Optimization algorithm to locate nodal points for the method of fundamental solutions

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Introduction

The Method of Fundamental Solutions (MFS) is one method that solves Laplace's equation. Laplace's equation is used to model many types of boundary value problems such as ideal fluid flow, ground water flow, and heat transfer. The MFS utilizes different families of analytic basis functions at source locations exterior the problem domain to approximate the potential on and in the problem domain. The basis functions used for this approximation are traditional source functions. The location for each source is determined by a novel nodal point optimization algorithm.

MFS Foundation

The MFS approximation, $\widehat{\omega}(x,y,z)$, is a linear summation of basis functions of the form,

$$\widehat{\omega}(x,y,z) = \sum_{j=1}^{n} c_j g_j(x,y,z),$$

where $\widehat{\omega}(x,y,z)$ is the approximate solution, c_j is the jth coefficient, $g_j(x,y,z)$ is the jth basis function, and n is the number of basis functions used to approximate the solution.

The basis functions in this research are, $g_j(x,y,z) = \frac{1}{R_j}$, where $R_j = \sqrt{(x-x_j)^2+(y-y_j)^2+(z-z_j)^2}$ and (x_j, y_j, z_j) is the nodal point source location.

Node Location Optimization Algorithm

1. Create a pool of node points located exterior of the problem domain. This set should be in no particular pattern.
2. Create a pool of collocation points located on the problem boundary. These are locations where the potential is known.
3. Create a one node approximation function for each combination of nodes and collocation points. Record the RMSE for each.
4. Select the node and collocation pair the produced the least RMSE error. Remove node and collocation point from test pool.
5. Create a two node model, utilizing the selected pair from step 4 as one of the node pairs.
6. Test all two node approximations for RMSE. The approximation function that results in the least RMSE becomes the best two node model.
7. Repeat steps 4-6 until the number of nodes desired in the model is reached.

