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BOUNDARY ELEMENT TECHNOLOGY XV

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Extending the complex variable boundary element method of three dimensions

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Abstract

The Complex Variable Boundary Element Method, or CVBEM, is extended to solve three (or higher) spatial dimensions. This new advance breaks down a barrier that has limited the CVBEM to two dimensions. The new 3DCVBEM is easy to apply, and does not require spatial transformation to particular spatial domain shapes. The 3DCVBEM can be applied to arbitrary spatial shapes. In this paper, the mathematical underpinnings are reviewed, and the associated numerical algorithm is considered. Application of the 3DCVBEM to 3D problem shape is considered for a Dirichlet problem.

1 Introduction

The Complex Variable Boundary Element Method [Refs. Hromadka and Guymon [3], Hromadka [4], Whitley and Hromadka [7]], or CVBEM, is a numeric technique used in solving, in an approximation sense, two-dimensional (2D) potential problems or 2D Poisson problems.

Recently, the 2D CVBEM has been extended to three-dimensional (3D) problems (see Hromadka, 2000) [2], and Hromadka and Whitley (2001a,d) [5], [6]. This was accomplished by applying the CVBEM to three coupled projections of the 3D problem domain, in orthogonal 2D planes, and then superimposing the resulting 2D CVBEM solutions. In this paper, the new 3D CVBEM is fully generalized by applying the CVBEM to

Theorem 1

Let f(z) be a function analytic in a disk $D(z_0, r)$, where it is not a polynomial; using Lemma 1 we can suppose that (1) holds. Let Ω be a domain in R^n, with $R^n - \Omega$ connected, which satisfies an external cone condition at each boundary point; and let g(x) be a continuous real-valued function on Γ and $\varepsilon > 0$ be given. There are complex constants c_j and vectors a^j and b^j in R^n, $j = 1,2,\ldots,N$ with

$$a^j \bullet b^j = 0 \text{ and } |a^j| = |b^j| < r, \tag{2}$$

for any r for which (2) implies

$$|a^j \bullet x + i\ b^j \bullet x| < \rho, \tag{3}$$

for all x in $\overline{\Omega}$, so that the function

$$h(x) = Re \sum_{j=1}^{N} c_j f(z_0 + (a^j \bullet x + i\, b^j \bullet x)), \tag{4}$$

defined and harmonic on $\overline{\Omega}$, satisfies

$$|h(x) - g(x)| < \varepsilon \text{ for } x \text{ in } \Gamma. \tag{5}$$

Consequently for all x in $\overline{\Omega}$, h(x) is within ε of the exact solution u(x) to the Dirichlet problem with boundary data g.

The proof of Theorem 1 (Whitley and Hromadka, 2001c) [1], shows that the vectors a^j and b^j appearing in the approximating sum may be taken to have a special form where a has only one non-zero coordinate and b^j is zero in that coordinate, a^j and b^j still being required to have the same length.

Using the above results, the CVBEM can be generalized into three or more geometric dimensions. In this paper, we will focus on the generalized 3-dimensional (3D) approach, and develop the corresponding numerical model to apply the 3D CVBEM to novel test problems.

4 Applications of the 3D CVBEM

In all of the application problems considered, a (small) set of five projection planes were used, all with the same 3D orientation about the problem domain's enclosing sphere. Additionally, although other Dirichlet problems were examined, only a single common Dirichlet problem is presented in this paper for brevity. The problem considered is a 2 source and 1 sink temperature problem where the sources and sink are located closely to the 3D problem boundary in order to more vigorously test the 3D CVBEM. The exact solution to the test problem is T(x,y,z) where

$$\begin{aligned} T(x,y,z) = &-500/[(x-5)^2 + (y-0)^2 + (z+2)^2]^{1/2} \\ &+ 10000/[(x-0)^2+(y+0.1)^2 + (z-5)^2]^{1/2} \\ &+ 100/[(x-0)^2 + (y-5)^2 + (z-6)^2]^{1/2} \end{aligned} \quad (6)$$

and (x,y,z) are the 3D coordinates. The problem boundary conditions are simply the above T(x,y,z) evaluated at the problem boundary's integration points. For each problem geometry, the exact solution results, from (6), and the 3D CVBEM results, are plotted along selected 2D surfaces of the 3D domains or boundaries. For each problem considered, only four CVBEM nodes are used, evenly spaced, on each projection plane; with five projection planes used in the approximation, a total of 20 CVBEM nodes are employed; this small number of nodes is shown to give high accuracy in the example below.

