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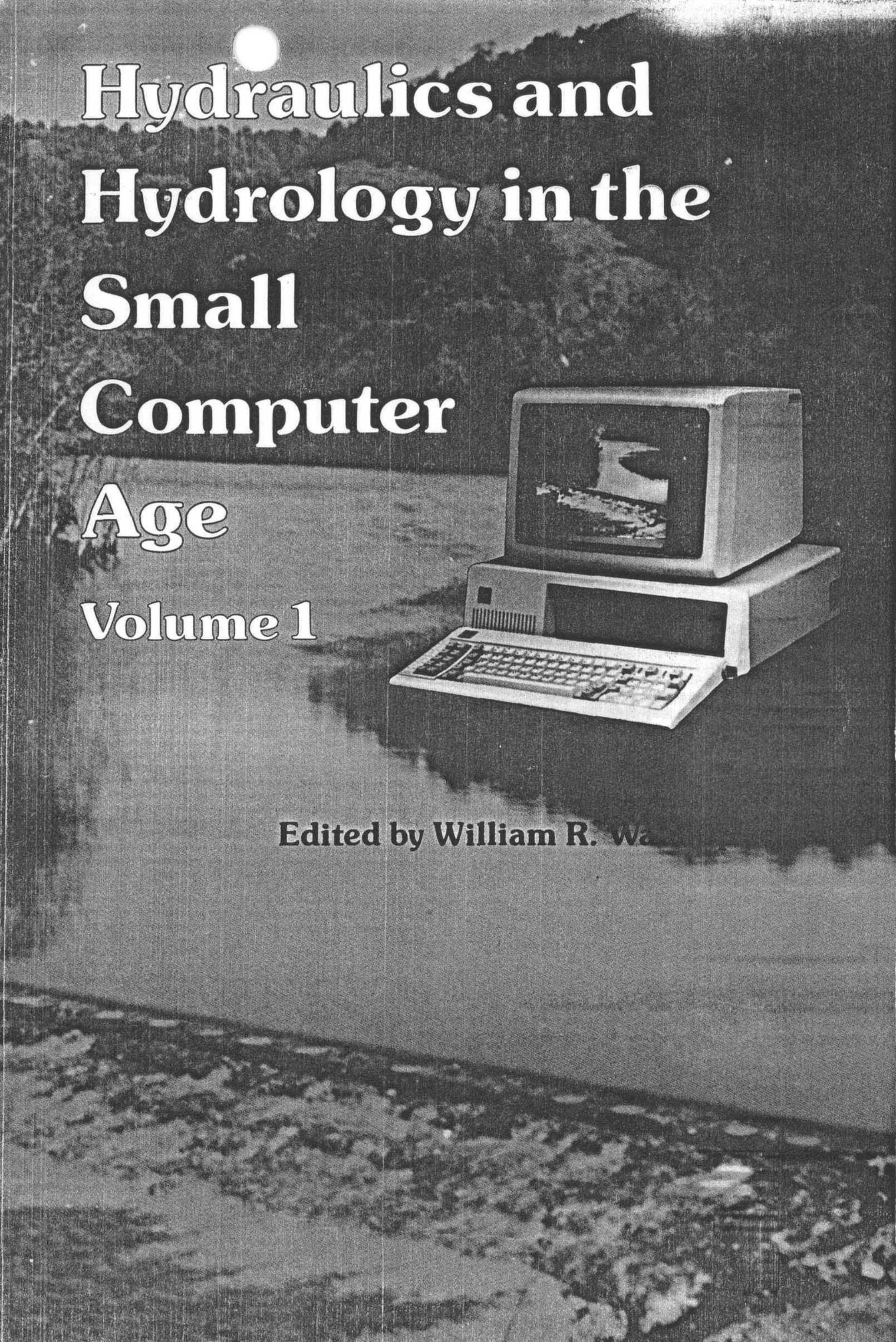

