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Comparison of Hydraulic and Hydrologic Routing Methods for Channel Flow
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ABSTRACT

The current use of channel routing methods in hydrologic models can be categorized into two groups; namely, hydraulic models based on the approximation of the one-dimensional form of the St. Venant equations, and hydrologic routing methods which empirically model the effects of hydrograph translation and peak flow attenuation. Similarly, hydrologic models can be roughly categorized into two groups of modeling approaches (for models which develop runoff hydrographs by the simulation of the rainfall-runoff process rather than the evaluation of statistical floodflows from streamgage data such as in the "balanced flood" theory) depending on whether hydraulic or hydrologic channel routing is used. Typically, the hydraulic modelers also use the hydraulic routing formulations to develop runoff hydrographs from the surface flow contributions whereas the hydrologic routing modelers uses empirical relationships such as the unit hydrograph method. Comparisons between hydraulic and hydrologic channel routing methods are rare. In this paper, the most commonly used hydraulic routing procedure - the kinematic wave - is compared to the commonly used hydrologic routing procedure - convex routing. The two methods are tested against channel routing predictions obtained from the diffusion approximation of the St. Venant equations. The test results indicate that for a wide range of channel routing conditions, such as occurs in urban watersheds, the simpler convex routing method provides significantly better approximations of channel routing effects than does the kinematic wave procedure. Because uses of kinematic wave models often are based on level of discretization which forces a considerable weight in the overall modeling accuracy to be placed on the channel routing approximation accuracy, it is
recommended that kinematic wave models re-evaluate use of the channel routing approximation procedure.

INTRODUCTION

Models of watershed runoff typically include a submodel for approximating the effects of unsteady flow in open channels (i.e. channel routing) for routing a runoff hydrograph through a channel reach. The various methods used to approximate the unsteady flow routing process can be grouped primarily into two categories: hydraulic routing methods which approximate the governing flow equations of continuity, energy and momentum; or hydrologic routing methods which represents the effects of translation and channel storage on the inflow runoff hydrograph. One of the most popular hydraulic methods used in watershed hydrologic models is the kinematic wave approach. One of the most popular (but simple) hydrologic channel routing models is the convex method.

In this paper, the standard kinematic wave routing method is compared to the standard convex routing method such as described and employed in the HEC-1 Kinematic Wave (KW) program (HEC TD-10, 1979) and the SCS Engineering Handbook (1972), respectively. Several watershed models use the KW method for channel routing such as used in the HEC-1 KW program and, therefore, the results of this study apply to KW programs in general. The focus of this paper is not only the validity and appropriateness of either routing method in the approximation of flow routing effects, but also the computational errors that are associated to either method. It is shown that except for those conditions where there is no attenuation or subsidence of the runoff hydrograph peak flowrate due to channel storage effects and where the inflow hydrograph includes a mild rising and falling limb, the KW model exhibits significant computational error and numerical-diffusion effects which depend on the user-specification of the KW modeling reach length and timestep sizes. In comparison, however, the simple convex hydrologic routing method shows only a small fraction of the irregularities associated with the KW modeling results.

As a result, the use of the KW approach may be questioned as being retrograde for use as a channel routing analog in hydrologic models.

LITERATURE REVIEW

Use of the KW channel routing technique is popular among many of the watershed models developed during the last decade. (To avoid confusion, the KW routing method is defined to be the technique described in the HEC TD-10, 1979.) However, the literature contains several examples of KW channel routing performance which indicate that this procedure may be of limited value in comparison to other methods. For example, Akan and
Yen (1981) show that their comparisons of KW routing results to
diffusion and fully dynamic computational solutions indicate
that the KW peak flowrate estimates and hydrograph timing differ
significantly from the other comparable modeling results.
Similar results were obtained by Katapodes and Schamber (1983)
which demonstrate the significant errors developed from the KW
flow routing approximation. In a test of the performance of KW
models where the standard KW model is "corrected" for dynamic
routing effects, Weinmann and Laurenson (1979) demonstrate the
significant errors developed from the standard KW approach.

The source of the KW errors can be grouped into two categories:
(1) error in the KW model fundamental assumptions, and (2) com-
putational errors from the finite-difference numerical solution
of the KW approach. Typically, both errors are "seen" together
and comparisons are reported in the literature which do not
isolate the two sources of error. For example, Doyle et al
(1983) write that "It has been shown repeatedly in flow-routing
applications that the kinematic wave approximation always pre-
dicts a steeper wave with less dispersion and attenuation than
may actually occur." Generally speaking, however, the KW does
not attenuate the peak flowrate; that is, modeled attenuation
of the hydrograph peak flowrate is under most circumstances a
result of the computational errors in approximating the KW flow
equations. Ponce et al (1978) write "...the kinematic model, by
definition, does not allow for subsidence." In consideration
of solving the KW flow equations by using the method of
characteristics, Strelkoff (HEC Research Document No. 24, 1980)
writes that "...kinematic waves can attenuate under certain
conditions. Such attenuation is enhanced by overflow into flood
plains, but can occur when kinematic shocks (as distinguished
from bores) are formed in the channel at the intersection of
the characteristics."

