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STUDENT CHAPTER

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USING A TWO-DIMENSIONAL DIFFUSION DAM-BREAK MODEL IN ENGINEERING PLANNING

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ABSTRACT

A simplification of the two-dimensional (2-D) continuity and momentum equations is the diffusion equation. This simpler dynamic model of a 2-D floodplain affords the hydraulic engineer a means to quickly estimate dam-break effects for diverging flow. To investigate its capability, the numerical model using the diffusion approach is applied to a hypothetical failure problem of a regional water reservoir. The model is based on an explicit, integrated finite-difference scheme, and the floodplain is simulated by a popular home computer which supports 64K FORTRAN. Though simple, the 2-D model can simulate some interesting flooding effects that a 1-D full dynamic model cannot.

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2. Orange County Environmental Management Agency

INTRODUCTION

One approach to studying flood wave propagation is to simply estimate a maximum possible flowrate and route this flow as a steady state flow through the downstream reaches. This method is excessively conservative in that all effects due to the time variations in channel storage and routing are neglected.

A better approach is to rely on one-dimensional (1-D) full dynamic unsteady flow equations (e.g. St. Venant eqs.). Some sophisticated 1-D models include more terms and parameters to account for complexities in prototype reaches which the basic flow equations cannot adequately handle. However, the ultimate limit of the 1-D model can only be broken by extending into two-dimensional (2-D) realm. There are some 2-D models employing full dynamic equations. Among them, one particularly aimed at flood flow analysis is by Katopodes and Strelkoff (3). Attendant with the increased power and capacity of 2-D full dynamic models, are the greatly increased boundary, initial, geometry and other input data and computer memory and resources requirements as well as computational effort. It is often claimed that the extra computational cost and effort in using a more sophisticated model is quite negligible compared with the total modeling cost and effort in the 1-D realm, the parallel in the 2-D realm seems to be premature at present.

A 2-D diffusion hydrodynamic model described in this paper appears to offer a simple and economic means for fast estimation of dam-break effects for diverging flood flow. It can be operated by a small home computer and yield respectable results. The study object here is to estimate the flood plain resulting from a hypothetical failure of the Orange County Reservoir in the City of Brea, California.

ONE-DIMENSIONAL DIFFUSION MODEL FOR DAM-BREAK FLOODS

Generally, the 1-D flow is modeled wherever there is no significant lateral variation in the flow. Land (4,5) examines four such dam-break models in their prediction of flooding levels and flood wave travel time, and compares the results against observed dam failure information. Ponce and Tsivoglou (6) examine the gradual failure of an earth embankment (caused by an overtopping flooding event) and present a detailed model of the total system: sediment transport, unsteady channel hydraulics, and earth embankment failure. Although many dam-break studies involve flood flow regimes which are truly two-dimensional (in the horizontal plane), the 2-D case has not received much attention in the literature. In addition to the aforementioned model of Katopodes and Strelkoff (3), which relies on the complete 2-D dynamic equations, Xanthopoulos and Koutitas (7) use the diffusion model to approximate a 2-D flow field. The model assumes that the flood plain flow regime is such that the inertia terms are negligible. In a 1-D model, Akan and Yen (1) also use the diffusion approach to model hydrograph confluences at channel junctions. In the latter study, comparisons of model results were made between the diffusion model, a complete dynamic wave model solving the total equation system, and the basic kinematic wave equation model. The comparisons between the diffusion model and the dynamic wave model were good for the study cases, showing only minor discrepancies.

MODEL ACCURACY IN PREDICTION OF FLOOD DEPTHS

In order to evaluate the accuracy of the proposed diffusion model in the prediction of flood depths, Land's model (4,5), referred in the preceding section, which solves 1-D full dynamic equations by an implicit finite scheme is identified as the USGS dynamic model K-634, has been used for comparison purposes. The study approach was to compare predicted flood depths for various channel slopes and inflow hydrographs using the above two models.

From the study (2) it is seen that the diffusion model provides estimates of flood depths that compare very well to the flood depths predicted from the K-634 model. Differences in predicted flood depths are less than 3-percent for the various channel slopes and peak flow rates considered.

In the following sections, the development of a 2-D dam-break flood model will be described. The model is based on a diffusion scheme in which gravity, friction, and pressure forces are assumed to dominate the flow equations. Earlier, Xanthopoulos and Koutitas (7) employed such an approach in the prediction of dam-break flood plains in Greece. Good results were also obtained in their studies applying the 2-D model to flows that are essentially of 1-D in nature. In the following, a finite difference grid model is developed which equates each cell-centered node to a function of the four neighboring cell nodal points.

