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Comparisons of Hydraulic Routing Methods for One-Dimensional Channel Routing Problems

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ABSTRACT

A crucial element in any model of watershed hydrology is the approximation used to model the one-dimensional flow of water in a reach of channel. Should the channel routing procedure be in error, then timing problems immediately arise where runoff hydrographs are combined incorrectly at channel confluences due to the incorrect arrival times estimated by the routing model. Consequently, as the level of discretization of the watershed is increased, the modeling accuracy becomes dominated by the accuracy of the channel routing approximation. In this paper, three hydraulic routing approximations of the fully dynamic one-dimensional form of St. Venant equations are considered; namely, the kinematic wave approximation, the diffusion approximation, and the approximation of the St. Venant equations by use of an implicit finite-difference algorithm developed by the U.S. Geological Survey Water Resources Division, the K-634 computer program. Tests are made for various channel reach slopes and severe inflow hydrographs. The test results indicate that the diffusion formulation is a good approximation of the K-634 modeling approach, and provides a considerable improvement over the often-used kinematic wave approach. Because of the advantages afforded by the diffusion routing approach over the kinematic wave approach, e. g. accommodation of backwater and reverse flow effects, wider range of applicability, and increased accuracy in the modeling of peak flow attenuation, it is recommended that hydrologic models based upon the kinematic wave approximation reconsider the use of the diffusion approximation.

ONE-DIMENSIONAL DIFFUSION HYDRODYNAMIC MODEL DEVELOPMENT

The mathematical relationships in a one-dimensional diffusion hydrodynamic (DHM) model are based upon the flow equations of continuity and momentum which can be rewritten (Akan and Yen, 1981) as

$$\frac{\partial Q_x}{\partial x} + \frac{\partial A_x}{\partial t} = 0 \qquad (1)$$

$$\frac{\partial Q_x}{\partial t} + \frac{\partial (Q_x{}^2/A_x)}{\partial x} + gA_x \left(\frac{\partial H}{\partial x} + S_{fx} \right) = 0 \tag{2}$$

where Q_x is the flowrate; x,t are spatial and temporal coordinates; A_x is the flow area; g is gravity; H is the water surface elevation; and S_{fx} is a friction slope. It is assumed that S_{fx} is approximated from Manning's equation for steady flow by (e.g. Akan and Yen)

$$Q_x = \frac{1.486}{n} A_x R^{2/3} S_{fx}{}^{1/2} \tag{3}$$

where R is the hydraulic radius; and n is a friction factor which may be increased to account for other energy losses such as expansions and bend losses. Letting m_x be a momentum quantity defined by

$$m_x = \left(\frac{\partial Q_x}{\partial t} + \frac{\partial (Q_x{}^2/A_x)}{\partial x} \right) \Big/ gA \tag{4}$$

then Eq. (2) can be rewritten as

$$S_{fx} = \left(\frac{\partial H}{\partial x} + m_x \right) \tag{5}$$

In Eq. (4), the subscript x included in m_x indicates the directional term.

Rewriting Eq. (3) and including Eqs. (4) and (5), the directional flow rate is computed by

$$Q_x = - K_x \left(\frac{\partial H}{\partial x} + m_x \right) \tag{6}$$

where Q_x indicates a directional term, and K_x is a type of conduction parameter defined by

$$K_x = \frac{1,486}{n} A_x R^{2/3} \Big/ \left| \frac{\partial H}{\partial x} + m_x \right|^{1/2} \tag{7}$$

In Eq. (7), K_x is limited in value by the denominator term being checked for a smallest allowable magnitude.

