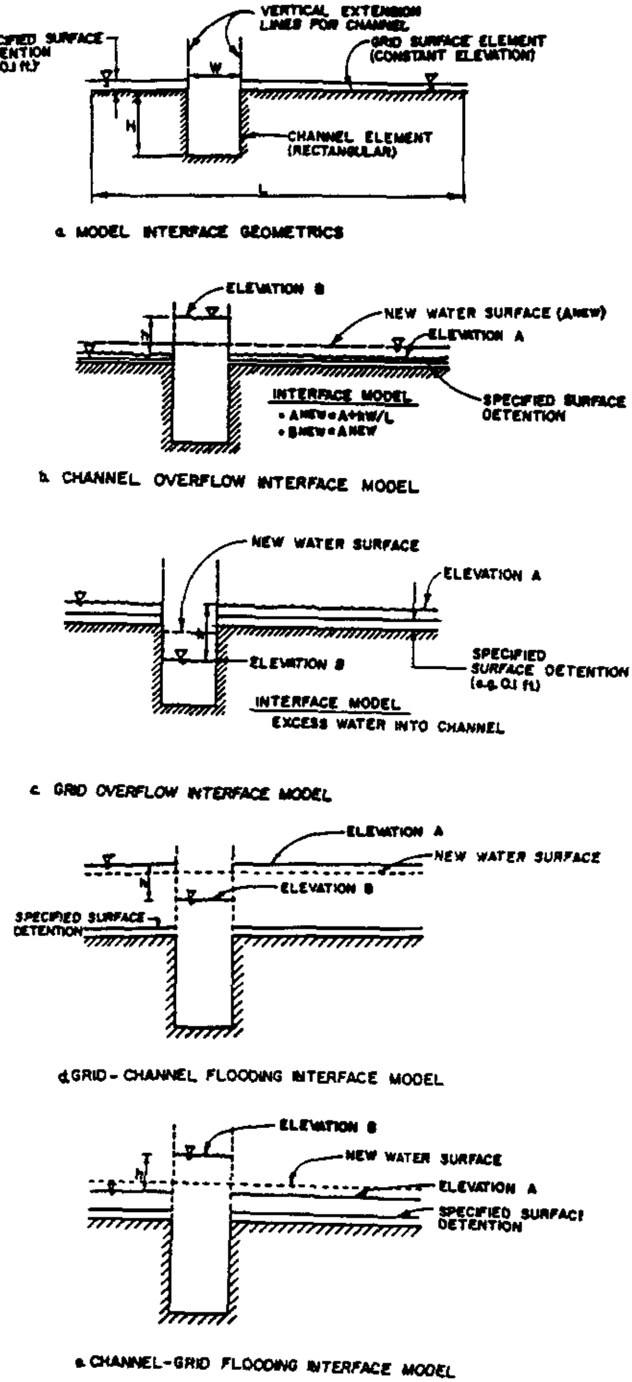

Figure 2 DHM interface model

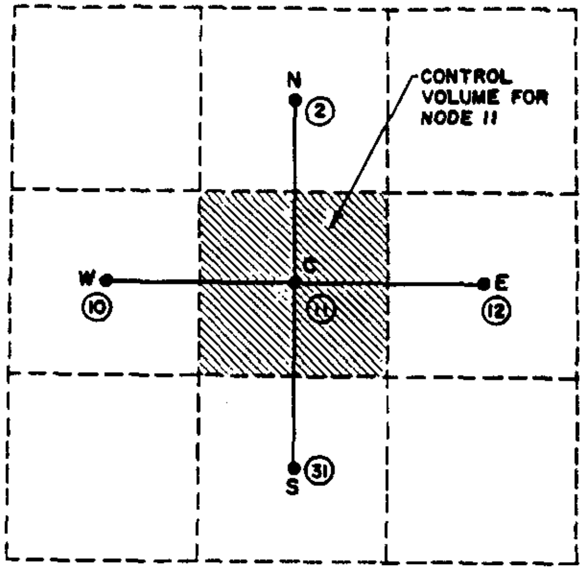

Figure 3. Grid element nodal molecule

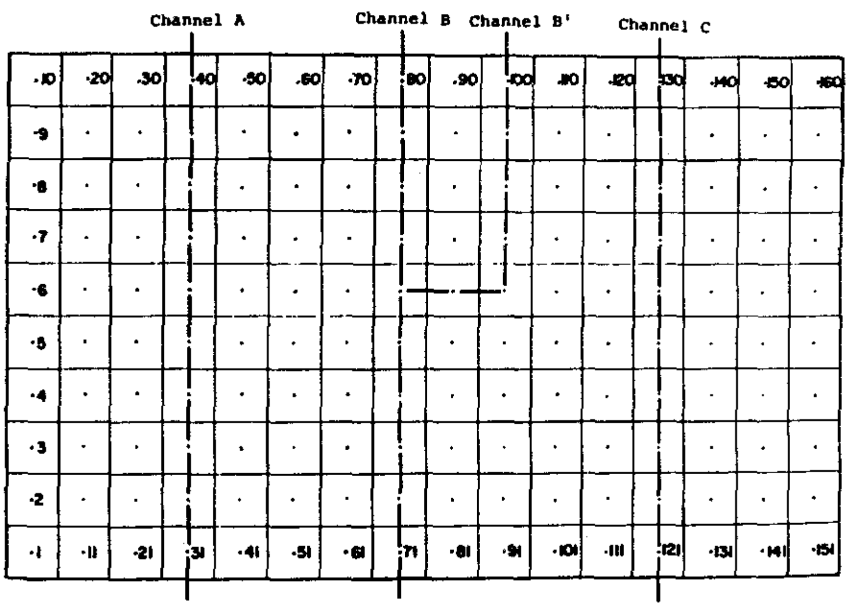