Therefore, attenuation of the hydrograph peak flowrate when
using the KW technique is essentially the result of computational
errors including numerical-diffusion, and not due to the appli-
cation of the KW flow equations. This paper focuses on the
magnitude and significance of these computational errors as
produced by the well-known HEC-1 KW program. (The HEC-1 KW
program is used for demonstration purposes only. Other KW
routing analogs will demonstrate the tendencies described
herein.) In this way, the second category of errors associated
with the KW method are evaluated. The first category of KW
errors (i.e. the appropriateness of the use of the KW flow
equations) is essentially addressed by the statement in Li
et al (1975) "The limitations of this new method are inherited
from the restriction resulting from the kinematic wave
approximation. That is, local and convective accelerations
must be negligible, and the water surface slope is nearly equal
to the channel bed slope."
STUDY PROCEDURE

The reported difficulties in the referenced KW model were investigated during the course of a study to evaluate the accuracy of hydraulic and hydrologic channel routing models. During the course of that study, the significance of the KW computational errors were evaluated and then separately studied to identify the implications, if any, in the use of a KW channel routing model in a hydrologic model setting.

Several test cases were considered involving various rectangular channel reach lengths, slopes, friction coefficients and base widths. In all cases, a runoff hydrograph shape typical of those anticipated for flood control studies was used. Use of a more peaked runoff hydrograph worsened the computational errors identified for the set of test cases reported upon in this paper.

Each test case involved a total channel length of 25,000 feet. Throughout the length, all channel properties are held constant. The inflow hydrograph was then routed through the channel using various (constant) channel segment (Δx) and timestep (Δt) sizes in the KW model. The convex method was then applied to the same problem conditions using identical channel segment sizes used for the KW model test, but with a constant timestep size of 5-minutes.

In the following are presented the set of test results involving the rectangular channel of 40 feet base width, a bottom slope of 0.0010, and a Manning's friction factor of n = 0.050. In this test, the largest magnitude of computational error was noted for the set of tests considered in our study.

CASE STUDY RESULTS

In HEC-1, the program selects Δx on the basis of Δt, or Δt is chosen on the basis of Δx. The routing reach is always divided into at least two segments, so that the maximum Δx is one-half of the reach length. Because the finite-difference solution used in the kinematic wave routing equations introduces numerical diffusion into computational results, noticeable differences in routed hydrographs occur as Δx is varied in the reach. Figure 1 contains KW model results for channel lengths of L = 5,000 and 10,000 feet for two modeling attempts each. For L = 10,000 feet, it is seen that depending on whether Δx = 2,500 or 5,000 feet, Qpeak (outflow) is 840 cfs or 680 cfs, respectively. A smaller Δx would result in a higher Qpeak (outflow) until the 940 cfs Qpeak (inflow) value is reached.

Figure 2 shows the KW model outflow hydrographs for various channel lengths L from L = 0 feet (i.e. the inflow hydrograph) to L = 25,000 feet. In all cases, Δx = 2,500 feet and Δt = 6 min. Again, the Qpeak (outflow) values of Fig. 2 would raise
Fig. 1. KW Outflow Hydrographs for $L = 5,000$ ft. and $10,000$ ft. (Inflow Hydrograph Shown in Fig. 2)

Fig. 2. Outflow Hydrographs for $\Delta x = 2,500$ ft., $\Delta t = 6$ min. for Various Channel Lengths ($L$).
(or lower) should a smaller (or larger) $\Delta x$ value be specified in the KW model. This is demonstrated by using a $\Delta x = 500$ feet and $\Delta t = 2$ min. such as shown in Fig. 3. Comparing Figs. 2 and 3 it is seen that using more computational effort in the KW model (i.e. decreasing $\Delta x$ from 2,500 feet to 500 feet) increases the $Q_{\text{peak}}$ (outflow) and also changes the hydrograph shape and time-to-peak.

Fig. 3. Using $\Delta x = 500$ ft. and $\Delta t = 2$ min. in the KW Model Test of Fig. 2.

Figure 4 summarizes the KW modeling results for the total channel length of 25,000 feet. From the figure it is seen that depending on whether $\Delta x = 2,500$ feet or 8,333 feet, $Q_{\text{peak}}$ (outflow) = 640 cfs or 400 cfs, respectively. Recalling the Fig. 3 value for $L = 25,000$ feet using $\Delta x = 500$ feet, $Q_{\text{peak}}$ (outflow) = 800 cfs. Again, use of still smaller $\Delta x$ would increase $Q_{\text{peak}}$ (outflow) to the 940 cfs $Q_{\text{peak}}$ (inflow) value.