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MODELING HYDRAULIC PROBLEMS USING
THE CVBEM AND THE MICROCOMPUTER

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ABSTRACT: As a new technique in the field of computational hydraulics, the Complex Variable Boundary Element Method (CVBEM) offers an effective and efficient means for modeling two-dimensional potential and related flow problems. The CVBEM is formulated on the basis of the H_n Approximative Function, which is derived from the Cauchy integral formula. Reductions in computational effort and computer resources requirements resulting from the boundary approach, simplification and improvement in the modeling process through complex variable integration, and coupled operation of error analysis and model adjustment, make use of a microcomputer (with visual and graphical capability) both convenient and desirable. The method has been applied to various hydraulic and hydrodynamic problems--surface water, ground water, and other flows--and has proven its accuracy, reliability and usefulness.

INTRODUCTION

In contrast to numerical methods using the domain approach in computational hydraulics, a few methods using the boundary approach have appeared in recent years, such as the Boundary Element Method (BEM) [(1)] and the Boundary Integral Equation Method (BIEM).[(5)] Another new modeling method belonging to the boundary approach has been developed recently. This method is called the Complex Variable Boundary Element Method (CVBEM).[(2)] While most prevailing boundary element techniques deal with real-variable integration along the boundary of the real-variable domain, this new modeling technique operates complex-variable integration along the boundary of the complex domain.[(2,4)] Use of the boundary approach results in a substantial reduction of computational effort as well as computer resources requirements. Recourse to complex variables for boundary integration also leads to a simplified and more efficient modeling process. Because of the above features, the modeling of hydraulic problems by the CVBEM is particularly suitable for a small computer such as a desk-top computer, a microcomputer or a personal computer.

The purposes of this paper are: (1) to outline the CVBEM to those hydraulic engineers who have not been acquainted with the method; (b) to set forth some of the more recent developments in the CVBEM modeling, with particular emphasis on those studied subsequent to Ref. (3); (c) to discuss features of the CVBEM as a new technique in the field of computational hydraulics; (d) to revisit some classical hydrodynamic, fluid mechanic and hydraulic problems using the CVBEM; and (e) to address the use of the microcomputer in CVBEM modeling.

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THE COMPLEX VARIABLE BOUNDARY ELEMENT METHOD (CVBEM)

The Cauchy integral formula

$$\omega(z_o) = \frac{1}{2\pi i} \int_{\Gamma} \frac{\omega(\zeta)\, d\zeta}{\zeta - z_o} \tag{1}$$

relates a function ω at the interior point z_o of a complex region Ω to the integration of the function on a simple closed boundary Γ. The formula (Eq. 1) implies that the value of a function that is analytic in a region is determined throughout the region by its values on the boundary.

In computational hydraulics, the prototype (closed) boundary is discretized into a number of (say, m) straight-line segments, Γ_j. The function along the boundary, $\omega(z)$, is then replaced by a continuous global trial function $G_n(z)$, which is to be integrated along the discretized boundary to yield the approximative function $\hat{\omega}(z_o)$,

$$\hat{\omega}(z_o) = \frac{1}{2\pi i} \sum_{j=1}^{m} \int_{\Gamma_j} \frac{G_n(\zeta)\, d\zeta}{\zeta - z_o} \tag{2}$$

in which G_n, a complex polynomial of degree n, is defined on each boundary element but joined together at each (vertex) node to form a continuous global function. If $n = 0$ or $n = 1$, G_n gives, respectively, a step-wise varied or linearly varied (between adjacent nodes) trial function. The case of $n = 1$, the most frequently used case, is assumed hereafter, unless otherwise stated.

Equation 2 (integrated in the usual positive sense) is referred to as the H_n Approximative Function. If in Eq. 2 all nodal function values, $\omega(z_j) = \phi(z_j) + i\psi(z_j)$; $z_j \in \Gamma_{j-1} \cap \Gamma_j$, are specified (denoted as $\bar{\omega}$, $\bar{\phi}$, $\bar{\psi}$), then $\hat{\omega}(z_o)$ may be readily obtained throughout the region including the boundary (it can be shown that the Cauchy principal values exist at singularities). In many engineering problems, however, only one of the two nodal values $\bar{\phi}$ and $\bar{\psi}$ is given at a node, and the other has to be evaluated. These m unknowns can be solved by setting up m equations as follows.

