Technical note: Estimating water shed S-graphs using a diffusion flow model

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A simplification of the two-dimensional (2-D) continuity and momentum equations is the diffusion equation. This simpler dynamic model of two-dimensional hydraulics affords the hydrologist a means to quickly estimate floodflow effects for overland flow. To investigate its capability, a numerical model using the diffusion approach is applied to a set of hypothetical watersheds in order to develop unit hydrographs. The model is based on an explicit, integrated finite-difference scheme, and the floodplain is simulated by use of topographic elevation and geometric data. Synthetic unit hydrographs (S-graphs) developed from use of the simple 2-D model show interesting correlations to the well-known S.C.S. unit hydrograph (S-graph).

INTRODUCTION

A frequently used technique for modelling watershed rainfall-runoff effects is the unit hydrograph approach. When given ample stream gauge data, an averaged S-graph can be developed for use in studying severe storm hydrology[4]. However when stream gauge data is inadequate, synthetic S-graphs are often employed based on similar watershed characteristics.

In this note is reported a technique for developing a synthetic S-graph for overland flow based on the diffusion model of the full hydrodynamic equations for two-dimensional flow. The results of this study show a strong correlation to the well-known S.C.S. unit hydrograph S-graph equivalent[4].

One approach to studying watershed overland flow flood wave propagation is to simply estimate a maximum possible flowrate and route this flow as a steady state flow through the downstream reaches. A better approach is to rely on one-dimensional (1-D) full dynamic unsteady flow equations (e.g. St. Venant equations). Some sophisticated 1-D models include more terms and parameters to account for complexities in prototype reaches which the basic flow equations cannot adequately handle. However, the validity of the 1-D model is questioned when studying flood wave propagation in a two-dimensional (2-D) domain. There are some 2-D models employing full dynamic equations. Among them, one particularly aimed at flood flow analysis is by Katopodes and Strelkoff[3]. Associated with the increased power and capacity of 2-D full dynamic models, are the greatly increased boundary, initial, geometry and other input data and computer memory and resources requirements as well as computational effort.

A 2-D diffusion hydrodynamic model described in this note[1,2,5] appears to offer a simple and economic means for the fast estimation of overland floodflow effects. It can be used to develop a synthetic S-graph using topographic elevation data and estimates of the Manning's friction factor. The study objective is to estimate the S-graph resulting from a severe runoff event over a two-dimensional domain where overland flow effects dominate the flow hydraulics.

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MATHEMATICAL DEVELOPMENT FOR TWO-DIMENSIONAL MODEL

The set of (fully dynamic) 2-D unsteady flow equations consists of one equation of continuity

$$\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial H}{\partial t}=0 \tag{1}$$

and two equations of motion

$$\frac{\partial q_x}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q_x^2}{h}\right)+\frac{\partial}{\partial y}\left(\frac{q_x q_y}{h}\right)+gh\left(S_{fx}+\frac{\partial H}{\partial x}\right)=0 \tag{2}$$

$$\frac{\partial q_y}{\partial t}+\frac{\partial}{\partial y}\left(\frac{q_y^2}{h}\right)+\frac{\partial}{\partial x}\left(\frac{q_x q_y}{h}\right)+gh\left(S_{fy}+\frac{\partial H}{\partial y}\right)=0 \tag{3}$$

in which q_x, q_y are flow rates per unit width in the x, y-directions; S_{fx}, S_{fy} represent friction slopes in x, y-directions; H, h, g stand for, respectively, water-surface elevation, flow depth and gravitational acceleration; and x, y, t are for spatial and temporal coordinates.

The above equation set is based on the assumptions of constant fluid density with zero sources or sinks in the flow field, of hydrostatic pressure distributions, and of relatively uniform bottom slopes.