Substituting the flow rate formulation of Eq. (6) into Eq. (1) gives a diffusion type of relationship

$$\frac{\partial}{\partial x} K_x \left(\frac{\partial H}{\partial x} + m_x \right) = \frac{\partial A_x}{\partial t} \tag{8}$$

The one-dimensional diffusion model of Akan and Yen (1981) assumes $m_x = 0$ in Eq. (7). Thus, the one-dimensional DHM is given by

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} = \frac{\partial A_x}{\partial t} \tag{9}$$

where K_x is now simplified as

$$K_x = \frac{1.486}{n} A_x R^{2/3} \Bigg/ \left| \frac{\partial H}{\partial x} \right|^{1/2} \tag{10}$$

For a constant channel width, W, Eq. (9) reduces to

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} = W \frac{\partial H}{\partial t} \tag{11}$$

However, it is noted that a family of models is given by Eq. (8) where m_x is defined by selecting from the possibilities

$$m_x = \begin{cases} \dfrac{\partial (Q_x{}^2/A_x)}{\partial x} \Bigg/ gA_x, & \text{(convective acceleration model)} \\[2ex] \dfrac{\partial Q_x}{\partial t} \Bigg/ gA_x, & \text{(local acceleration model)} \\[2ex] \left(\dfrac{\partial Q_x}{\partial t} + \dfrac{\partial (Q_x{}^2/A_x}{\partial x} \right) \Bigg/ gA_x, & \text{(coupled model)} \\[2ex] 0\,, & \text{(DHM)} \end{cases} \tag{12}$$

ONE-DIMENSIONAL ANALYSIS

In order to evaluate the accuracy of the diffusion model of Eq. (11) in the prediction of flood depths, the U.S.G.S. fully dynamic flow model K-634 (Land, 1980a,b) is used to determine channel flood depths for comparison purposes. The K-634 model solves the coupled flow equations of continuity and momentum by an implicit finite difference approach and is considered to be a highly accurate model for many unsteady flow problems. The study approach is to compare predicted: (1) flood depths, and (2) discharge hydrographs from the K-634, the DHM (Eq. 11), and KW for various channel slopes and inflow hydrographs.

It should be noted that different initial conditions are used for these two models. The U.S.G.S. K-634 model requires a base flow to start the simulation, i.e., the initial depth of water can not be zero. Next, the normal depth assumption is used to generate an initial water depth before the simulation starts. These two steps are not required by the DHM nor the KW.

In this case study, two hydrographs are assumed; namely, peak flows of 120,000 cfs and 600,000 cfs. Both hydrographs are assumed to increase linearly from zero to the peak flow rate at time of 1-hour, and then decrease linearly to zero at time of 6-hours (see Fig. 1 inset). The study channel is assumed to be a uniform rectangular section of Manning's n equal to 0.040, and various slopes S_0 in the range of $0.001 \leq S_0 \leq 0.01$. Figure 1 shows the comparison of modeling results. From the figure, various flood depths are plotted along the channel length of up to 10-miles. Two reaches of channel lengths of up to 30-miles are also plotted in Fig. 1 which correspond to a slope $S_0 = 0.0020$. In all tests, grid spacing was set at 1000-foot intervals.

From Fig. 1 it is seen that the diffusion model provides estimates of flood depths that compare very well to the flood depths predicted from the K-634 model. Differences in predicted depths are less than 3-percent for the various channel slopes and peak flow rates considered. In contrast, the KW develops poorer estimates. Figures 2 and 3 show comparisons between the DHM, the KW, and K-634 model for water depths and outflow hydrographs at 5 and 10 miles downstream from the dam-break site. Again, the DHM provided a significantly better approximation of routing effects than KW.

Grid Spacing Selection

The choice of timestep and grid size for an explicit time advancement is a relative matter and is theoretically based on the well-known Courant condition (Basco, 1978). The choice of grid size usually depends on available topographic data for nodal elevation determination and the size of the problem. The effect of the grid size (for constant timestep for 7.2 seconds) on the diffusion model accuracy can be shown by example where nodal spacings of 1000, 2000 and 5000-feet are considered. The predicted flood depths varied only slightly from choosing the grid size between 1000-feet and 2000-feet. However, an increased variation in results occurs when a grid size of 5000-feet is selected. Figure 4 shows the computed flood depths in comparison

