Two- and Three-Dimensional Finite Element Groundwater Flow Models with Isothermal Freezing and Thawing

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1. INTRODUCTION

During the summer, cadets at the United States Military Academy at West Point can choose to participate in an AIAD program. For the project under consideration, the Advanced Individual Academic Development (AIAD) program provides cadets with an opportunity to observe and implement concepts from their course work in computational engineering mathematics with an immersion experience during a summer internship. The AIAD program enables cadets to work side by side with leaders in government and industry both stateside and abroad. The West Point Department of Mathematical-Sciences offers several AIAD opportunities that engage cadets with challenging aspects of topics including involving computational engineering mathematics. In the current project, ongoing work is culminated involving application of the finite element method (FEM) applied towards solving heat and groundwater flow transport in algid climates where soil water freezes and thaws isothermally. A two- and three-dimensional FEM computer program is developed for application towards engineering and planning studies where freezing and thawing of algid soils is of concern.

OBJECTIVES OF PROJECT

The target project is to be completed during the summer 2016 session. The funding for the project and coordination is through the AIAD program at West Point and is an element of the computational engineering mathematics theme at the Department of Mathematical-Sciences. This particular topic involves aspects of both engineering-mathematics and computational geosciences.

The main objectives of this project are fivefold:

(1) provide the theoretical and modeling background used in a two-dimensional and three-dimensional model of heat and soil-water flow, coupled by soil-water freezing and thawing

(2) present the major modeling assumptions used in the computer program, and the various components of the program in order to aid the end-user in the use of the model.

(3) present the necessary parameters and data requirements used in the computer model.

(4) discuss the computer model output product, and methods to interpret the results.

(5) present the data input sequence to the computer model.

PROJECT COMPUTER PROGRAM DOCUMENTATION ORGANIZATION

The Documentation report is organized into 5 chapters and 1 Appendix:

<u>CHAPTER 1.</u> Provides an introduction to the project and report.

<u>CHAPTER 2.</u> Develops the Nodal Domain Integration (NDI) model of two-dimensional soil-water flow in saturated and unsaturated soils.

<u>CHAPTER 3.</u> Develops the NDI model for heat flow in three dimensions. The third dimension is included in this NDI development (an NDI analog for radial coordinates is presented in Appendix A) in order to provide for the extension of the phase change model to other coordinate systems.

<u>CHAPTER 4.</u> Develops the two-dimensional soil-water phase change model used to couple the heat and soil-water flow during soil-water freezing and thawing.

<u>CHAPTER 5.</u> Presents a summary of the two-dimensional freezing/thawing model and the modeling input-output characteristics. Provides documentation to the computer program input requirements.

<u>Appendix</u> Expands upon the heat flow modeling approach developed in chapter 3 to radial coordinates. The theory developed in the appendix applies to both heat and soil-water flow in the selection of mass-lumping factors for use in NDI domain models.

COMPUTATIONAL MATHEMATICAL MODEL DESCRIPTION

A problem of interest is the design and maintenance of roadways constructed in algid climates where freezing and thaw degradation results in collapsing the roadway embankments and possible roadway failure. Analysis of such embankments to estimate the maximum penetration of the freezing front provides information for use in designing mitigation methods to protect from thaw degradation impacts. The typical roadway embankment under study has a two-dimensional geometry as depicted in Figure 1. In Figure 1, the problem domain is discretized into triangular shaped finite elements. For three-dimensional problems, tetrahedrons are the basis elements used in the finite element method ("FEM") model. These tetrahedron elements are subsequently further resolved from modeling "bricks" used in model definition in order to simplify model buildup. Figure 2 shows the partitioning of a triangular finite element into "nodal domains" or, similarly termed "finite volumes (areas)" or "control volumes". The shaded area indicates that portion of the partitioned finite element associated by the phase change algorithm to model node j. The assembly of all associated FEM partitioned areas to node j results in the total nodal domain or control volume associated in the algorithm to node j (see Figure 3). Figure 4 depicts the variation of the state variable over the nodal domain for node j. For modeling soil water phase change within the node j nodal domain, the basis function used is the step function as depicted in Figure 5. Figure 6 shows the occurrence of phase change occurring at both the upper and lower boundary regions of the node j nodal domain. Heat flux estimates are computed using the flux directions as depicted in Figure 7. Figure 7 applies to both the heat transport as well as soil water flow regimes with respect to model node j. Figure 8 defines the computer model outcome estimates for frozen soil water content associates with node k. Sharp interface displacement geometry used in the isothermal phase change algorithm follows the geometry depicted in Figure 9. The computer model calculates volumetric quantities, such as volumetric ice content. The end-user interprets these quantities to develop locations of the evolving freezing front.