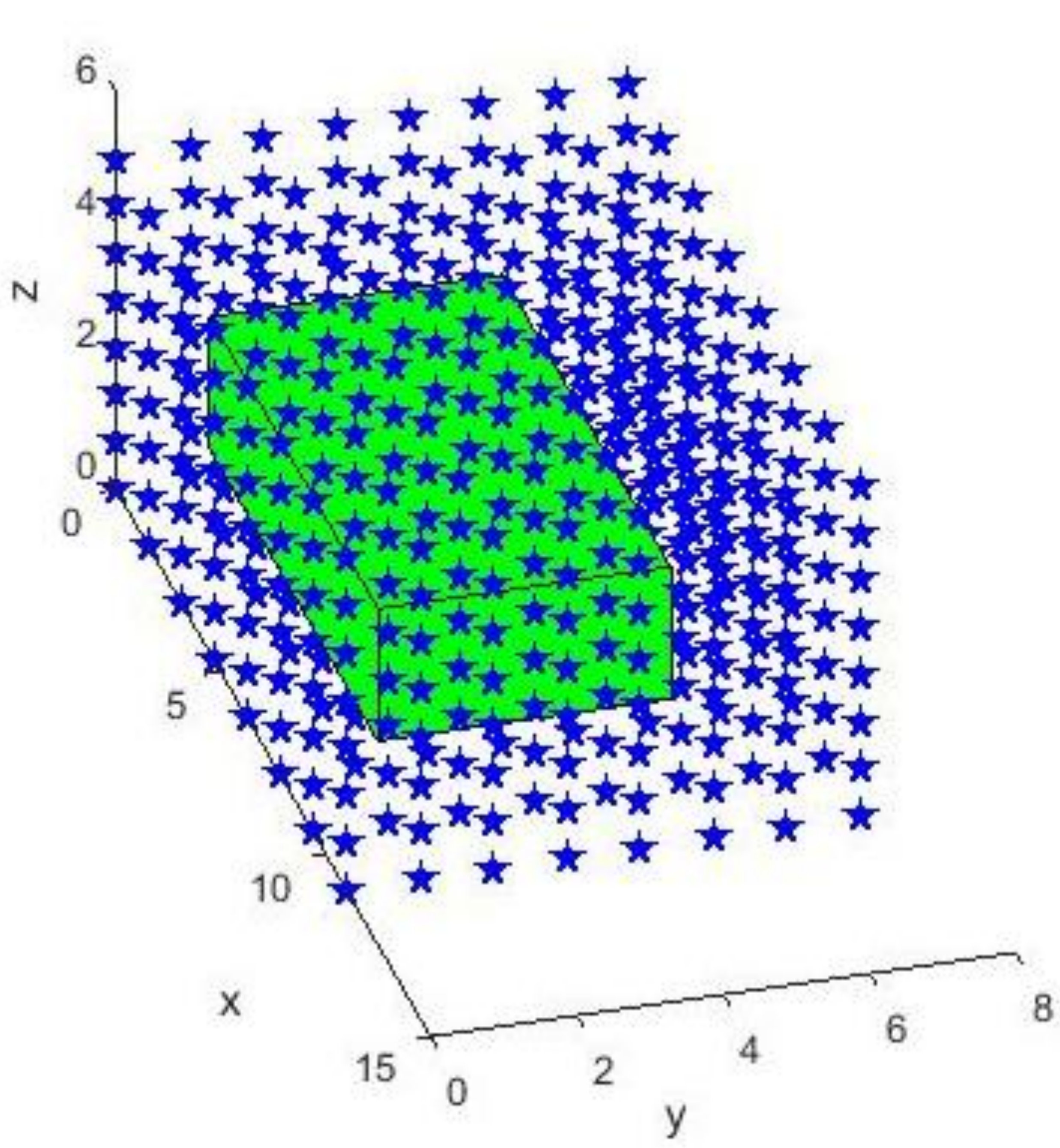

Figure 1: Candidate nodal point locations.

Example Problem

Domain: $\Omega = \{(x,y,z) \in R^3 \mid 1 \le x \le 9, 1 \le y \le 5, \ 1 \le z \le 3\}$
PDE: $u_{xx} + u_{yy} + u_{zz} = 0$; BC: $u(x,y,z) = x^2 + y^2 - 2z^2$