Figure 4. DHM model discretization of a hypothetical watershed

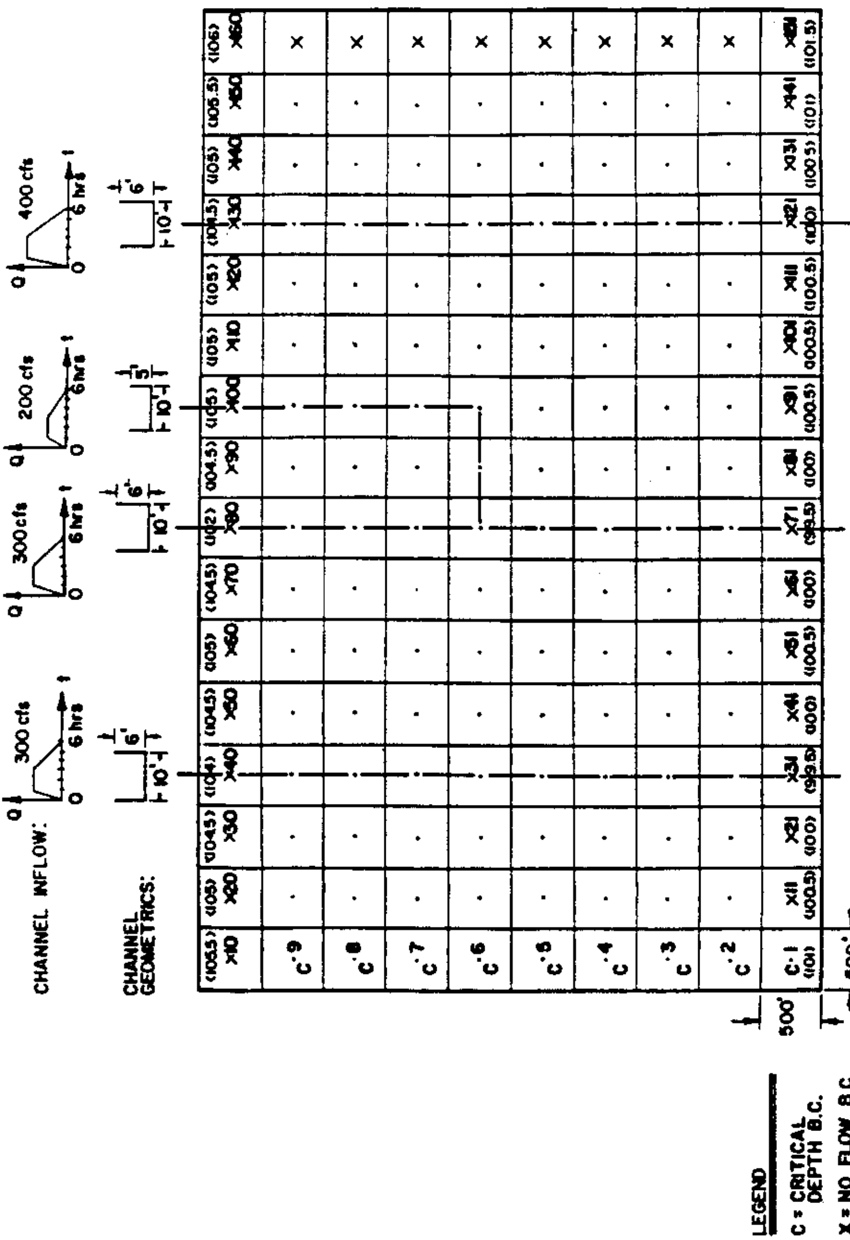

Figure 5 Inflow and outflow boundary conditions for the hypothetical watershed model.