COMPUTER PROGRAM SOURCE CODE

The Documentation includes the FORTRAN source code for the computer models. Also provided in this report is the documentation for the program PROTO0 which can be obtained from the U.S. Army Corps of Engineers, Cold Regions and Research Laboratory (CRREL). Program PROTO0 provides a streamlined input setup for typical roadway and highway cross sections and embankments. Problem geometry and mesh generation, as well as initial and boundary conditions are guided input.

REFERENCES

1. Hromadka II, T.V., McInvale, D., Gatzke, B., Phillips, M., Espinosa, B. 2014, A Cumulative Departure Model of the Cryosphere during the Pleistocene. ASCE Journal of Cold Regions Engineering, published online April 2014.

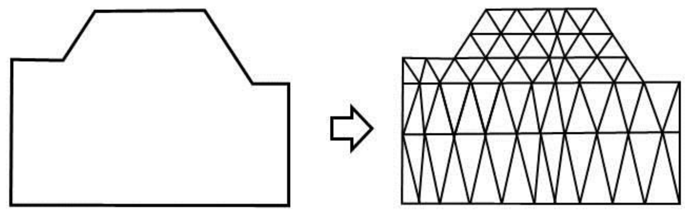

FIGURE 1. DISCRETIZATION OF ROADWAY EMBANKMENT DOMAIN INTO FINITE ELEMENTS.

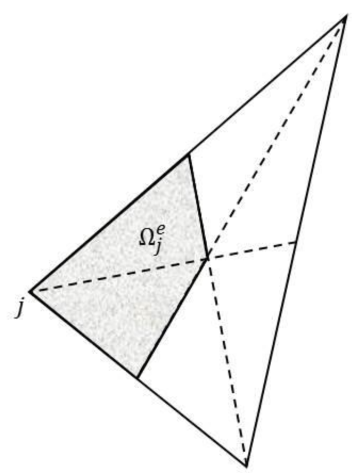

FIGURE 2. PARTITION OF FINITE ELEMENT INTO NODAL DOMAINS OR FINITE VOLUMES (AREAS).

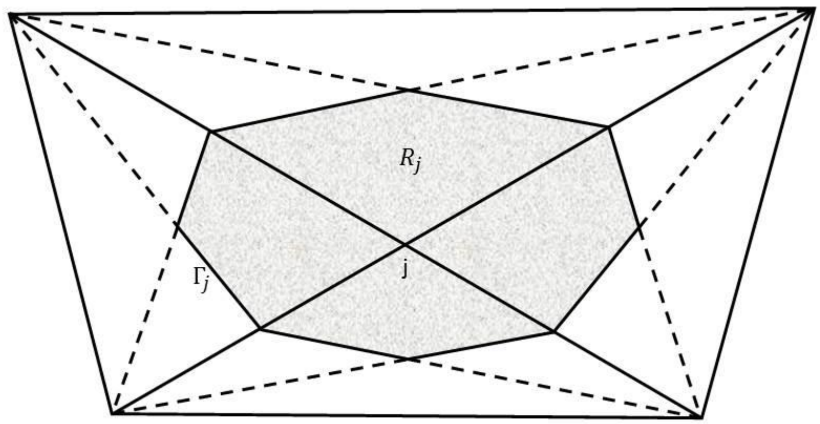

FIGURE 3. NODAL DOMAIN ASSOCIATED TO NODE J.

