Real Time Boundary Element Node Location Optimization

Presenter and Author: Samuel Smith

Co-Authors: Baxter, Robert; Menges, Joshua; Dr. Hromadka II, T.V.;

Accepted for Publication in the Journal of Engineering Analysis and Boundary Elements

Abstract

Boundary Element Method (BEM) computer models typically involve use of nodal points that are the locations of singular potential functions such as the logarithm or reciprocal of the Euclidean distance function. Recent research on the types of basis functions used in a BEM approximation has shown that considerable improvement in computational accuracy and efficiency can be achieved by optimizing the location of the singular basis functions with respect to possible locations on the problem boundary and also locations exterior of the problem boundary. To develop such optimum locations for the modeling nodes (and associated singular basis functions), the approach presented here is to develop a Real Time Boundary Element Node Location module that enables the program user to click and drag nodes throughout the exterior of the problem domain. The provided module interfaces with the Complex Variable Boundary Element Method (CVBEM) program, built within computer program Mathematica, so that various types of information flows to the display module as the node is moved, enabling the user to determine "optimum" nodal locations in real time.

Ten Step Boundary Condition Problem:

As a demonstration problem, a ten-step Dirichlet type boundary condition is used that introduces singularities in the boundary conditions at the interfaces between the boundary condition stepped values. Because the CVBEM results in an approximation function that is analytic everywhere except along branch cuts and at nodal points, and furthermore is continuous everywhere except at the branch cuts but is continuous at the nodal points, then the imposed singularities of the boundary condition cannot be matched but will be approximated as a typical Fourier series representation along a stepped target function. This behavior (similar to a Fourier series) is the result of the approximation process involved with the Hilbert Space setting. Figure 2 displays a typical comparison between the ten-stepped problem boundary conditions along the problem boundary and the corresponding CVBEM approximation function evaluated along the problem boundary, for a single node CVBEM approximation function with nodes located as shown in Figure 1. As more nodes are introduced into the CVBEM approximation function, the resulting comparison between CVBEM and given boundary condition values will converge similar to what is seen as more terms are added to a generalized Fourier series approximation (see Figures 3-8). This is evidenced by the decreasing L_2 and L_∞ norms as the number of nodes increases.