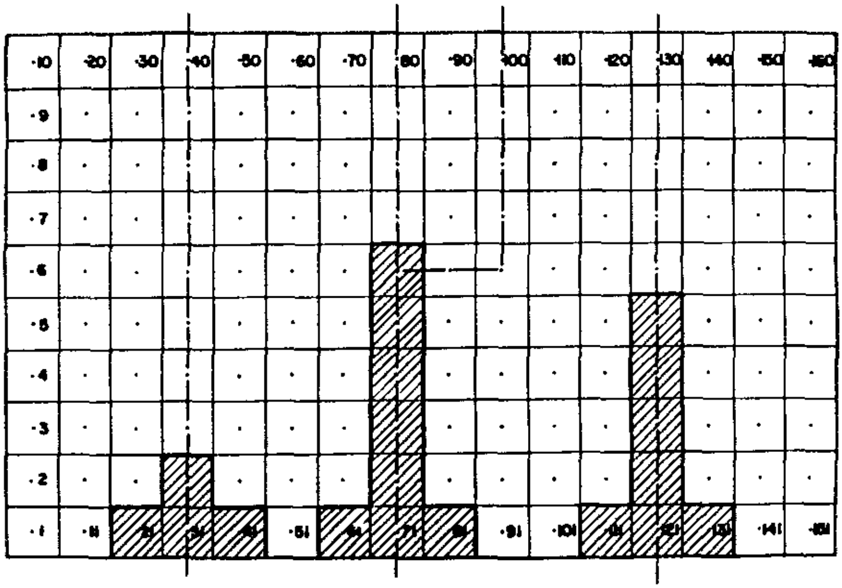

Figure 6. DHM modeled floodplain at time = 1-hour

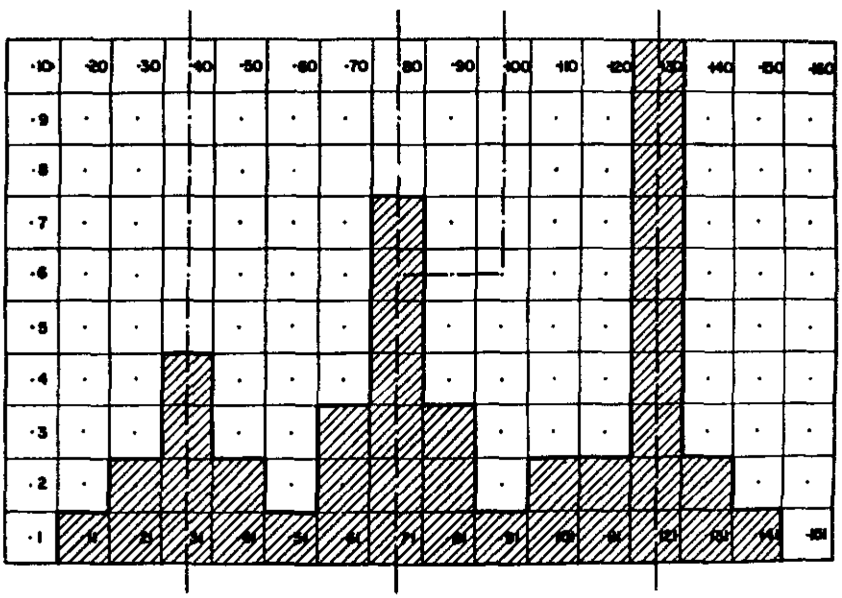

Figure 7. DHM modeled floodplain at time = 2-hours