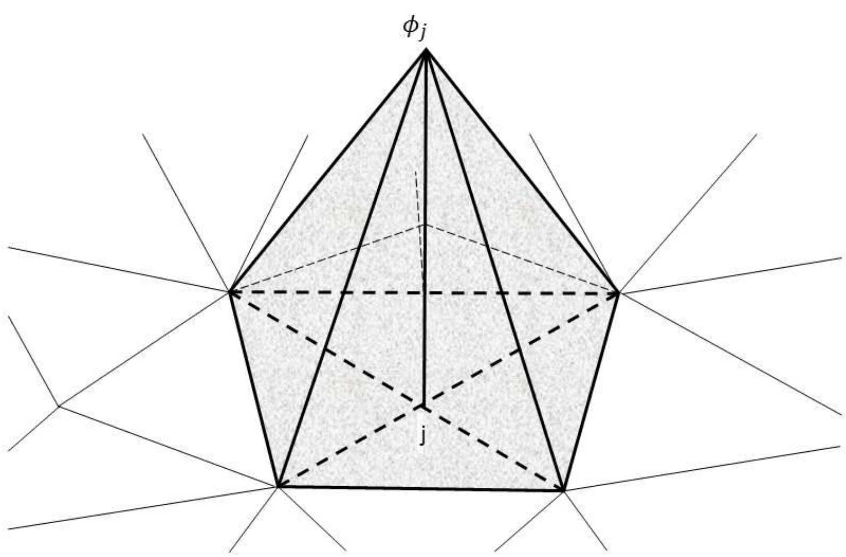

FIGURE 4. NODE J BASIS FUNCTION DEPICTION OF STATE VARIABLE VALUE.

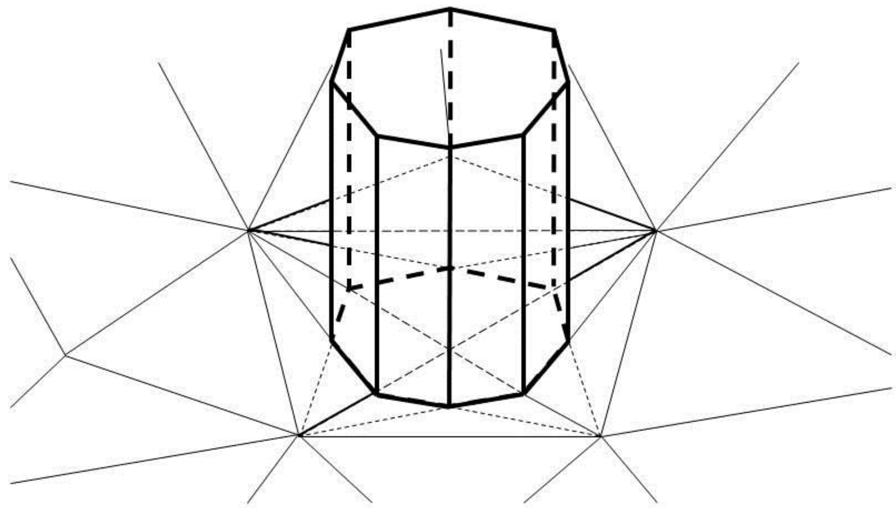

FIGURE 5. NODE J NODAL DOMAIN ICE VOLUME TRACKING.