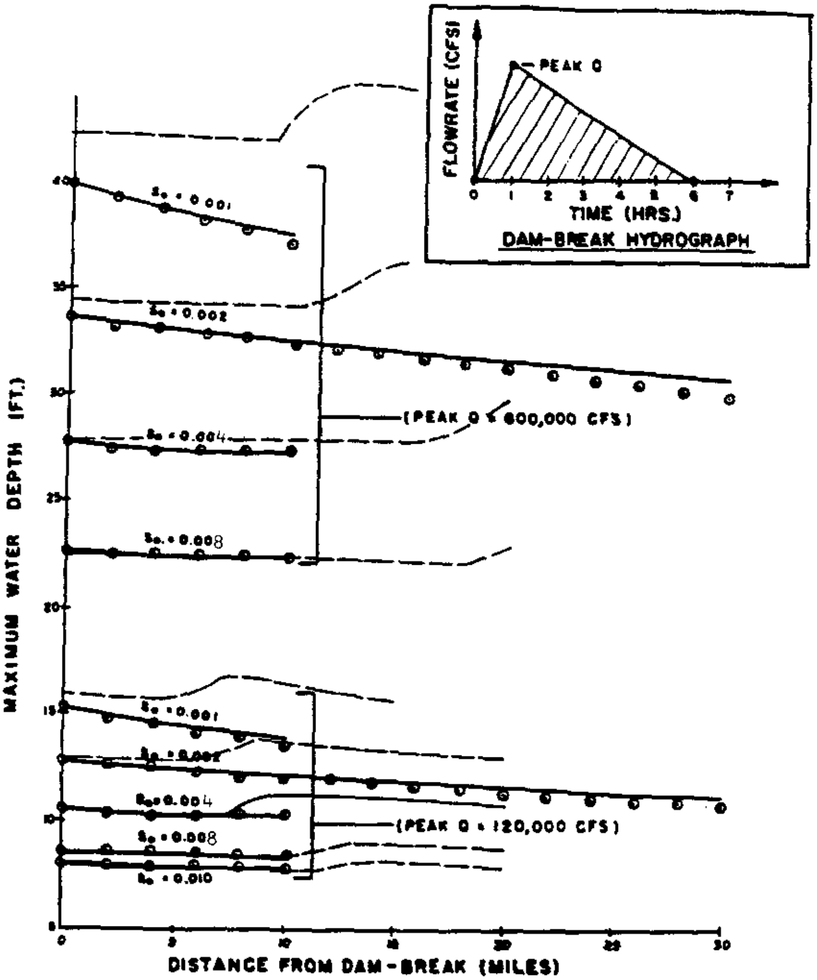


Fig. 1. Diffusion Model (•), kinematic routing (dashed line) and K-634 model results (solid line) for 1,000-foot width channel, Manning's n = 0.040, for various channel slopes, S_o.

to the K-634 modeling results (Fig. 1) for the considered grid sizes, and the peak flow rate test hydrograph of 600,000 cfs.

Because the algorithm presented is based upon an explicit time-stepping technique, the modeling results may become inaccurate should the timestep size versus grid size ratio become large. A simple procedure to eliminate this instability is to half the timestep size until convergence in computed results is achieved. Generally, such a timestep adjustment may be directly included in the DHM model computer program. For the cases considered in this section, timestep sizes of 7.2 seconds was found