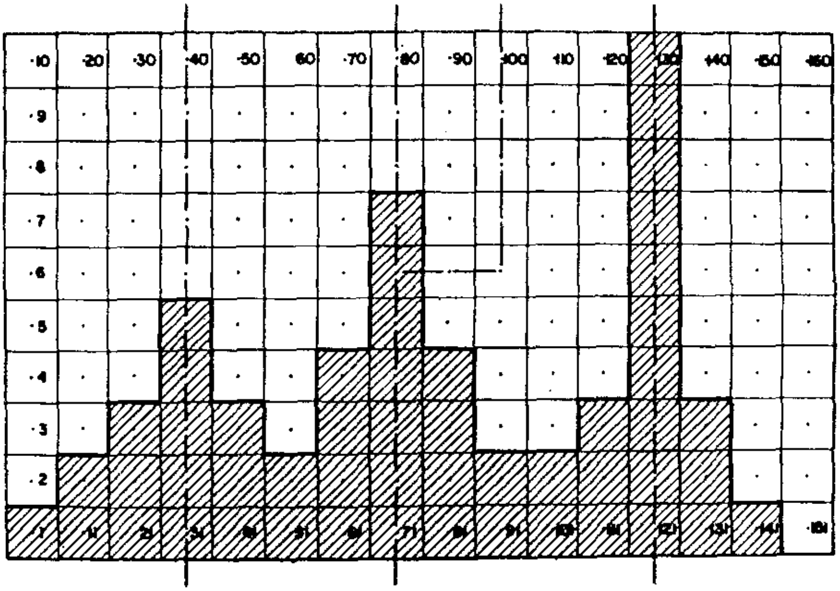

Figure 8. DHM modeled floodplain at time = 3-hours

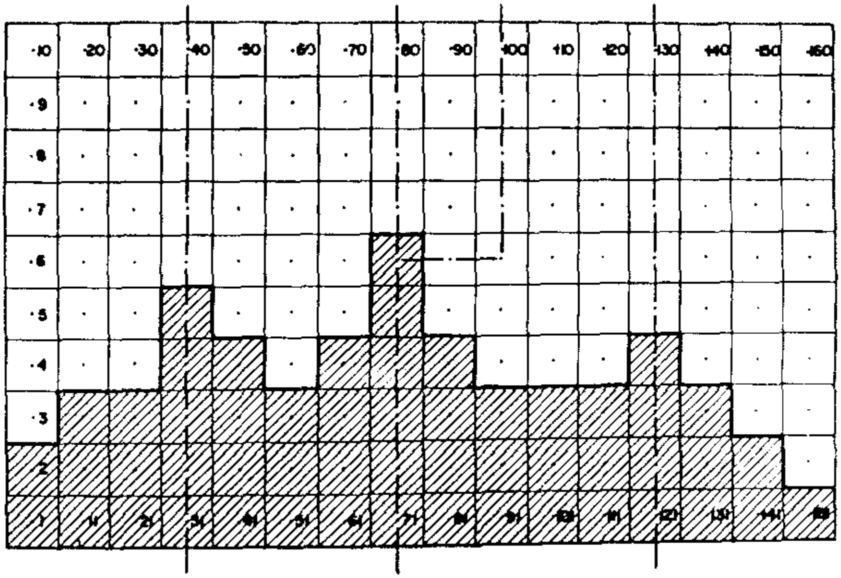

Figure 9. DHM modeled floodplain at time = 5-hours