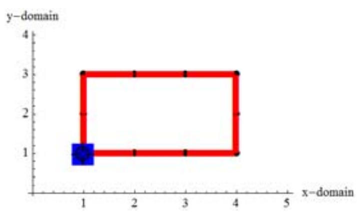

Figure 1: Single Node at the Bottom-Left Vertex

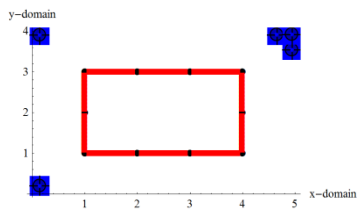

Figure 5: Five Node Model

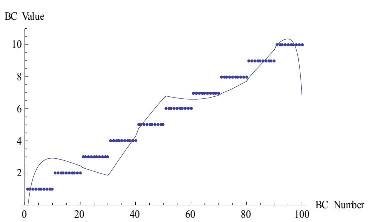

Figure 2: Fit of Function to Boundary Conditions
L_2=86.2094 L_∞=4.75691

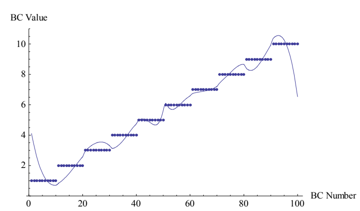

Figure 6: Fit of Function to Boundary Conditions
L_2=48.2625 L_∞=3.38377

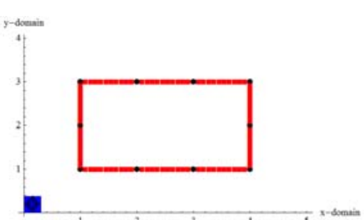

Figure 3: Adjusted Nodal Location

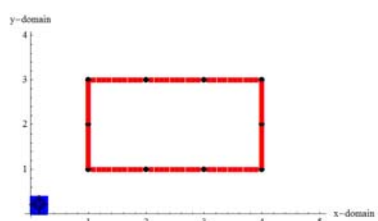

Figure 7: Optimized Clustered 10 Node Model

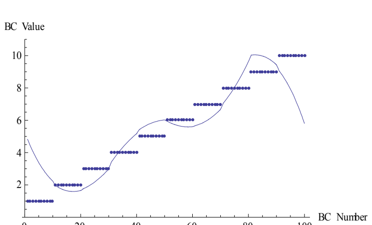

Figure 4: Fit of Function to Boundary Conditions
L_2=99.0174 L_∞=4.18971

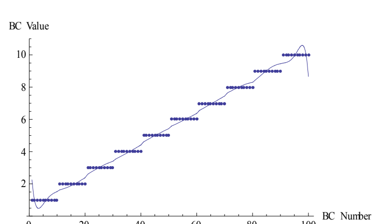

Figure 8: Fit of Function to Boundary Conditions
L_2=26.5259 L_∞=1.26019

"Optimal" Locations

The finalized so-called "optimal" locations are concluded based upon the three types of error measures used in the nodal point location optimization. Although the search for optimal nodal point locations is not at all exhaustive in the current paper, sufficient extent exterior of the problem domain is examined such as to demonstrate the utility of nodal point location optimization using the three approximation error measures considered (graphical, L_2, and L_∞). Of course, other measures of approximation error may result in different concluded optimal locations. Further research is needed to assess better error measures to base the process used for optimizing nodal point locations.

Assessment of a Given Path

The general procedure used in this paper for searching for optimal nodal locations (that reduce all three considered approximation error measures) is to introduce a nodal point upon the problem boundary and then to move the target node into the exterior of the problem domain further and further away until the three error measures appear to not be significantly improving the approximation. Similarly, the target node is also moved into the interior of the problem domain. Once that particular line of search is assessed, the target node is then re-introduced into the CVBEM approximation function, but this time at a different location on the problem boundary, and the previously mentioned procedure repeated. After examining all the considered pathways, the optimal node location is concluded. Then, a new target node may be introduced and the entire process repeated. In the problems considered, movement of the target node did not appear to follow generalized trends such as linearly decreased approximation error versus departure distance from the problem boundary.

LIST OF REFERENCES

[1] C.A. Brebbia and S. Walker, 1980, Boundary Element Method, Pentech Press, London.

[2] T. V. Hromadka and R. J. Whitley, 1998, Advances in the Complex Variable Boundary Element Method, Springer-Verlag, New York, 400 pages.

[3] T. V. Hromadka and G. L. Guymon, 1984, A Complex Variable Boundary Element Method: Development, International Journal for Numerical Methods in Engineering, Vol. 20, 25-37.

[4] T. V. Hromadka, 2002, A Multi-Dimensional Complex Variable Boundary Element Method, WIT Press, Southampton, England, 217 pages.

[5] P.A. Ramachandran, Method of Fundamental Solutions: Singular Value Decomposition Analysis, Communications in Numerical Methods in Engineering, Vol. 18, No. 11, pp. 789-801, November 2002.

[6] T. V. Hromadka and G. L. Guymon, 1984, Complex Polynomial Approximation of the Laplace Equation, ASCE Journal of Hydraulic Engineering, Vol. 110, No. 3, 329-339.

[7] A.W. Bohannon and T.V. Hromadka II, 2009, The Complex Polynomial Method with a Least-Squares Fit to Boundary Problems, Engineering Analysis with Boundary Elements, DOI 10.1016/j.enganabound.2009.02.005.

[8] The Wolfram Technology Guide, 2008 Wolfram Research, Inc., 25 Sep. 2008 at http://www.wolfram.com/technology/guide/.