Should the HEC-1 KW model user input input $\Delta t$, the results of the $L = 25,000$ feet case study vary according to Fig. 5. Again, as $\Delta t \rightarrow 0$, then $\Delta x \rightarrow 0$ and $Q_{\text{peak}}$ (outflow)$\rightarrow Q_{\text{peak}}$ (inflow).

Figure 6 summarizes the HEC-1 KW channel routing $Q_{\text{peak}}$ (outflow) values for various L lengths and for an input $\Delta t$ value of 6 minutes. Recalling that $Q_{\text{peak}}$ (inflow) = 940 cfs, the shaded area shown on Fig. 6 is the KW $Q_{\text{peak}}$ (outflow) values possible depending on the $\Delta x$ value chosen.

The convex routing model was also used to approximate the unsteady flow problems attempted by the KW model. Typically, the convex model performed "poorest" when the KW model did and, therefore, examination of the computational error for the same set of test problems described for the KW model is appropriate. Because the
Fig. 4. KW Results for L = 25,000 ft.

Fig. 5. Effect of $\Delta t$ Input in HEC-1 KW Model 
(L = 25,000 ft.)
($\Delta x$ Selected by HEC-1 Program).

convex model demonstrated only a small fraction of the variation in results that the KW model demonstrated, the convex modeling results are shown in tabled form.
Fig. 6. Variation in KW and Convex Method Modeling Results of Qpeak (outflow) for Various L Values from 6,000 ft. to 12,000 ft.

In Table 1 are contained the Qpeak (outflow) values from use of the convex model for the inflow hydrograph of Fig. 2, and for various values of L. Three cases are considered for Δx values; namely, Q1 values indicate three reaches composed of 2:10,000-foot lengths and 1:5,000-foot length; Q2 values indicate 5:5,000-foot lengths and Q3 values indicate 25:1,000-foot lengths. For all tests, a Δt of 5 minutes was used. Also included in the table is an additional convex test case for a different set of channel conditions which results in considerably higher channel flow velocities. It is readily seen that after 25,000 feet, the convex routing method involves computational errors due to the selection of Δx values of the order of 5 percent.

Figure 7 summarizes the range of computational results from the HEC-1 KW model (where the program selects the computational parameters); the convex routing method (for a constant timestep of 5-minutes). The illustrated range of channel lengths vary from 0 to 25,000 feet.

From Fig. 7, the convex method shows a variation of 5 percent. In contrast, the KW model shows a variation of over 130 percent for L = 25,000 feet depending on the Δx values selected.
### TABLE 1: CONVEX MODEL Q\text{peak} (OUTFLOW) RESULTS

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**Fig. 7.** Variation in KW and Convex Method Modeling Results of Q\text{peak} (outflow) for Various L Values from 0 to 25,000 ft.
DISCUSSION OF RESULTS

From the preceding results it is seen that the arbitrary use of the KW method to model unsteady flow in open channels is subject to considerable scrutiny due to the potential wide variation in results possible by the selection of $\Delta x$ or $\Delta t$ values. This "range of results" impacts the very credibility in using KW for channel routing hydrologic models. A possible remedy in using the standard KW approach (such as in HEC-1) may be to require that all users choose $\Delta x$ values sufficiently small as to guarantee a good solution of the KW assumption; but in that case, $Q_{\text{peak (outflow)}} = Q_{\text{peak (inflow)}}$ due to the lack of subsidence of the peak flowrate fundamental to the KW formulation. But most channel routing conditions that occur in practice do exhibit peak flow attenuation due to channel storage effects and, therefore, use of the KW would contradict the fundamental channel routing characteristics. In order to avoid the significant numerical diffusion problem, KW should only be used when there is negligible peak attenuation in the channel. But in that case, simple hydrograph translation would be a simpler method to use than KW.

The convex routing method, on the other hand, is simple to apply, does not demonstrate the computational deficiencies to the magnitude exhibited by the HEC-1 KW model, approximates peak attenuation, and yet also accommodates translation without peak attenuation for high velocity flows.

Based on the observed computational errors of the KW channel routing method, the limitations fundamental to use of the KW method, and the computational effort needed to approach a true KW hydrograph routing approximation, we submit that use of the KW method for channel routing needs a re-evaluation for use in hydrologic models unless guidelines are developed to control the arbitrary use of KW in design studies.

CONCLUSIONS

The HEC-1 KW model is studied to evaluate the significance of computational errors due to the choice of the computational effort used to approximate the unsteady flow effects in channel routing. It is shown that the selection of the computational effort used in the finite difference approximation of the KW relationship may have a significant impact on the KW modeling results, and that the simple convex hydrologic routing method demonstrates but a small fraction of the variations in results demonstrated by the KW model used. Even when restricting the use of KW to "fast flow" situations, the simpler convex method appears to provide more accurate estimates of peak flow attenuation than does the KW. It is recommended that hydrologic models which use the standard KW method for channel routing be re-evaluated as to their credibility and reliability in their use in the typical flood control design setting of practising engineers.
REFERENCES