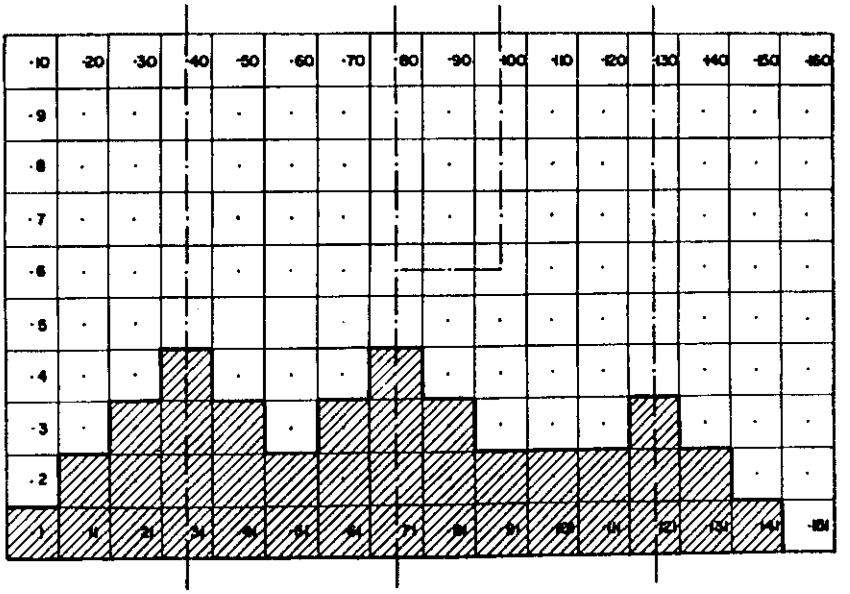

Figure 10. DHM modeled floodplain at time = 7-hours

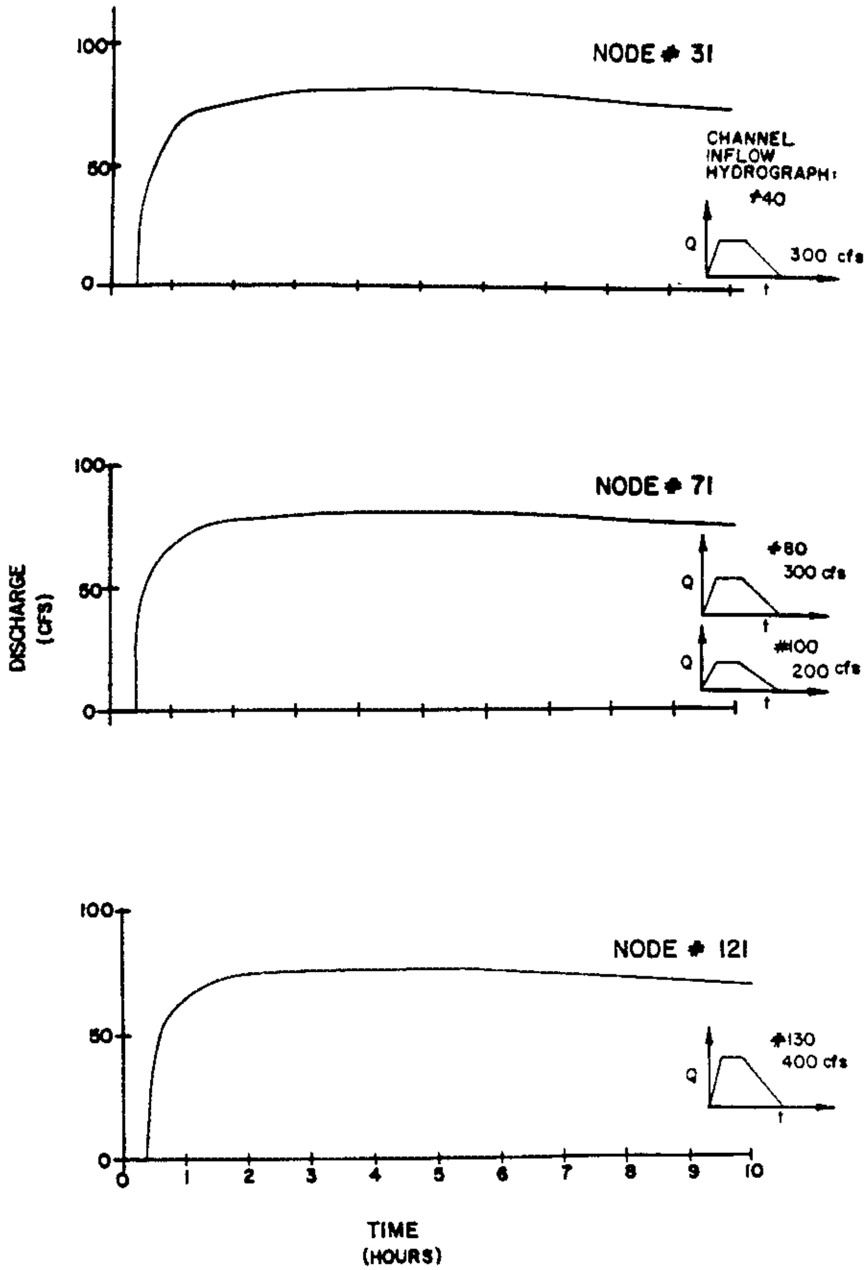

Figure 11. Bridge flow hydrographs assumed outflow relation: (Q = 10d)