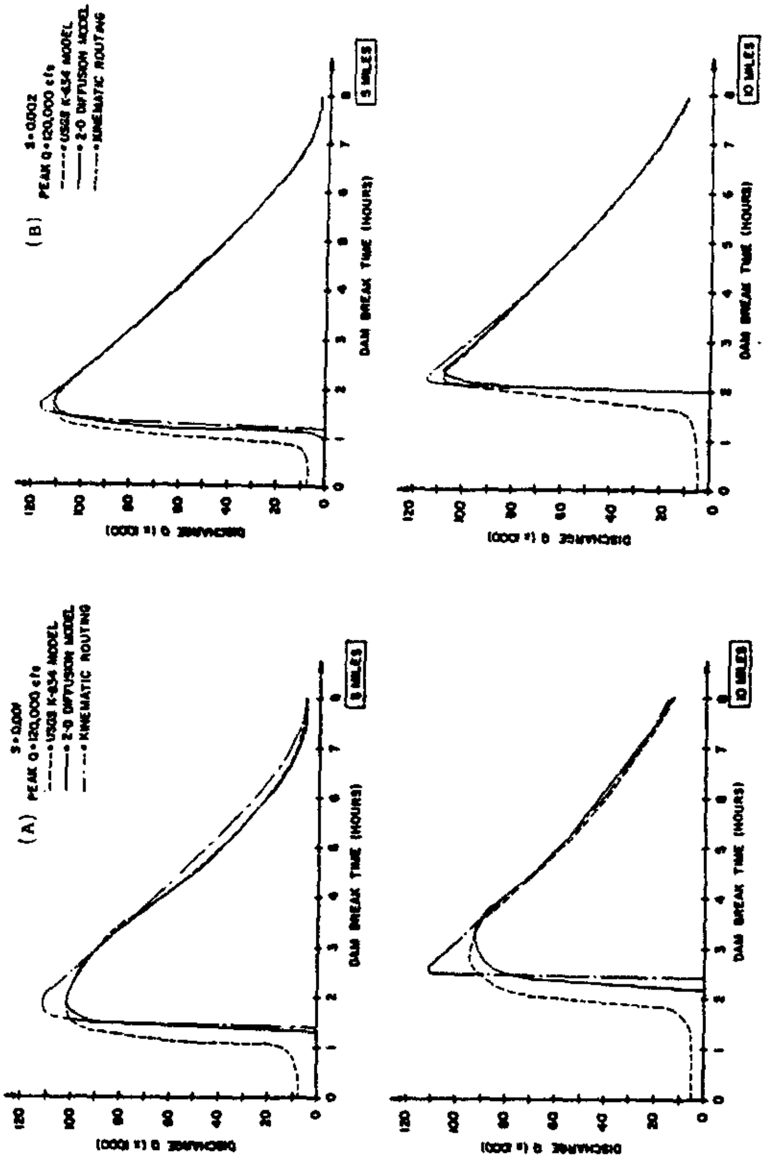


Fig. 2. Comparisons of Outflow Hydrographs at 5 and 10 miles downstream from the dam-break site.

