UNITED STATES MILITARY ACADEMY
WEST POINT

Applying the Method of Fundamental Solutions to Algid Soil Freezing Fronts

ARL-USMA Tech Symposium

Samuel Humphries, John Lee, William Nevils, Matthew Fauerbach, Kameron Grubaugh, Bryce Wilkins, Dr. Theodore Hromadka

October, 2019

DEPARTMENT OF MATHEMATICAL SCIENCES
"To Infinity and Beyond"
USMA

Problem Introduction

- Goals:
  - Model a 2-D advancing freezing using the method of fundamental solutions (MFS)
  - Upgrade technology
- Quasi steady-state temperatures along boundary

Problem Motivation

- Danger posed to roadway embankments
- Interested in:
  - Locating freezing fronts
  - Estimating heat flux values
- Advantages of MFS:
  - Adaptive
  - More convenient
- Advantages of Matlab

Project Process

- Identify research goals
- Achieve goals
- Conference presentation
- Defend research
- Continue work

UNITED STATES MILITARY ACADEMY WEST POINT

Methodology: Method of Fundamental Solutions

- Goal: Approximate a function over a given domain from known data points i.e. fluid flow, heat transfer
- Known points on boundary P_j (collocations)
- Nodes outside problem domain B_j with coefficients c_j
- Use least squares to solve for c_j coefficients to reduce error
- Approximation function is $\sum_{j=1}^{n} c_j * B_j(P_j)$

DEPARTMENT OF MATHEMATICAL SCIENCES USMA

Methodology: Application to Freezing Fronts

- Consider a heat flow problem defined on domain Ω with an exterior boundary Γ
- Objective equation; solve for *s* (spatial term):

$$L\frac{ds}{dt} = \sum_i Q_{ni}$$

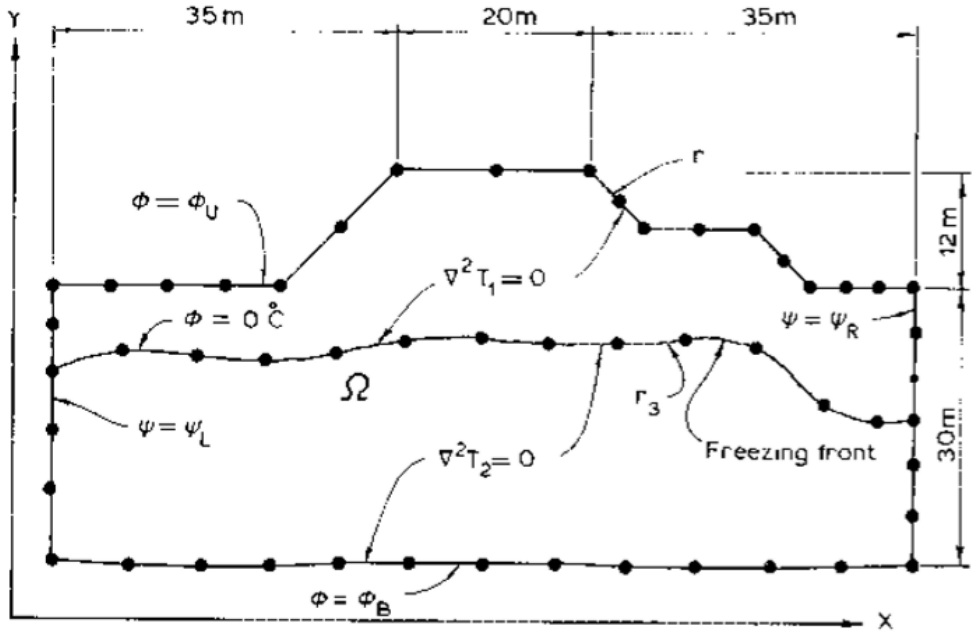

Figure: Roadway Embankment Problem and Nodal Placement

Methodology: Application to Freezing Fronts

- Heat flux across interior given by:

$$\vec{Q} = -K\vec{\nabla}\varphi$$

where φ(P) is the flow potential at any point P.

- For 2-D problems:

$$Q = Q_x + iQ_y = -K\frac{\partial\varphi}{\partial x} - iK\frac{\partial\varphi}{\partial y}$$

- φ(P) satisfies the Laplace Equation:

$$\nabla^2\varphi(P) = 0$$

- Seek to find:

$$u(P) = \frac{\partial\varphi}{\partial x} = \varphi_x(P) \text{ and } v(P) = \frac{\partial\varphi}{\partial y} = \varphi_y(P)$$

where $P \in \Omega$.

Methodology: Application to Freezing Fronts

- Approximation Function:

$$\varphi_n(c, B; P) = \sum_{j=1}^{n} c_j \log|B_j - P| \quad P \in \Omega$$

- Choose c_j and B_j to minimize least squares functional

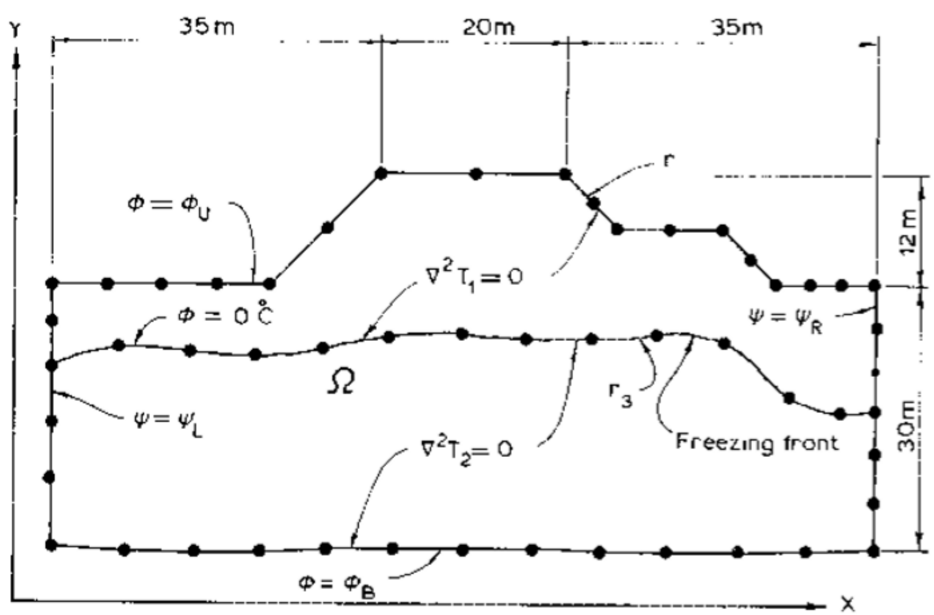

Figure: Roadway Embankment Problem and Nodal Placement

Methodology: Application to Freezing Fronts

- Velocity Components:

$$u(P) = u_n(c, B; P) = \frac{\partial \varphi_n}{\partial x} = \sum_{j=1}^{n} c_j \frac{(x - a_j)}{|B_j - P|}$$

$$v(P) = v_n(c, B; P) = \frac{\partial \varphi_n}{\partial y} = \sum_{j=1}^{n} c_j \frac{(y - b_j)}{|B_j - P|}$$

- Objective Equation; solve for *s:*

$$L\frac{ds}{dt} = \sum_i Q_{ni} = \sum_i \left(-K\frac{\partial \varphi_{ni}}{\partial x} - iK\frac{\partial \varphi_{ni}}{\partial y}\right) =$$

$$\sum_i \left(-Ku_{ni}(P) - iKv_{ni}(P)\right)$$

Anticipated Results

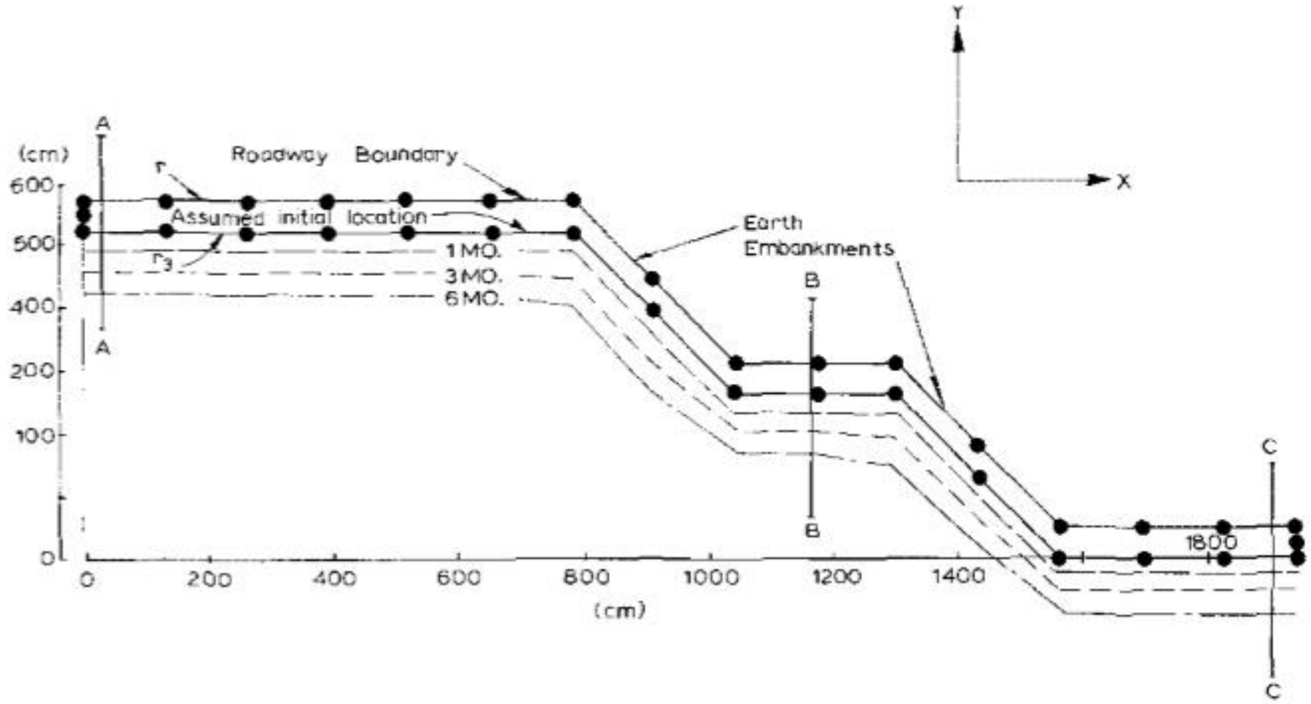

Figure: Estimated Freezing Front Locations

Follow-on Work

The Conversion

- Upgrade technology to Matlab
  - Improve computational efficiency
  - Include MFS in conjunction with Complex Variable Boundary Element Method (CVBEM)

Acknowledgments

We would like to thank Dr. Hromadka for his continued support and funding on this research topic as well as Bryce Wilkins and Kameron Grubaugh for their help advising our research.

QUESTIONS?