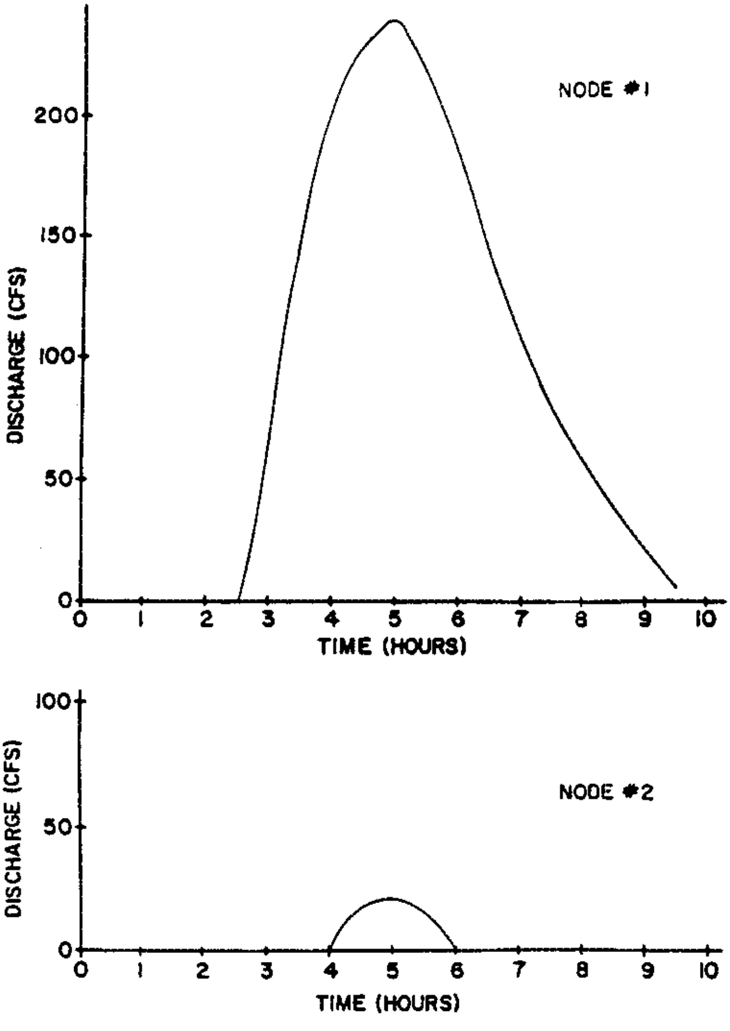

Figure 12. Critical outflow hydrographs for flood plain

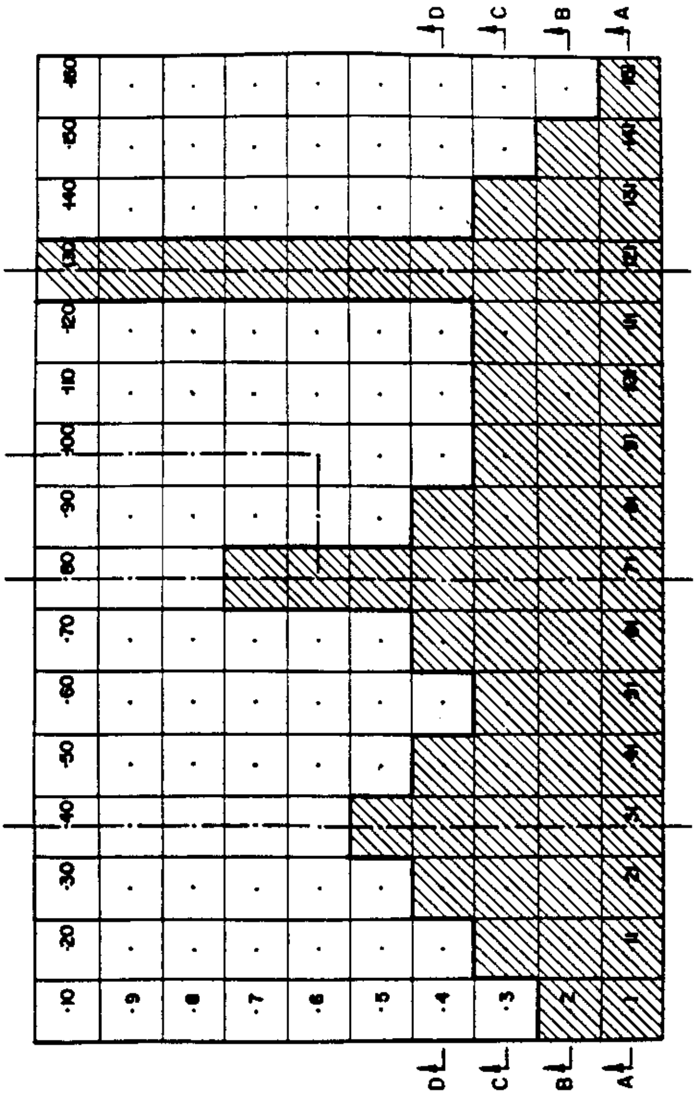