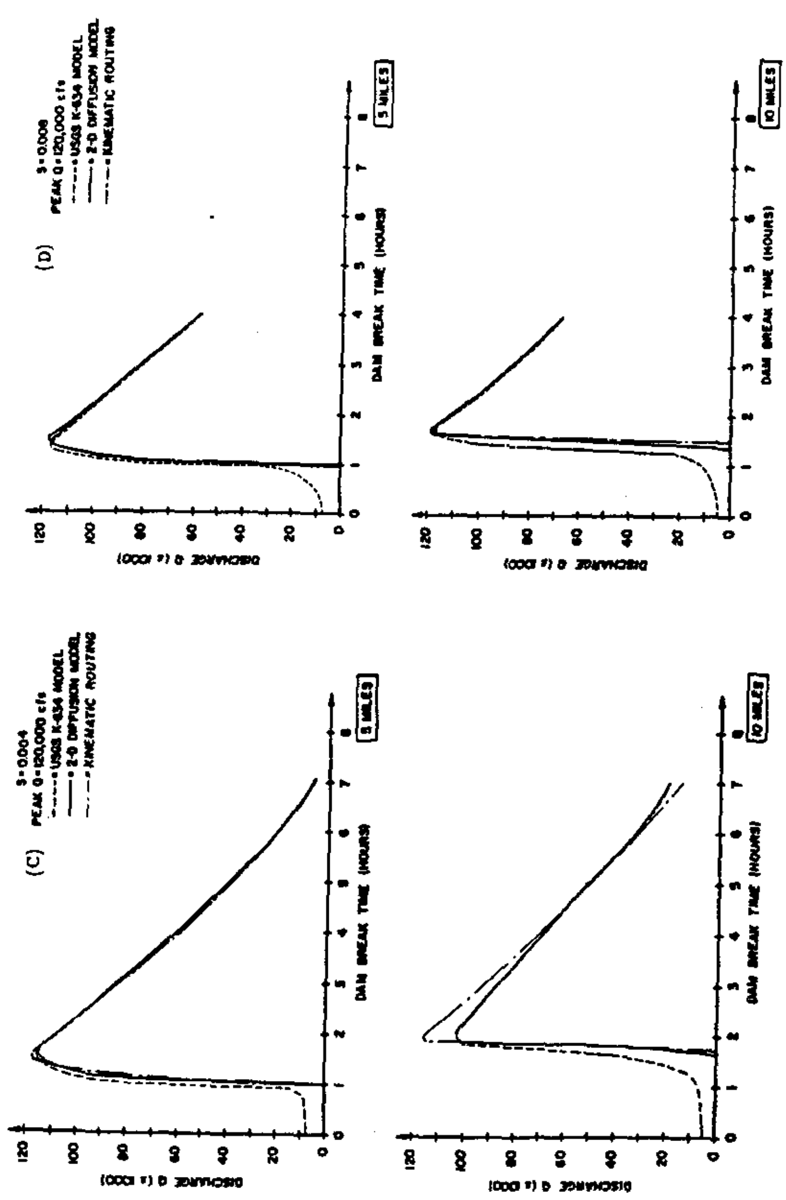
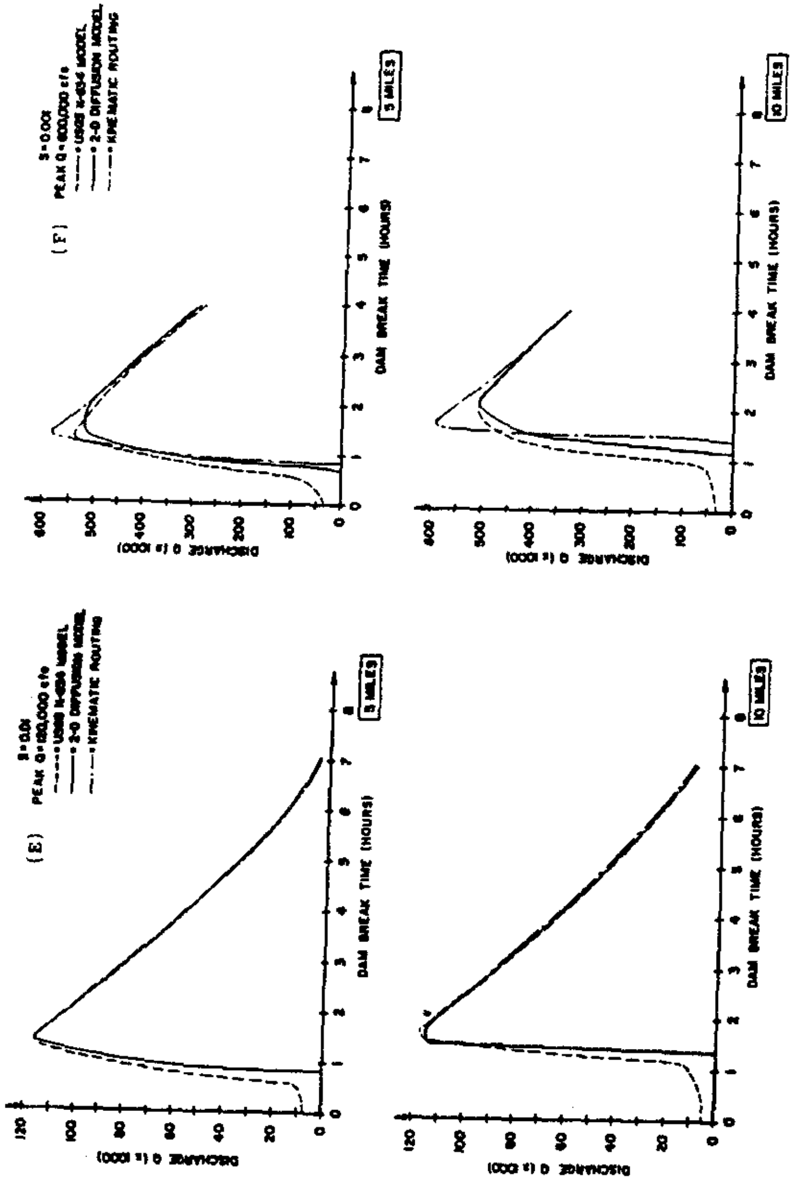


Fig. 2 (continued)