The local and convective acceleration terms can be grouped together such that equations 1, 2 and 3 are rewritten as

$$m_z+\left(S_{fz}+\frac{\partial H}{\partial z}\right)=0, \qquad z=x,\ y \tag{4}$$

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where m_z represents the sum of the first three terms in equations (3) and (3) divided by gh. Assuming the friction slope to be approximated by steady flow conditions, the Manning's formula in the US customary units can be used to estimate

$$q_z = \frac{1.486}{n} h^{5/3} S_{fz}^{1/2}, \qquad z = x, y \tag{5}$$

Equation (5) can be rewritten as

$$q_z = -K_z \frac{\partial H}{\partial z} - K_z m_z, \qquad z = x, y \tag{6}$$

where

$$K_z = \frac{1.486}{n} h^{5/3} \Bigg/ \left| \frac{\partial H}{\partial S} + m_S \right|^{1/2}, \qquad z = x, y \tag{7}$$

The symbol S indicates the flow direction which makes an angle of $\theta = \tan^{-1}(q_y/q_x)$ in the positive x-direction.

Values of m are assumed negligible by several investigators[1,2,5], resulting in the simple diffusion model,

$$q_z = -K_z \frac{\partial H}{\partial z}, \qquad z = x, y \tag{8}$$

The proposed 2-D flood flow model is formulated by substituting equation (8) into equation (1),

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} = \frac{\partial H}{\partial t} \tag{9}$$

NUMERICAL MODEL FORMULATION (GRID ELEMENT)

For uniform grid elements, the integrated finite difference version of the nodal domain integration (NDI) method is used. For grid elements, the NDI nodal equation is based on the usual nodal system shown in Fig. 1. Flow rates along the boundary Γ are estimated using a linear trial function assumption between nodal points.

For a square grid of width δ,

$$q|_{\Gamma_E} = -(K_x|_{\Gamma_E})(H_E - H_C)/\delta \tag{10}$$

where

$$K_x|_{\Gamma_E} = \begin{cases} \left(\dfrac{1.486}{n} h^{5/3}\right)_{\Gamma_E} \Bigg/ \left| \dfrac{H_E - H_C}{\delta \cos\theta} \right|^{1/2}; & \bar{h} > 0 \\ 0; & \bar{h} \leqslant 0 \text{ or } |H_E - H_C| < 10^{-3} \end{cases} \tag{11}$$

In equation (11), h and n are both the average of the values of C and E, i.e. $h = (h_C + h_E)/2$ and $n = (n_C + n_E)/2$. (Additionally, the denominator of K_x is checked such that K_x is set to zero if $|H_E - H_C|$ is less than a tolerance such as 10^{-3} ft.)

The model advances in time by an explicit approach

$$H^{i+1} = K^i H^i \tag{12}$$

where the assumed input flood flows are added to the specified input nodes at each timestep. After each timestep, the hydraulic conduction parameters of equation (11) are reevaluated, and the solution of equation (12) reinitiated. Using grid sizes with uniform lengths of 1000-feet, timesteps of size 5.0 sec were found satisfactory. Verification of the 2-D hydrodynamic model is given in Hromadka[2] for the class of problems involving severe peaked flood hydrographs such as those resulting from dam-breaks.