First, let the interior point z_o in Eq. 2 approach nodal point z_j. Then, because $G_n(z)$ is analytic on the boundary, the integrand of Eq. 2 can be integrated on all boundary elements except Γ_{j-1} and Γ_j, whose intersection point, z_j, is a singularity. But, the Cauchy principal value can be found for the integration along these two elements, thus permitting the completion of the closed boundary integration. By such an approach, an equation of the following form results.

$$\begin{aligned} \hat{\omega}(z_j) &= \hat{\phi}(z_j) + i\hat{\psi}(z_j) \\ &= \frac{1}{2\pi i} \sum_{k=1}^{m} \int_{\Gamma_j} [\alpha_\phi(\bar{\phi}_k, \bar{\phi}_{k+1}, \zeta) + i\alpha_\psi(\bar{\psi}_k, \bar{\psi}_{k+1}, \zeta)] \frac{d\zeta}{\zeta - z_j} \end{aligned} \tag{3}$$

in which α_ϕ, α_ψ are both functions of real variables.

Repeating the same process for all nodal points, $j = 1,2,\ldots,m$; 2m equations, expressed in matrix form below, can be derived.

$$\underset{(m\times1)}{[\hat{\phi}]} = \underset{(m\times2m)}{C_R}\underset{(2m\times1)}{\begin{bmatrix}\bar{\Phi}\\ \\ \bar{\Psi}\end{bmatrix}} \quad ; \quad \underset{(m\times1)}{[\hat{\psi}]} = \underset{(m\times2m)}{C_I}\underset{(2m\times1)}{\begin{bmatrix}\bar{\Phi}\\ \\ \bar{\Psi}\end{bmatrix}} \qquad (4a;4b)$$

Here, C_R and C_I are mx2m matrices composed of real numbers. Inasmuch as one half of the nodal variables ($\bar{\Phi}_j$, $\bar{\Psi}_j$), whether all in one type or a mixture of the two, will be known, the system, Eq. 4, reduces to one half of its original size; i.e. m equations with m unknowns, thus rendering the system determinate.

DEVELOPMENT IN CVBEM MODELING

It is possible to extend the basic CVBEM solution scheme to more complicated physical conditions and more difficult hydraulic subjects. A number of developments have been made to advance CVBEM modeling techniques in various aspects.

Solution Techniques: There are two schemes to solve the system of m equations (reduced from Eq. 4); one by collocating explicitly the known nodal values, assuming here $\bar{\Phi}$ is known and $\bar{\Psi}$ is unknown, $[\bar{\Phi}] = C_R\{\bar{\Phi}, \bar{\Psi}\}$, or collocating implicitly the unknown nodal values, $[\bar{\Psi}] = C_I\{\bar{\Phi}, \bar{\Psi}\}$. Around these two basic schemes or variations of them, various numerical tools and techniques have been developed and applied or are currently under testing.

Trial Functions: The solution accuracy of the potential function by the boundary element approach heavily depends on the type of trial functions. Although G_1 trial function (linear global trial function) has been the main tool in the CVBEM modeling, higher order trial functions (G_n function, $n>1$) are also being explored. Other improvements include the use of variable-definition trial function, i.e., using more than one kind of trial function within a boundary element; for instance, a combinatory use of constant and linear function within Γ_j.

Upper Half Plane Boundary Value Problems: The CVBEM application can be extended to a complex field with non-closing boundary by introduction of the Schwarz-Christoffel transformation. This will provide access to solution of some intriguing hydrodynamic problems.

Poisson Equation: Solutions of physical problems described by the Poisson equation $\nabla^2\phi(x,y) = f(x,y)$ can be achieved by finding a particular solution to the equation, then solving the corresponding Laplace equation $\nabla^2\phi(x,y) = 0$ by the CVBEM, and finally superposing the two solutions.