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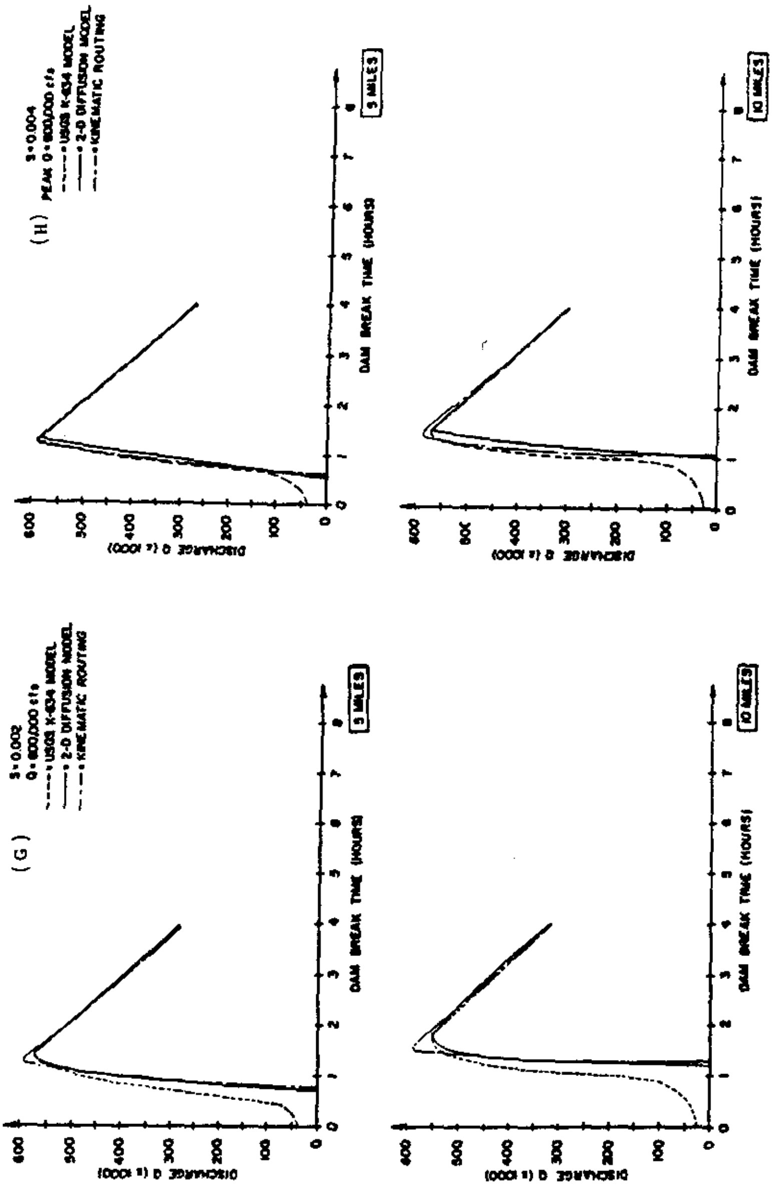

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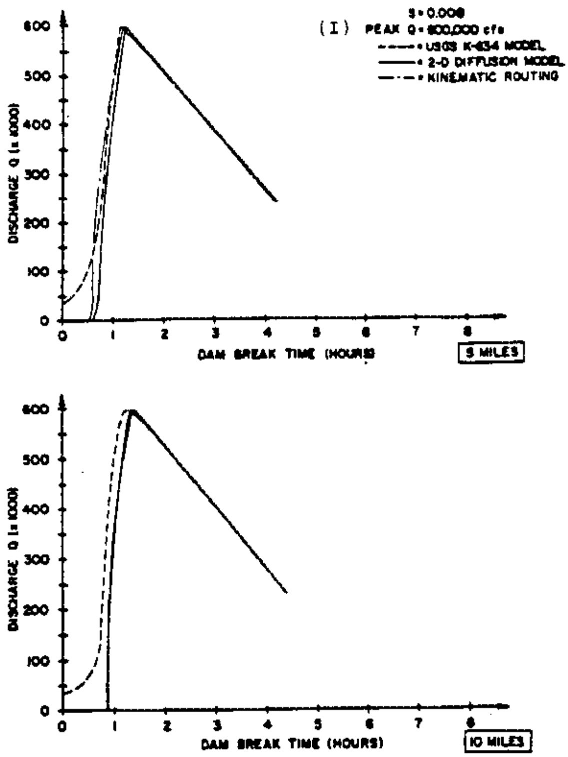


Fig. 2. (continued)

to be adequate when using the 1000-feet to 5000-feet grid sizes.