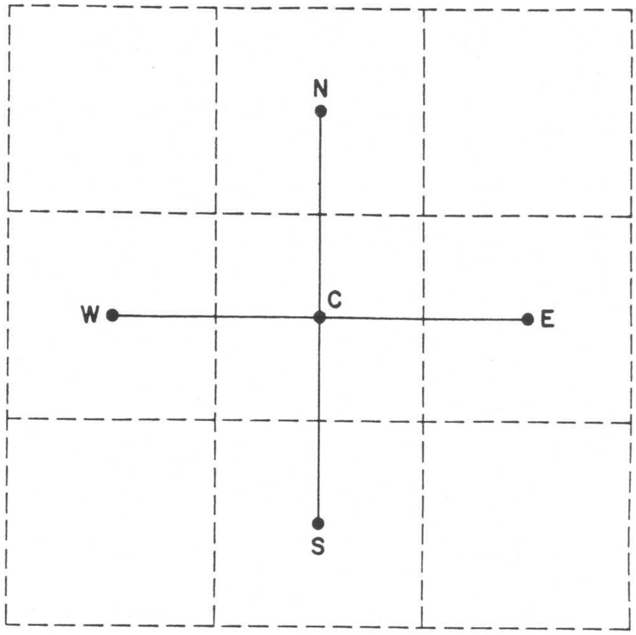


Fig. 1. Two-dimensional finite difference analog

DEVELOPMENT OF SYNTHETIC S-GRAPHS

The diffusion model can be used to develop a synthetic S-graph for a watershed where overland flow is the dominating flow effect. The watershed is discretized by uniform node-centred grid elements (see Fig. 1). The data required are an elevation and a Manning's friction factor for each nodal point.

To develop the S-graph, a uniform effective rainfall is assumed to occur over the entire watershed. For each timestep (e.g. 5-seconds), an incremental volume of water is added directly to each grid-element based on the rainfall intensity (a constant in this application), resulting in an equivalent increase in the nodal point depth of water. Because the discretized model is two-dimensional, the runoff flows to the selected point of concentration by simulating overland flow in the watershed.

The following applications show S-graphs developed for several watersheds with various cross-slopes, channel slopes, areas and friction factors. Figure 2 shows the watershed discretization used for the S-graph development shown in Fig. 3. Included in Fig. 3 are the S.C.S. S-graphs for a triangular and a curvilinear unit hydrograph representations. It is seen that the diffusion model closely matches the S.C.S. S-graph. Figure 4 shows other S-graphs developed for different watershed configurations and conditions. From the figure, all S-graphs have a strong similarity to the S.C.S. S-graph.

MODELLING SEVERE STORM RUNOFF

The 10 square mile Cucamonga Creek watershed (California) is shown discretized by 1000-foot grid elements in Fig. 5. A design storm (Fig. 6a) applied by the US Army Corps of Engineers and resulting runoff hydrograph is shown in Fig. 6b. Also shown in Fig. 6b is the corresponding diffusion model response for this canyon flow. From the figure, the diffusion model develops runoff quantities which are in good agreement with the values computed using a derived unit hydrograph.