MATHEMATICAL DEVELOPMENT FOR TWO-DIMENSIONAL MODEL

The set of (fully dynamic) 2-D unsteady flow equations consist of one equation of continuity

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial H}{\partial t} = 0 \qquad (1)$$

and two equations of motion

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_x^{\,2}}{h}\right) + \frac{\partial}{\partial y}\left(\frac{q_x q_y}{h}\right) + gh\left(S_{fx} + \frac{\partial H}{\partial x}\right) = 0 \qquad (2)$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial}{\partial y}\left(\frac{q_y^{\,2}}{h}\right) + \frac{\partial}{\partial x}\left(\frac{q_x q_y}{h}\right) + gh\left(S_{fy} + \frac{\partial H}{\partial y}\right) = 0 \qquad (3)$$

in which q_x, q_y are flow rates per unit width in the x,y-directions; S_{fx}, S_{fy} represent friction slopes in x,y-directions; H, h, g stand for, respectively, water-surface elevation, flow depth, and gravitational acceleration; and x,y,t are for spatial and temporal coordinates.

The above equation set is based on the assumptions of constant fluid density with zero sources or sinks in the flow field, of hydrostatic pressure distributions, and of relatively uniform bottom slopes.

The local and convective acceleration terms can be grouped together such that Eqs. 1, 2, and 3 are rewritten as

$$m_z + \left(S_{fz} + \frac{\partial H}{\partial z} \right) = 0, \ z = x,y \tag{4}$$

where m_z represents the sum of the first three terms in Eqs. (2) and (3) divided by gh. Assuming the friction slope to be approximated by steady flow conditions, the Manning's formula in the U. S. customary units can be used to estimate

$$q_z = \frac{1.486}{n} h^{5/3} S_{fz}{}^{1/2}, \quad z = x,y \tag{5}$$

Equation 5 can be rewritten as

$$q_z = - K_z \frac{\partial H}{\partial z} - K_z m_z, \quad z = x,y \tag{6}$$

where

$$K_z = \frac{1.486}{n} h^{5/3} \Big/ \left| \frac{\partial H}{\partial S} + m_S \right|^{1/2}, \quad z = x,y \tag{7}$$

The symbol S indicates the flow direction which makes an angle $\theta = \tan^{-1} (q_y/q_x)$ +x-direction.

Values of m are assumed negligible by several investigators (1,2,7), resulting in the simple diffusion model,

$$q_z = - K_z \frac{\partial H}{\partial z}, \quad z = x,y \tag{8}$$

The proposed 2-D flood flow model is formulated by substituting Eq. 8 into Eq. 1,

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} = \frac{\partial H}{\partial t} \tag{9}$$

NUMERICAL MODEL FORMULATION (GRID ELEMENTS)

For uniform grid elements, the numerical modeling approach used is the integrated finite difference version of the nodal domain integration (NDI) method. For grid elements, the NDI nodal equation is based on the usual nodal system shown in Fig. 1. Flow rates along the boundary Γ are estimated using a linear trial function assumption between nodal points.

For a square grid of width δ,

$$q\Big|_{\Gamma_E} = - \left(K_x\Big|_{\Gamma_E}\right) \left(H_E - H_C\right) \Big/ \delta \tag{10}$$

where

$$K_x\Big|_{\Gamma_E} = \begin{cases} \left(\dfrac{1.486}{n} h^{5/3}\right)_{\Gamma_E} \Bigg/ \left|\dfrac{H_E - H_C}{\delta \cos \theta}\right|^{1/2} ; & \bar{h} > 0 \\ 0 ; \quad \bar{h} \le 0 \text{ or } \left|H_E - H_C\right| < 10^{-3} \end{cases} \tag{11}$$

In Eq. 11, h and n are both the average of the values at C and E, i.e. $h = (h_C + h_E)/2$ and $n = (n_C + n_E)/2$. Additionally, the denominator of K_x is checked such that K_x is set to zero if $|H_E - H_C|$ is less than a tolerance such as 10^{-3} ft.