Conclusions and Discussions

For the runoff hydrographs considered and the range of channel slopes modeled, the simple diffusion dam-break model of Eq. (11) provides estimates of flood depths and outflow hydrographs which compare favorably to the results determined by the well-known K-634 one-dimensional dam-break model. In this study, the difference between the two modeling approaches is found to be less than a 3 percent variation in predicted flood depths.

The presented diffusion routing model is based upon a straightforward explicit timestepping method which allows the model to operate upon the nodal points without the need to use large matrix systems. Consequently, the model can be implemented on most currently available microcomputers.

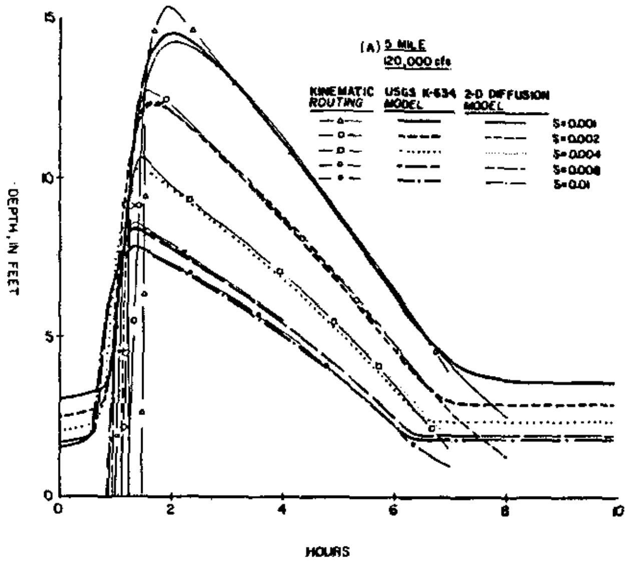


Fig. 3. Comparisons of depths of water at 5 and 10 miles downstream from the dam-break site

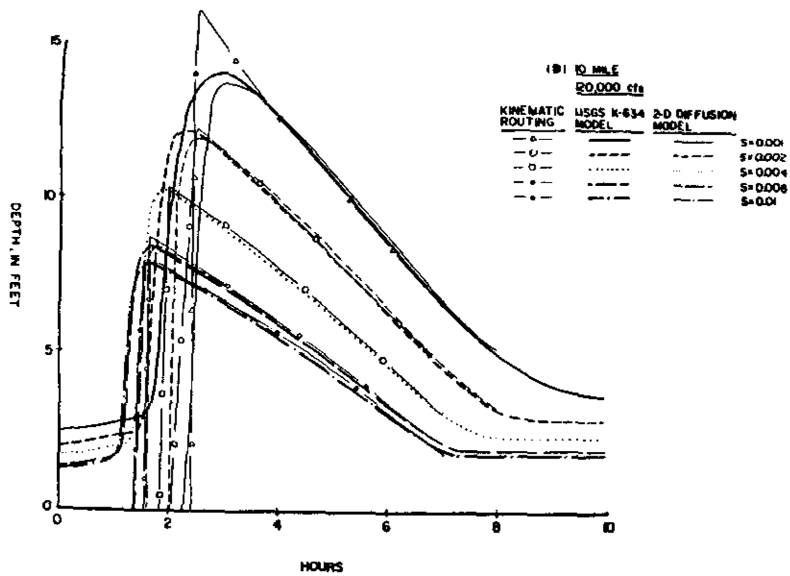
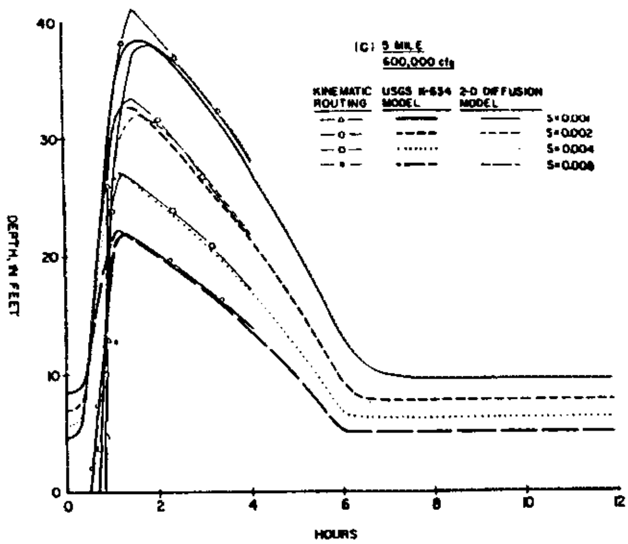