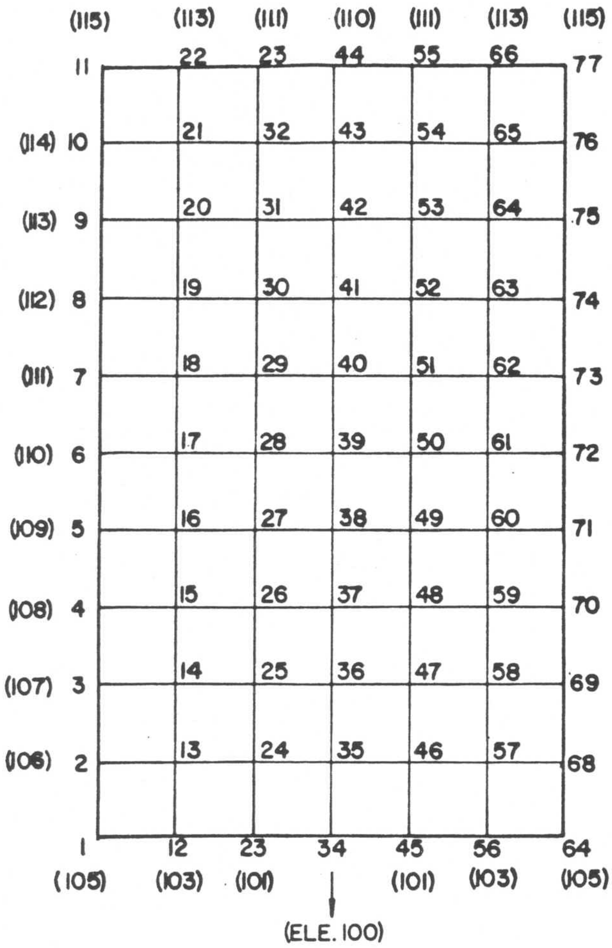


Fig. 2. Test watershed discretization

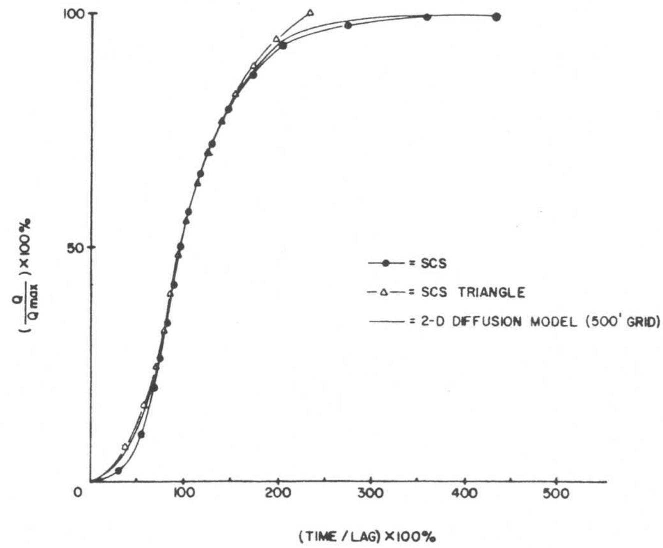


Fig. 3. Diffusion model produced S-graph and S.C.S. S-graphs

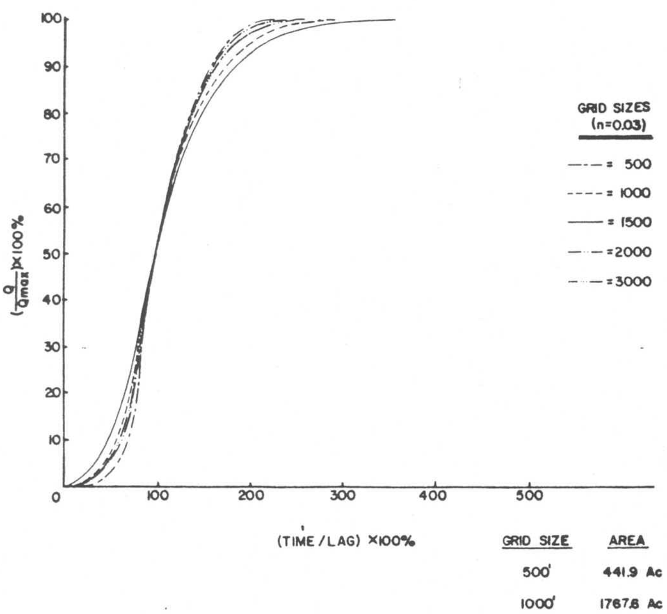


Fig. 4a. Diffusion model produced S-graphs for various grid sizes (nodal elevations held constant)