The model advances in time by an explicit approach

$$\underset{\sim}{H}^{i+1} = \underset{\sim}{K}^{i}\ \underset{\sim}{H}^{i} \tag{12}$$

where the assumed input flood flows are added to the specified input nodes at each timestep. After each timestep, the conduction parameters of Eq. 11 are reevaluated, and the solution of Eq. 12 reinitiated. Using grid sizes with uniform lengths of one-half mile, timesteps of size 3.6 sec were found satisfactory. Verification of the 2-D model is given in Hromadka (2). Because the two-dimensional model is explicit in time-advancement, home computers can be used to execute the simulation.

APPLICATION OF THE 2-D MODEL TO A HYPOTHETICAL DAM FAILURE

Orange County Reservoir: Orange County Reservoir, which was constructed during 1940-41, is located approximately 2.5 miles northeast of the City of Brea in Orange County. It has a storage capacity of 212 acre-feet and serves primarily as a regulating facility on the Orange County Feeder. This reservoir also provides water storage which can permit continued service for the cities of Fullerton, Anaheim, and Santa Ana when the northerly portion of the Orange County Feeder is shut down.

Flood Plain Discretization and Parameters: Using current USGS topographic quadrangle maps (photo-revised, 1981), a 500-foot grid control volume area-averaged ground elevation was estimated based on the topographic map. A Manning's roughness coefficient of n = 0.040 was used throughout the study. (Canyon reaches, n = 0.030; grassy plains, n = 0.050.)

Modeling Assumptions: In addition to those already stated, other major assumptions made in this study are: (1) Friction resistance is modeled by Manning's formula. (2) All in-situ storm drain systems provide negligible draw off of the dam-break flows. This assumption accommodates a design rain storm in progress during the dam failure. (3) All canyon damming effects due to culvert crossings provide negligible attenuation of dam-break flows. This assumption is appropriate due to the simultaneous occurrence of a design storm and due to deposition of sediment transported from the failing earth dam. (4) The reservoir failure yields an outflow hydrograph shown in Fig. 3.

Dam Failure Mode: The Orange County Reservoir is an earth dam lined along the interior with concrete. In the event of a failure, an erosion process will allow the escape flow rate to increase gradually rather than suddenly as would occur for a rigid structure. To estimate a probable peak outlet flow, Q_p, an iteration method is used until a balance between the estimated outlet hydrograph Q_p is made to the resulting flow rate as a function of the remaining stored waters. The ultimate outlet geometry is assumed to be a V-shaped massive failure with side slopes at a 45-degree incline. Flows are then based on critical depth, with a free outlet to the steep downstream canyon reaches.

Based on the above assumptions, the outlet flowrate for a ponded depth h (feet) is given (for the ultimate dam-break failure geometry) by $Q_p = 2.472h^{2.5}$ cfs. The reservoir rating curve relating basin depth to volume is given in Table 1.

Table 1. Assumed Orange County Reservoir Volume and Dam-Break Outflow

Depth (ft)	Volume (af)	Q_p (Dam-Break) (cfs)	Depth (ft)	Volume (af)	Q_p (Dam-Break) (cfs)
2	4.6	14	10	26.4	782
4	9.5	79	20	62.0	4,420
6	14.8	218	30	108.4	12,200
8	20.3	447	40	166.7	25,000

For the assumed outlet hydrograph shape width Q_p occurring at 20-minutes after dam-failure, the volume drained by time 20-minutes is given by $V_d = 0.01377Q_p$ (acre-feet). The estimate of Q_p is provided by the iteration shown in Table 2.

Table 2. Dam-Break Q_p Estimation (20-Minute Peak Time)

Assumed Depth (ft)	Q_p[1] (cfs)	Volume Drained[2] (af)	Volume Left[3] (af)	Depth (ft)
20.0	4,420	60.9	151.0	37.0
26.0	8,520	117.3	94.7	27.5
26.5	8,936	123.1	88.9	26.0

Notes: 1: $Q_p = 2.472h^{2.5}$; 2: $V_d = 0.01377Q_p$ af; 3: $V_{left} = (212 - V_d)$af

DAM-BREAK FLOOD PLAIN ESTIMATION

In this study, an Apple IIE with two disc-drives, which supports 64K FORTRAN, is used to execute the 2-D model. The dam-break hydrograph of Fig. 3 was used as the inflow to the grid network of Fig. 2. The resulting flood plain is shown in Fig. 4. Also shown in this figure is comparison of the flood plain to a previous study (Metropolitan Water District of Southern California, 1973). The main differences in estimated flood plains is due to the dynamic nature of the diffusion model which accounts for the storage effects due to flooding, and the attenuation of a flood wave because of 2-D routing effects.