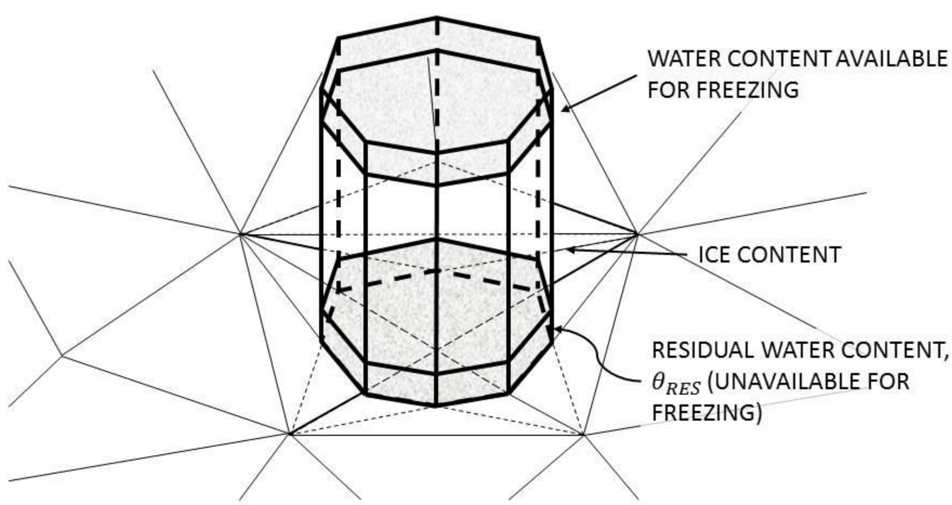

FIGURE 6. NODE J DEFINITION OF PHASE CHANGE ACCUMULATION OF FROZEN SOIL WATER.

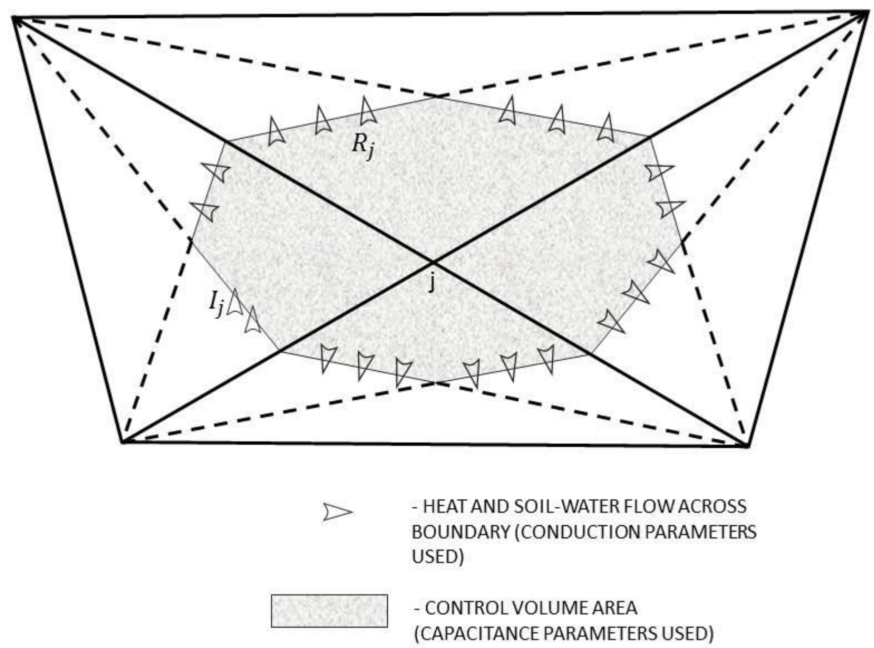

FIGURE 7. NODE J HEAT FLUX TRANSPORT ACROSS NODAL DOMAIN CONTROL SURFACE (BOUNDARY).