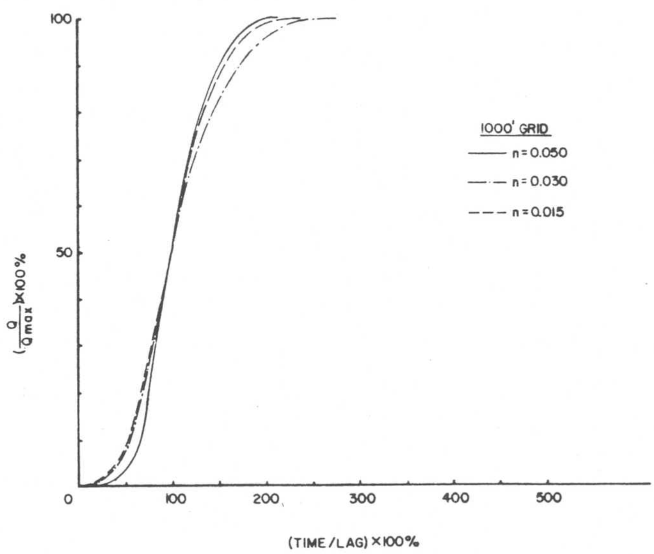


Fig. 4b. Diffusion model produced S-graph for various Manning's friction factors

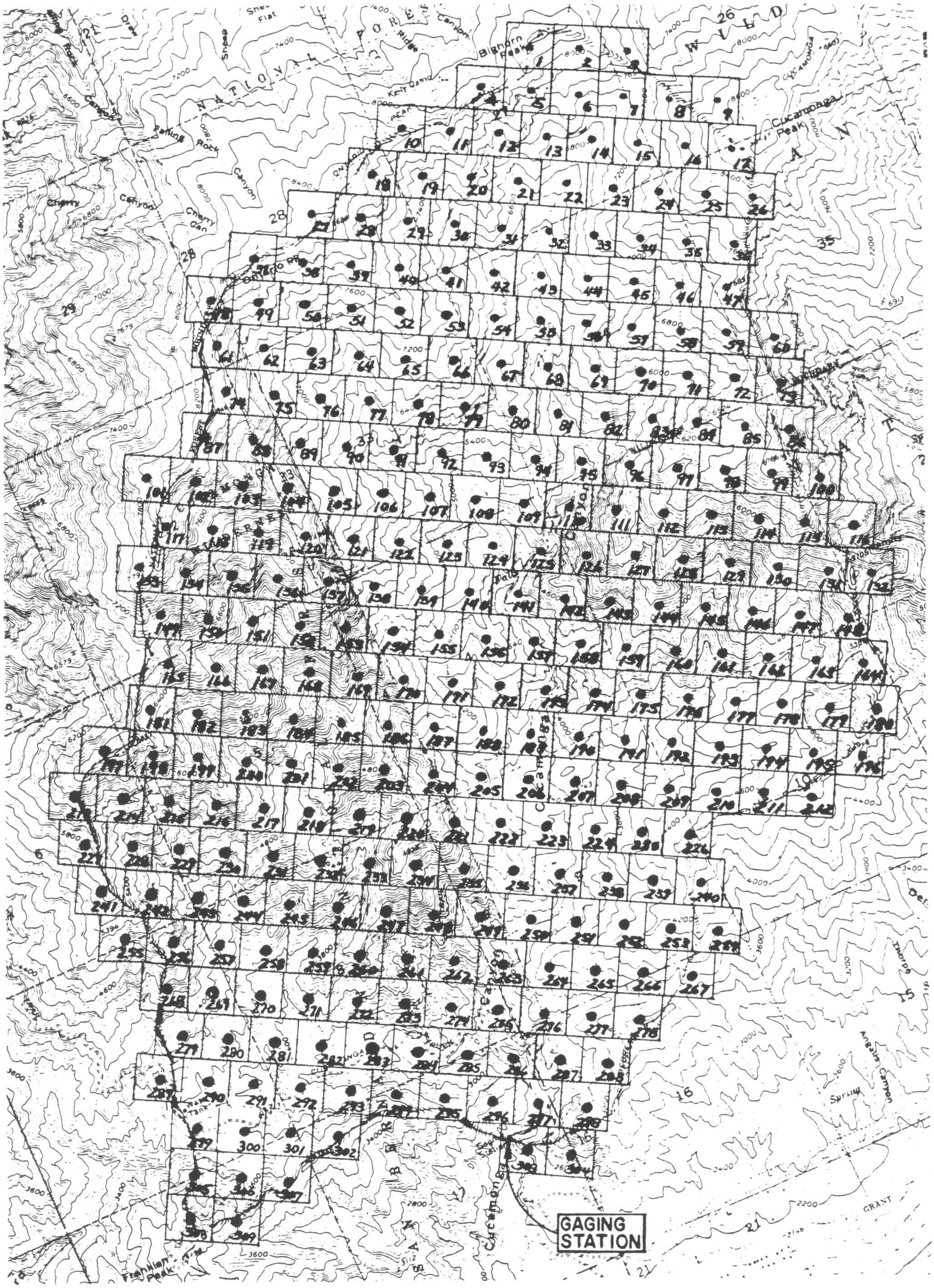
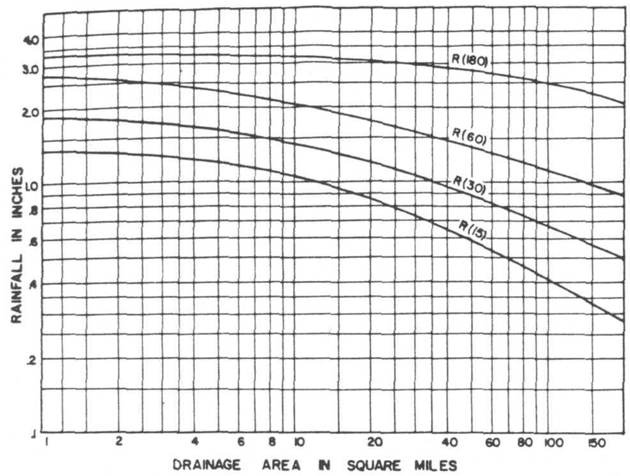


