A Computational Model of Groundwater Mound Evolution Using the Complex Variable Boundary Element Method and Generalized Fourier Series

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STUDENT ARTICLE

Abstract

Overview

An emerging issue in topics of computational engineering mathematics is the general use of groundwater computational models for solving problems in groundwater flow. Although computational groundwater models are useful for understanding and visualizing groundwater flow, computational errors can often result in significant design errors. In this work, an important problem in groundwater flow planning and design is considered, namely, the modeling of unsteady groundwater flow in a groundwater mound located below a groundwater recharge basin.

The actual problem setting is recast into two prototype problems that are suitable for computational assessment. The two test problems are mathematical formulations of unsteady groundwater flow and are designed to assess (1) the ability of computational groundwater models to develop descriptions of the potential surface (groundwater surface), and (2) the ability to develop the associated streamline vector trajectories. Testing the accuracy of computational groundwater models may lead to increased confidence in computational results and may possibly facilitate identifying computational modeling issues before the modeling outcomes move forward towards design and planning actions.

In addition to providing and explaining the two proposed test problems, we also propose a numerical solution technique for this problem. The numerical scheme uses the standard procedure of resolving the global initial-boundary value problem into a steady-state component and a transient component. The steady-state component is modeled by application of the Complex Variable Boundary Element Method (CVBEM), and the transient component is modeled by application of an approximation function that is a linear combination of basis functions that are the product of a two-dimensional Fourier sine series and an exponential function. The accuracy of this coupled procedure is proposed as a benchmark standard for comparison with other computational models.

Test Problem A: Potential Surface Modeling

The computation of a changing water table due to the creation of a groundwater mound is important since such mounds can rise sufficiently such that the mound reaches the base of the recharge basin and interferes with the recharge process. Another possible complication (among others) is the mobilization of pollutants that are stored in subsurface soils, and the transport of these pollutants to other locations.

In Test Problem A, the steady-state flow situation is modeled as flow around a 90-degree bend. The initial condition is specified as the superposition of the background flow (steady-state flow) and a two-dimensional single-peaked mound geometry. This geometry is picked for its simplicity, however, it is noted that a wide variety of geometric shapes can be specified. For model times $t>0$, the mound is continuously reducing in spatial coverage due to downward and lateral drainage of the stored groundwater in the mound. This flow regime is associated with difficult-to-solve spatial distributions for both the potential and streamline functions, and hence provides a possibly interesting case where the analysis must predict the dissipation of the flow regimes corresponding to the specified background flow superimposed with the groundwater mound.

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Test Problem B: Streamline Development

Another situation often encountered in groundwater flow analysis is the identification of sources of groundwater contamination. Computational models are typically developed and applied to the flow situation to estimate flow streamlines from which possible locations for sources of groundwater contamination are identified. The analysis procedure presented in Test Problem B uses the flow field streamline function trajectories to work upstream along streamlines toward possible locations for contaminant sources. Here, the initial condition is specified as the superposition of the background flow (which is planar in this case) and a two-dimensional single-peaked mound. It is assumed that a groundwater recharge mound is present, however the steady-state flow situation is a plane rather than fluid flow around a 90-degree bend.

Assumptions

The problem domain is assumed to be homogeneous and isotropic, thus reducing the need for parameter specification in the problem formulation. An initial condition for the groundwater surface is defined that is approximately the situation encountered, and includes a groundwater mound that is draining into the groundwater regime located below the mound. The flow situation involves unsteady flow of the groundwater mound resolving itself towards the steady-state conditions of the problem. Therefore, the conceptual problem can be decomposed into two components: (1) an unsteady flow component of the groundwater mound reducing in vertical extent over time, with flow moving into the underlying groundwater regime modeled by a two-dimensional Fourier sine series, and (2) a steady-state component representing the considered test situation after the groundwater mound has fully drained into the groundwater regime, which is modeled by the Complex Variable Boundary Element Method procedure using complex variable monomials as basis functions (see Wilkins et al, 2016).

Evaluation Procedure for Common Numerical Methods

Typical groundwater computational models involve thousands, or even more, of finite elements or finite difference grid nodes. Here, the procedure used to test the veracity of such large models is to apply test problems designed to represent realistic and important problems in groundwater flow modeling. Generally, a portion of the target computational model is isolated and the test situation is applied and examined. It is assumed that if the computational model performs adequately for the test situations, that the computational model will perform similarly well for the actual problems at hand.

A Numerical Scheme Using the CVBEM and Two-Dimensional Fourier Sine Series

The numerical scheme proposed in this work involves decomposing the global initial-boundary value problem into two sub-problems, namely, a steady-state component and a transient component. The steady-state component is governed by the Laplace partial differential equation (PDE) and the transient component is governed by the diffusion partial differential equation. The steady-state component is solved by application of the CVBEM and the transient component is solved by application of a linear combination of basis functions that are the product of a two-dimensional Fourier sine series and an exponential function. It can be shown that the global solution, which is the sum of the modeling outcomes from the steady-state and transient sub-problems, satisfies the governing PDE.

The boundary conditions of the global BVP are satisfied by the CVBEM approximation of the steady-state solution. Thus, the unsteady component is specified with homogeneous boundary conditions. The initial condition of the transient problem is specified as the difference between the global initial condition and the CVBEM approximation of the steady-state situation. Consequently, when the two outcomes are summed, the global initial condition is appropriately modeled.

Numerical Test Modeling Results

The spatial domain is identical for both test situations considered in this work. For the planar steady-state of Test Problem B, the specified groundwater mound is superimposed upon a steady-state condition of a geometric plane. Such a geometry may be appropriate for many problems even outside of the testing situation because within a relatively small distance of the mound, the groundwater regime may be significantly linear. For Test Problem A, the background groundwater flow is assumed to be similar to a flow field in a 90-degree bend. This more difficult flow regime is solved by the complex variable quadratic polynomial function and the potential is readily determined to be the real part, or x^2-y^2 in the first quadrant of the two-dimensional coordinate axis plane.

Conclusion

This work shows that test problems can be applied to large-scale groundwater models to check the modeling veracity of the groundwater model utilized. Such tests are a necessary but not sufficient condition in order to develop an accurate model of the problem being assessed.

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