Figure 13. Maximum water depth at different cross-sections

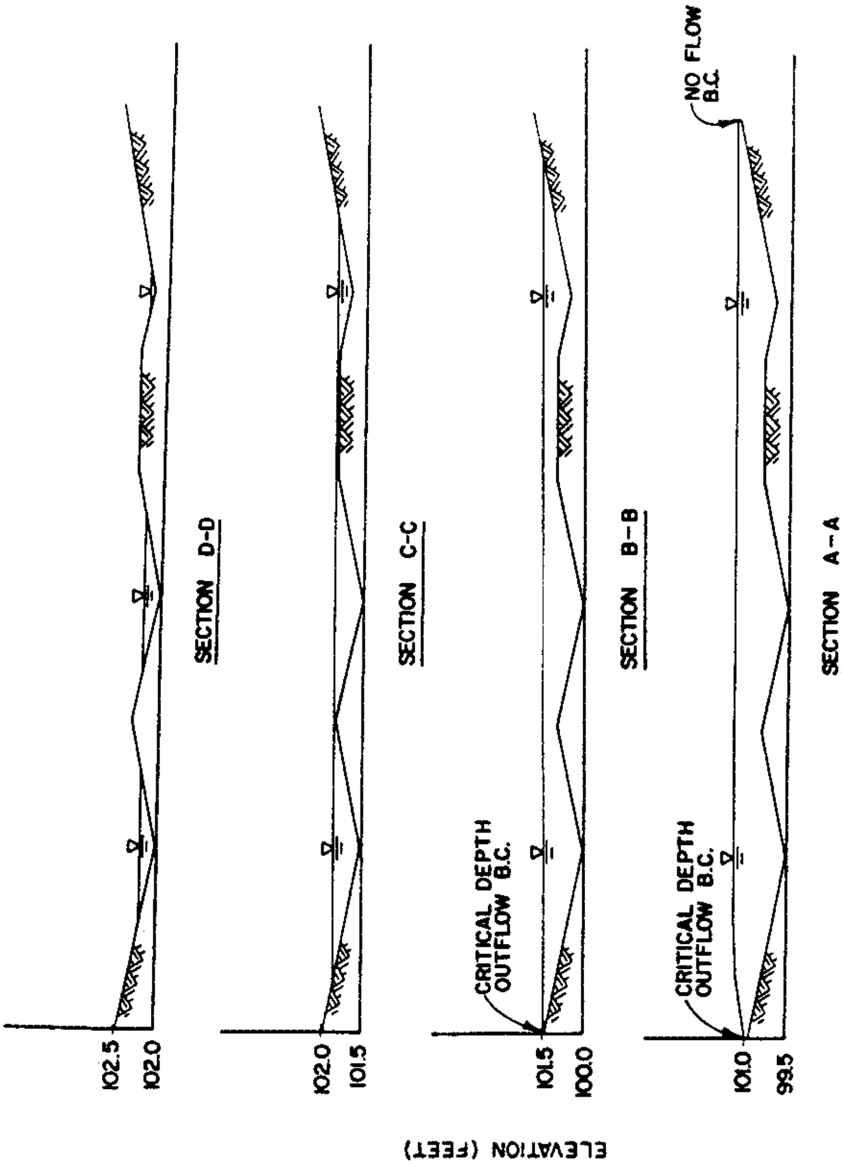

Figure 14. Maximum water profiles