Fig. 5. Cucamonga Creek discretization



HYETOGRAPH COMPUTATION

UNIT PERIOD	AMOUNT	
1	.07	(R(180)-R(60))
2	.05	(R(180)-R(60))
3	.11	(R(180)-R(60))
4	.05	(R(180)-R(60))
5	.20	(R(180)-R(60))
6	.22	(R(180)-R(60))
7	.14	(R(180)-R(60))
8	.16	(R(180)-R(60))
9	.48	(R(60)-R(30))
10	.52	(R(60)-R(30))
11	1.00	(R(15))
12	1.00	(R(30)-R(15))

LOCAL PROJECT STORM
DEPTH AREA DURATION CURVES

Fig. 6a. Design storm

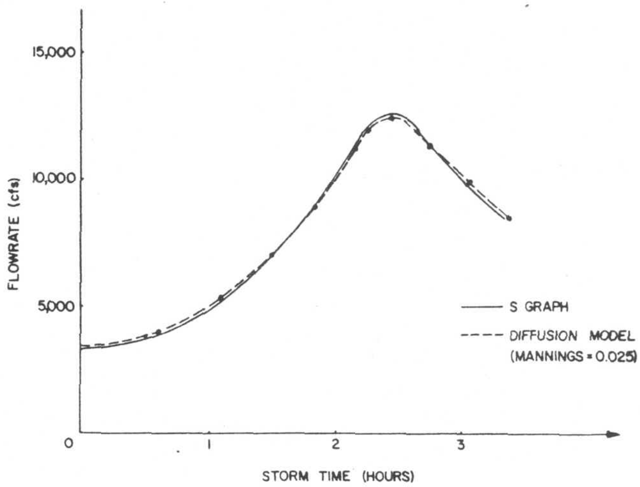


Fig. 6b. Modelled runoff hydrographs

CONCLUSIONS

Interesting results have been obtained using a diffusion model of two-dimensional hydrodynamics for overland flow. The synthetically derived S-graphs show striking similarities to the well-known S.C.S. S-graph. The diffusion model affords benefits to the hydrologist in that two-dimensional flow effects are modelled, and data requirements are minimal; topographic elevations and Manning's friction factors.

Further research is on-going to examine the sensitivity of model results to watershed shape factors, and the use of diffusion model in a complete, physically based hydrologic watershed model.

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