CONCLUSIONS

A 2-D diffusion dam-break model is used to estimate the flood plain resulting from a hypothetical failure of the Orange County Reservoir north of the City of Brea, California. From the study, it is concluded that the estimated flood plain is reasonable. It is based upon a modeling approach which accounts for the time varying effects of routing, storage, and an earth dam failure process. The diffusion approach develops an easy-to-use 2-D flooding model, and provides fast, economical and practical estimates of flood depths, widths, and times of travel for planning-study purposes. Because the method is simple, studies can be easily performed using a home computer.

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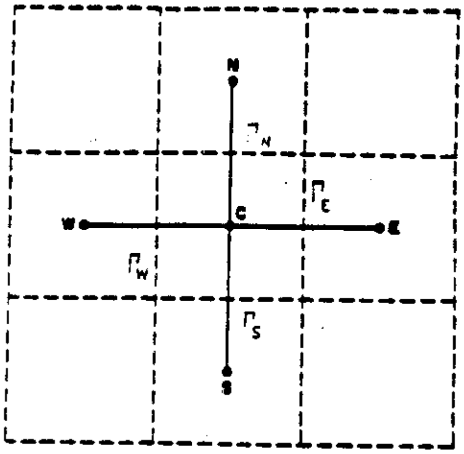

Figure 1. Grid Element Nodal Molecule

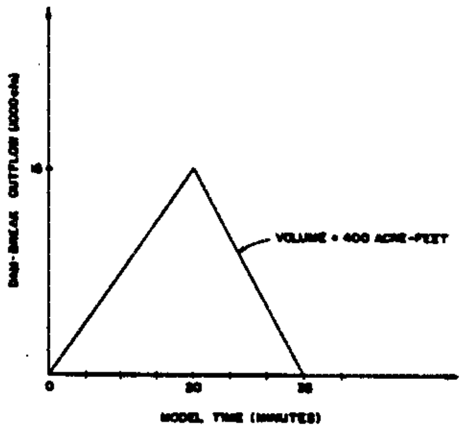

Figure 3. Study Dam-Break Outflow Hydrograph

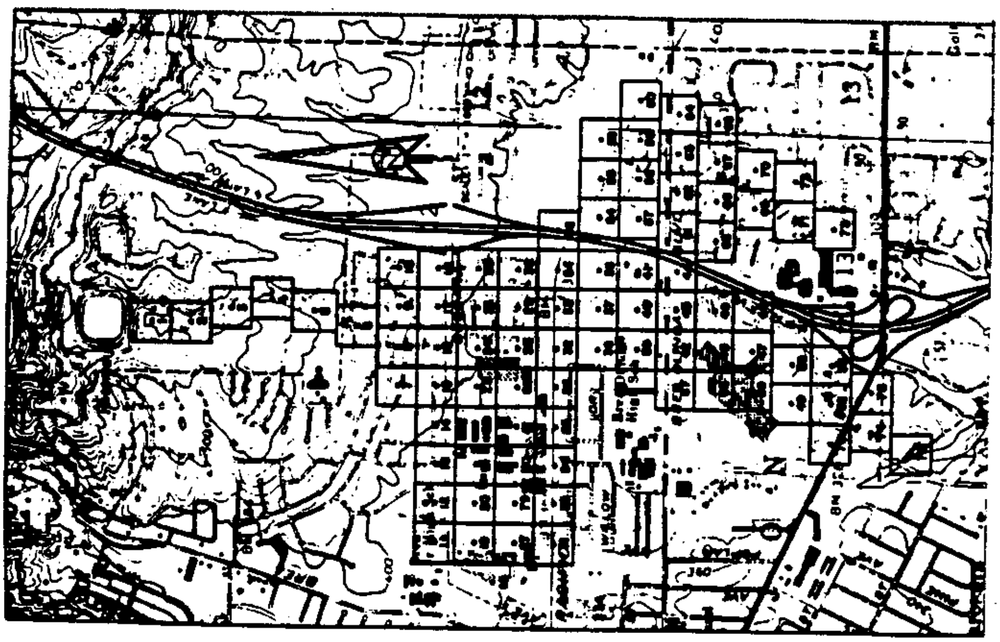

Figure 2. Domain Discretization

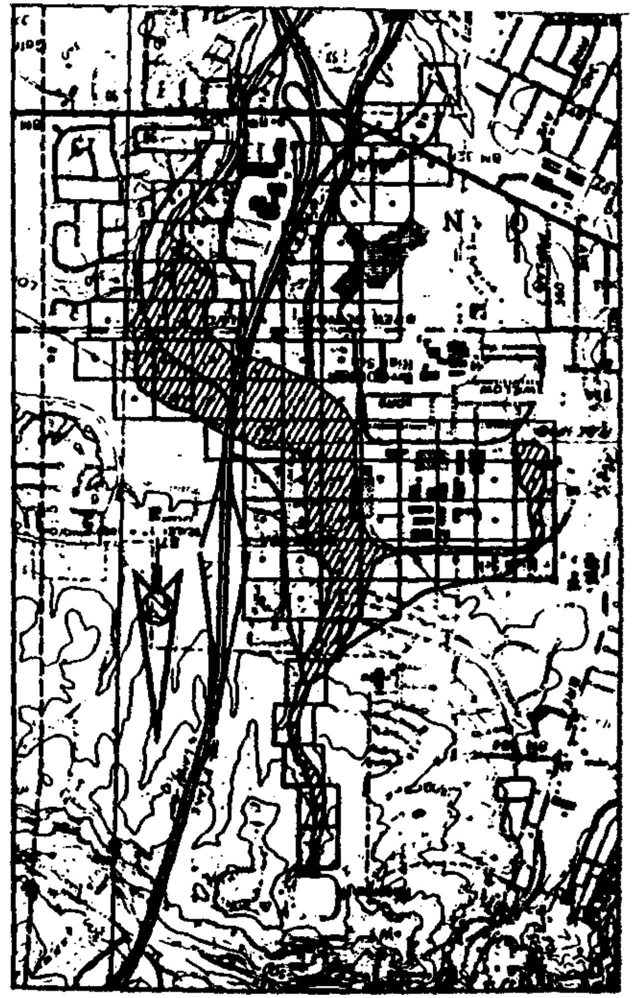

Figure 4. Comparison of Flood Plain Results (Metropolitan Water Study Shown Hatched)

QUESTION by
Violet Chu: How do you describe your collection system and how do you account for backwater?

ANSWER:

1. The collection system is described (modeled) by the overland flow governed by the diffusion formulation of the St. Venant equations.

2. Backwater is modeled (inherently) based on the most current nodal depths. It is important to note that a major feature of the diffusion equation over kinematic routing (typically used in watershed models such as the USGS Rainfall-Runoff Model) is that flows are not directed to occur in a specified route, and backwater effects are included.

QUESTION by
Martin Price: Regarding the proposed watershed model:

1. What numerical boundary value technique and/or algorithm could be used to model tidal effects?

2. Would the proposed watershed modelling technique have the capability of simulating hydraulic structures (e.g., lift stations, weirs, storage basins, etc.)?