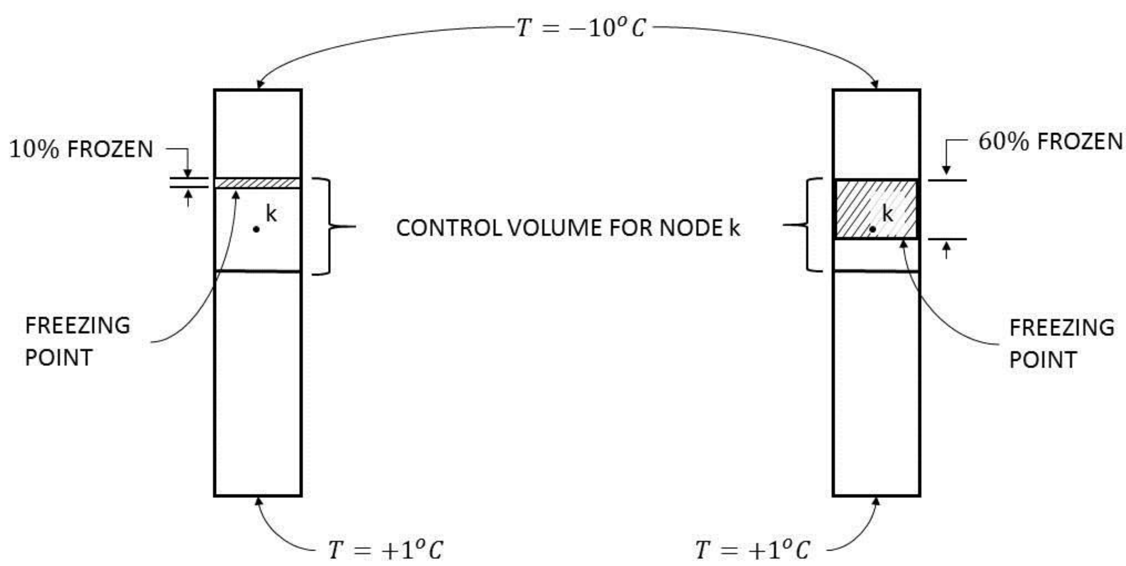

FIGURE 8. PHASE CHANGE ACCOUNTING ALGORITHM DEPICTION.

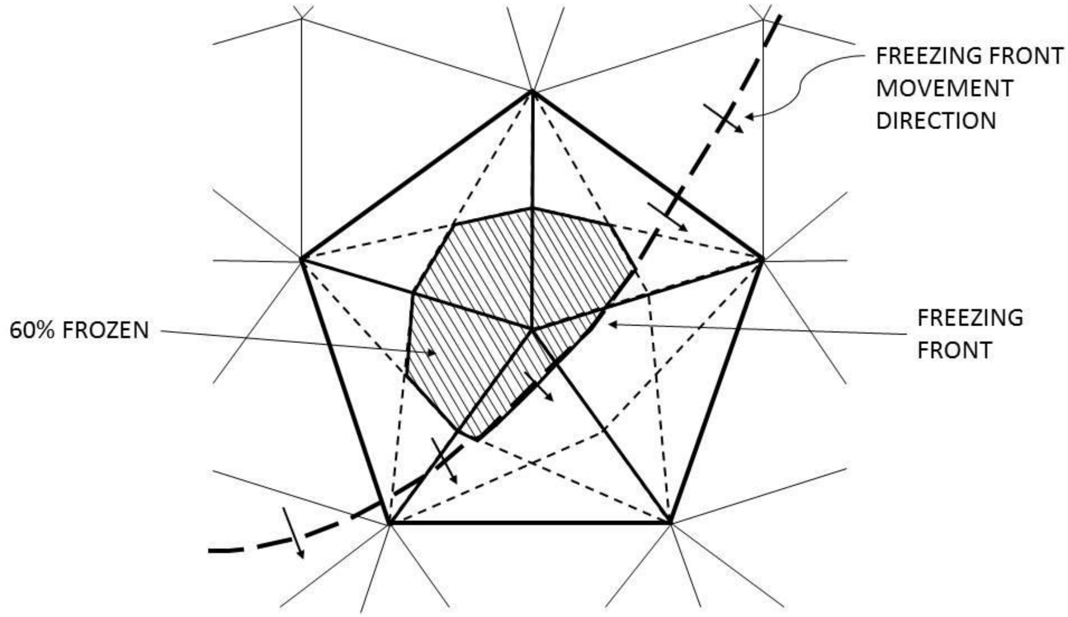

FIGURE 9. PHASE CHANGE MOVING BOUNDARY SHARP INTERFACE DEPICTION OF ALGORITHM.