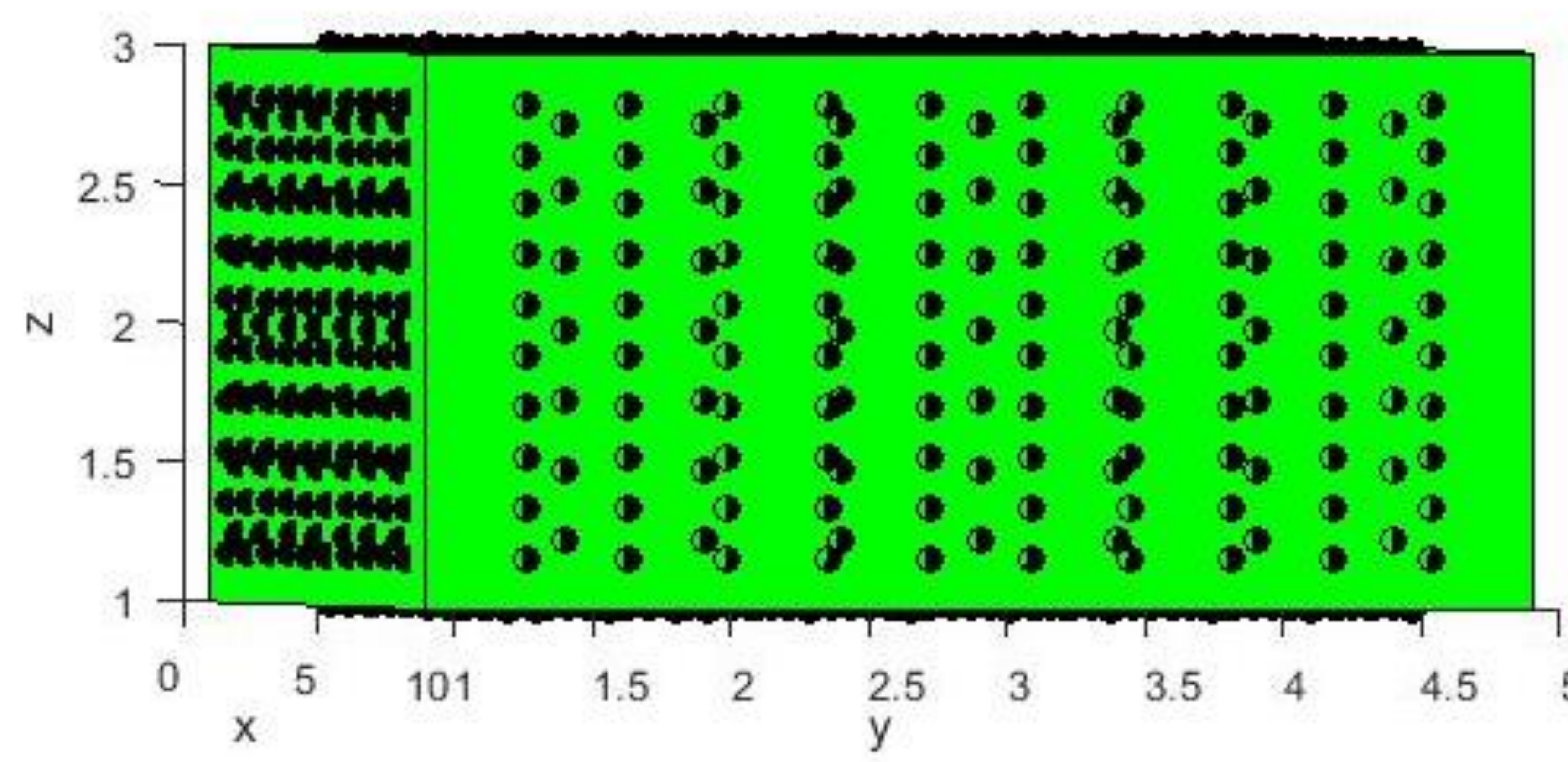

Figure 2: Candidate collocation point distribution

Results

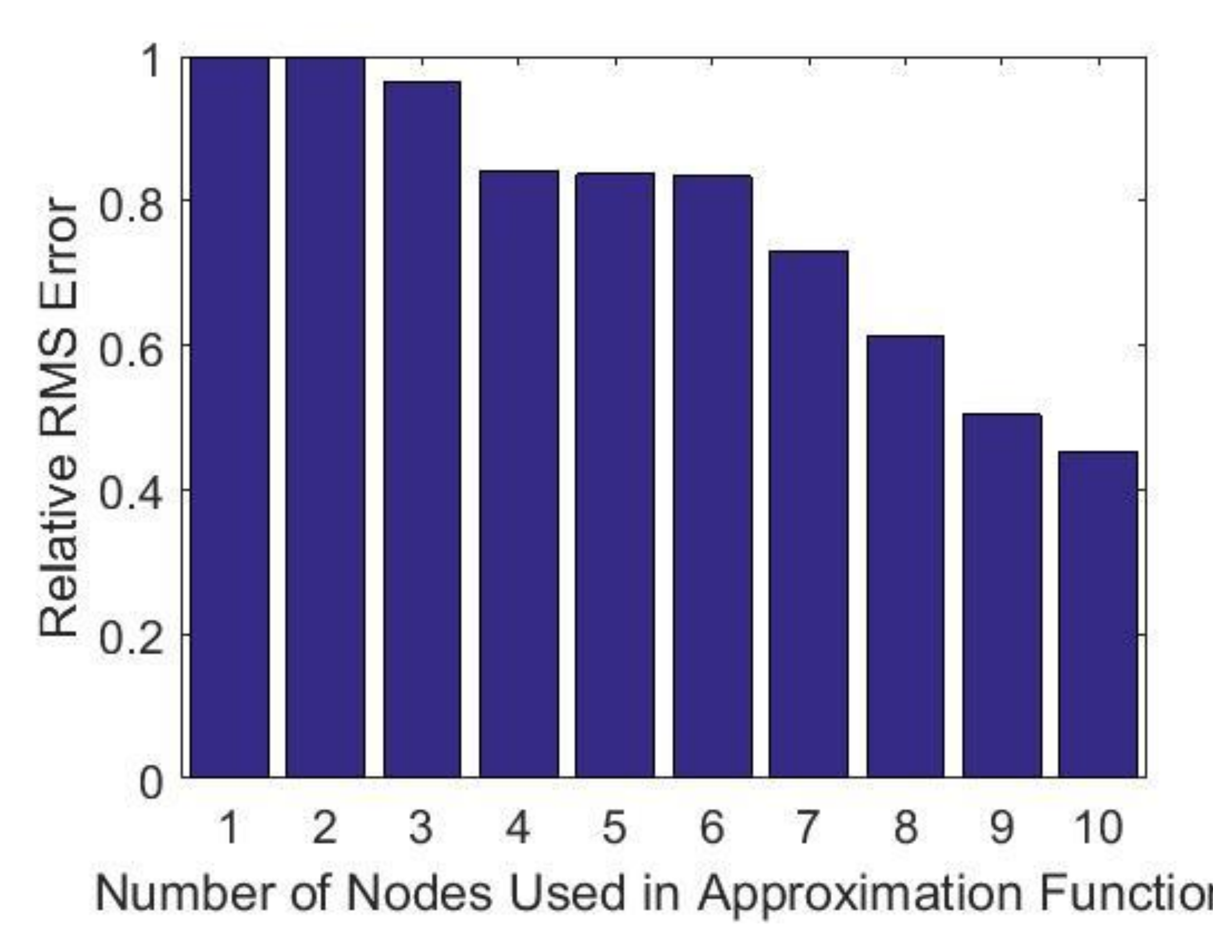

Figure 2: Error comparison utilizing optimum source locations.

Ordered Pair	Col. Location	Node Location	RMS Error
1	(3,3,3)	(0.01,0.01,0.01)	1
2	(1,2,1.5)	(0.01,0.01,2.505)	0.997893
3	(1,2,2)	(0.01,7,0.01)	0.965495
4	(1,3,2.5)	(0.01,3.505,5)	0.840827
5	(1,3,1.5)	(0.01,7,2.505)	0.837547
6	(1,4,2.5)	(0.01,3.505,2.505)	0.83462
7	(3,1,1.5)	(11,3.505,2.505)	0.729709
8	(1,4,1.5)	(5.505,3.505,5)	0.612669
9	(7,1,2.5)	(0.01,0.01,5)	0.503244
10	(3,1,2)	(11,3.505,0.01)	0.452256

Table 1: Optimum node and collocation point ordered pairs.

Conclusion

The location of modeling nodes are treated as additional degrees of freedom in the computational modeling effort to reduce computational error in in achieving problem boundary conditions. As expected, the use of the presented algorithm improved computational modeling accuracy. The over-arching conclusion can be made that the associated increase in available degrees of freedom provides significant additional opportunities in reducing computational error.

Acknowledgments

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