3. Compare and contrast a finite difference solution to a finite element solution for the "DAMBRK" problem including relative numerical errors, solution stability and practicality of developing a "user friendly" computer program?

4. Does the finite element (i.e., triangular domains) description introduce a possibility of producing an ill-conditioned or unsolvable problem? Is there an inherent limitation on local topographic elevation changes?

ANSWER:

1. Tidal effects and other control depths are modeled by specifying the nodal depth at the required node. Because the diffusion model is driven by the water surface gradients, this boundary condition is conveniently accommodated as a function of time.

2. Weirs are modeled as flow through critical depth. Storage basins are handled immediately due to changes in neighboring element elevations (and hence ponding occurs.) Hydraulic structures would require additional submodels such as found in XTRAN. The current diffusion model is still under rapid development and lacks many important hydraulic features.

3. Both the finite-difference and finite element models are unified by the Nodal Domain Integration method. Therefore, answers are similar except for the mass-weightings assigned to nodal points.

   Solution stability is a valid concern with the current model due to the explicit time-advancement used. Consequently, small timesteps are required to ensure stability.

   A "user-friendly" version is a worthwhile endeavor which needs to be considered in the near future.

4. Again, the finite element or finite difference models are unified in this model. However, triangles with unusual diameter ratios (long, narrow triangles) do introduce instability, requiring much smaller timesteps than a model based on regular elements. Elevation changes are modeled as straight-line interpolation, with all the associated limitations in representing topography.