Source and Sink Problems: Fluid flows including sources and sinks are approached by superposing the classical (analytical) complex-variable solutions of sources and sinks and CVBEM solutions of background flows and boundary conditions.

Nonhomogeneous Domain: A nonhomogeneous domain can be divided into subregions each of which is considered homogeneous. The CVBEM is then applied to each subregion, and the interface boundary conditions are satisfied by iterative use of the approximative boundary approach.

Gradually Varied Unsteady Flow: Some classes of very gradually varied unsteady flow can be modeled by the CVBEM, such as soil-water phase-change problems. Potential flow solution is performed at prescribed time intervals with renewed boundary conditions at each time step.

Axisymmetric Flow: Because axisymmetric flow on a plane is essentially a one-dimensional (1-D) flow, adding the vertical dimension permits extension of the CVBEM to three-dimensional (3-D) space.

In addition to the foregoing CVBEM developments and advancements, a considerable amount of study has been made on error analysis in CVBEM modeling. The extent of the study, the significance of the CVBEM error analysis, and its impact on the model refinement make this topic worthy of separate treatment.

CVBEM APPROXIMATION ERROR

If $\omega(z)$ is the solution of the boundary-value problem (Laplace equation) in a closed complex region, an error function is defined on this region by $e(z) = \omega(z) - \hat{\omega}(z)$. The approximative function errors are due to the incorrect basis functions assumed on each Γ_j. Extensive studies of error analysis and methods of reducing $e(z)$ on the boundary have been made. Error criteria are often derived for proposed trial functions by finding the maximum bound $|M|$ on the boundary Γ, say occurring in Γ_j, and developing mathematical expressions in terms of $|M|$, min $|z|$ ($= D_j$), etc. Sensitivity analysis is also conducted, sometimes including error effects at one location induced by an error at another nodal point.

To improve the trial function and reduce integration errors, several techniques can be experimented with, such as adding nodal points in the vicinity of the maximum nodal-value error, using an iteration technique, increasing the degree of the basis polynomial, applying a variable-definition trial function, and so forth. The CVBEM offers highly useful methods of examining approximation error which aid in better $\hat{\omega}(z)$ function development. These methods include: (a) examining and correcting errors of known boundary conditions, (b) comparing and adjusting the approximative boundary constructed from the computational results to match with the true boundary, (c) weighting the error by plotting error area along the boundary for more effective error correction, and (d) dealing with the total error, $|G(z) - \hat{\omega}(z)|$, for inclusion of error contribution for both the potential and stream functions.

In short, error analysis in CVBEM modeling is more than just a tool for model assessment for accuracy and reliability as in many other numerical models; it is an inseparable part of model development.

CHARACTERISTICS OF THE CVBEM

From what has previously been stated, the CVBEM model is basically for 2-D potential flow simulation. It is a generalization of the Cauchy integral formula into a boundary integration method. Some of the important features of the CVBEM are: (1) the generated approximative functions are analytic and exactly satisfy the 2-D Laplace equation throughout the enclosed region; (b) the integration of the boundary integrals is exact; (c) the approximation is made only at the boundary; i.e., the foregoing three features signify that $\hat{\omega}(z)$ is exact on $\Omega\cup\hat{\Gamma}$, but $\hat{\Gamma}$ is an approximation of Γ and $G_n(z)$ is an approximation of $\omega(z)$ on Γ;-- *for these reasons, the CVBEM is often classified as a semi-analytical method*; (d) mathematical means can be devised to evaluate approximation error, which can be visually illustrated; (e) the CVBEM model, with its extremely high accuracy, can be used to calibrate or verify domain-approach numerical models; and (f) it is particularly suitable for microcomputer model development, implementation, and operation.

MODELING OF HYDRAULIC PROBLEMS

Scores of hydrodynamic and hydraulic problems can be described by potential theory and thus may be approached, solved, or modeled by the CVBEM.