Fig. 3 (continued)



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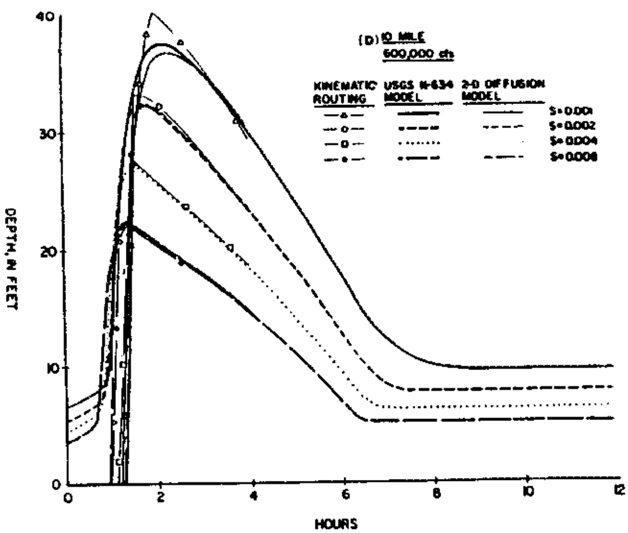

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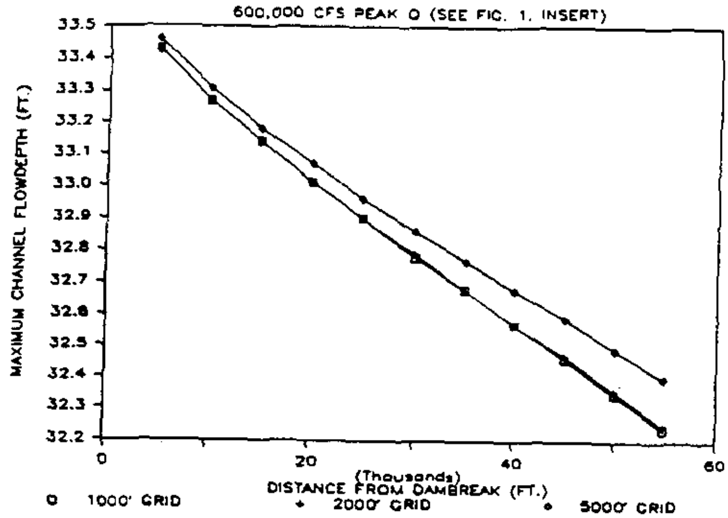

Fig. 4. Effects of Differing Grid Size

The diffusion model of Eq. (11) can be directly extended to a two-dimensional model by adding the y-direction terms which are computed in a similar fashion as the x-direction terms. The resulting two-dimensional diffusion model is tested by modeling the considered test problems in the x-direction, the y-direction, and along a 45-degree trajectory across a two-dimensional grid aligned with the x-y coordinate axis. Using a similar two-dimensional model, Xanthopoulos and Koutitas (1976) conceptually verify the diffusion modeling technique by considering the evolution of a two-dimensional flood plain which propagates radially from the dam-break site.

From the above conclusions, use of the diffusion approach of Eq. (11) in a two-dimensional DHM may be justifiable due to the low variation in predicted flooding depths (one-dimensional) with the exclusion of the inertia terms. Generally speaking, a two-dimensional model would be employed when the expansive nature of flood flows is anticipated. Otherwise, one of the available one-dimensional models would suffice for the analysis.

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