Ideal-fluid Flow: Many two-dimensional ideal-fluid flows in the classical hydrodynamics are tractable by the CVBEM. These include problems that were solved analytically with the traditional complex variable methods, including conformal mapping, the Schwarz-Christoffel transformation, sources and sinks, and so forth. The scope of approachable problems is further extended to those that previously defied analytical solutions such as those with geometrical shapes of the flow boundary too complex to reduce algebraically to a tractable form, or those with flows solvable only by flow net technique, the relaxation method, or iterative schemes.

Some examples of recent modeling activity include description of a flow net for steady flow of an infinite fluid around a cylinder, flow nets for flow over a spillway and under a sluice gate, the flow pattern over a mound of an arbitrary shape, the flows around a corner and around a bend, and flows involving, or being influenced by, sources and sinks.

Groundwater Flow: For flows in which Darcy's law is applicable, the CVBEM is, of course, applicable and very useful. As in the real-variable boundary element methods, subsurface flows were among the first to be explored with the CVBEM. A number of CVBEM models have been developed in this field ranging from Darcian flows, sources and sinks, seepage flow beneath a dam, contaminant transport and others.

Some examples of recent modeling are: steady advective contaminant transport, injection well-discharge well problems, nonhomogeneous-density flows, and axisymmetric flows.

Others: Heat flow, Fickian diffusion, soil-water phase change, thawing/freezing around underground pipes, and other types of potential flow can be and have been studied.

USE OF MICROCOMPUTERS IN CVBEM MODELING

In view of those CVBEM features as mentioned above, namely, the reduced computational effort and diminished computer resources demands, a simple and efficient modeling process, and the need for coupled and interctive operations between error analysis and model adjustment, a small computer with a CRT display unit or a monitor, such as a desk-top computer, a microcomputer, or a personal computer (home computer), is quite suitable for modeling by the CVBEM.

As a matter of fact, the development of the CVBEM model was initiated with a microcomputer in mind. Many programs and routines have been written on this basis. These programs are designed for and are capable of simulating a large variety of hydraulic problems, some of which have been identified in the preceding section. Various computer output formats, including computer graphics, have been designed and implemented on microcomputers and their display screens. These facilities enable the user to evaluate computational errors and the agreement of approximative boundaries, and make the results visible in both graphical and tabular forms for direct interactive model adjustment.

Improvement and advancement of model capability is continuously being made. Because of the limited software availability in the average home computer, the more basic FORTRAN IV (FORTRAN 66) is used in all programming. (In the initial stage of the model development an Apple II with 64K memory was used. For those examples given in Ref. (3) only average 40% of the maximum core memory was needed.) However, in accordance with expanding microcomputer capability and its double role as a terminal to a large computer, more sophisticated versions using FORTRAN 77 are also being developed. An added benefit of adopting FORTRAN 77 is the movement toward language standardization, inasmuch as earlier home computers and microcomputers relied on numerous dialects. These new programs are well structured, are better in both style and efficiency, incorporate many new features of FORTRAN 77, are more lavishly commented, and, above all, contain the latest CVBEM model developments.

CONCLUDING REMARKS

A new modeling method called the Complex Variable Boundary Element Method (CVBEM) has recently been developed. The CVBEM is programmed on the basis of the H_n Approximative Function, which is derived from the Cauchy integral formula. Substantial reduction in computational effort and computer resources requirements through the boundary approach, and simplification and efficiency improvement in the modeling process due to the use of complex variables for boundary integration, make the CVBEM particularly suitable for adapting to a microcomputer.

Through numerous applications to 2-D potential flows in hydraulics, modeling by the CVBEM is found to be useful, efficient, accurate and reliable for such classes (potential flow) of problems in surface water, ground water, interface flows, coupled flows of heat and water, nonhomogeneous flows and others. It is also found that for the interactive operation of error analysis and model adjustment--a special feature of the CVBEM modeling--the use of a microcomputer with its attendant features and facilities is not merely helpful but almost indispensable.

Improvement and advancement of modeling capability in both numerical mathematics and software development need to be, and are being, made constantly, to meet the ever-increasing demands of knowledge in water resources problems and to keep pace with rapidly advancing computer science. Ongoing research should further advance these two aspects of the CVBEM model.

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