Solid Sphere
For a sphere, of radius 2.5 units, the temperature results can be plotted on the sphere's surface for the "north hemisphere" and the "south hemisphere". Figure 1 shows the sphere's orientation, the integration points, and the locations of the sink and two sources. The exact solution, and the CVBEM approximation, respectively, are depicted on Figures 5a,b, for the north hemisphere. Similar depictions are provided on Figures 5c,d, for the south hemisphere.

Conclusions

The CVBEM has been successfully generalized for solving 3D Dirichlet problems based on new mathematical results. In this paper, the mathematical underpinnings of the 3D CVBEM are reviewed, and several 3D geometries are examined. From the results and the experience gained from the applications, the new 3D CVBEM appears to have considerable promise for research and industrial calculations.

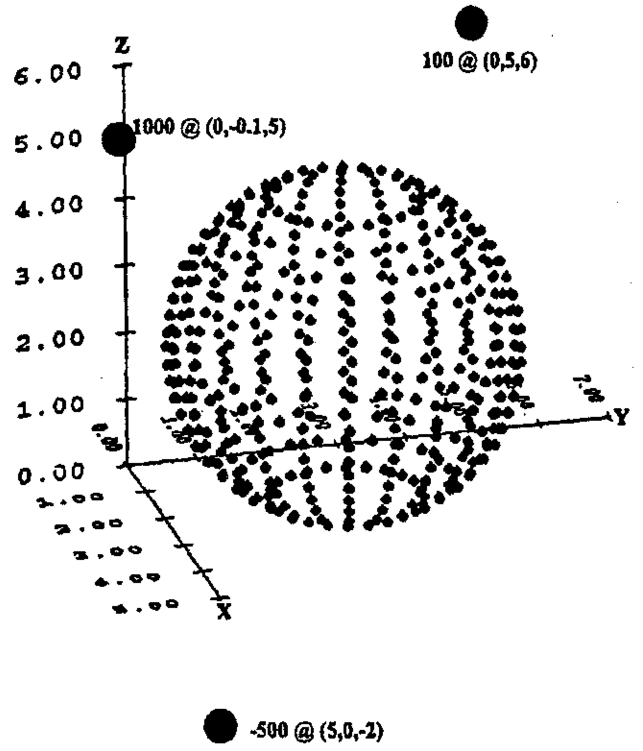


Figure 1: Sphere geometry showing integration points, and sink and sources

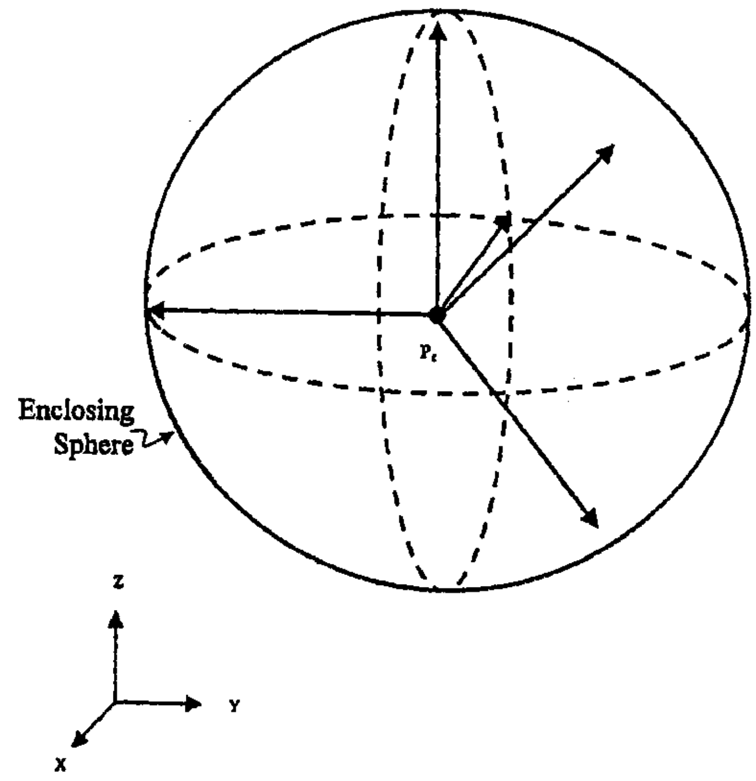
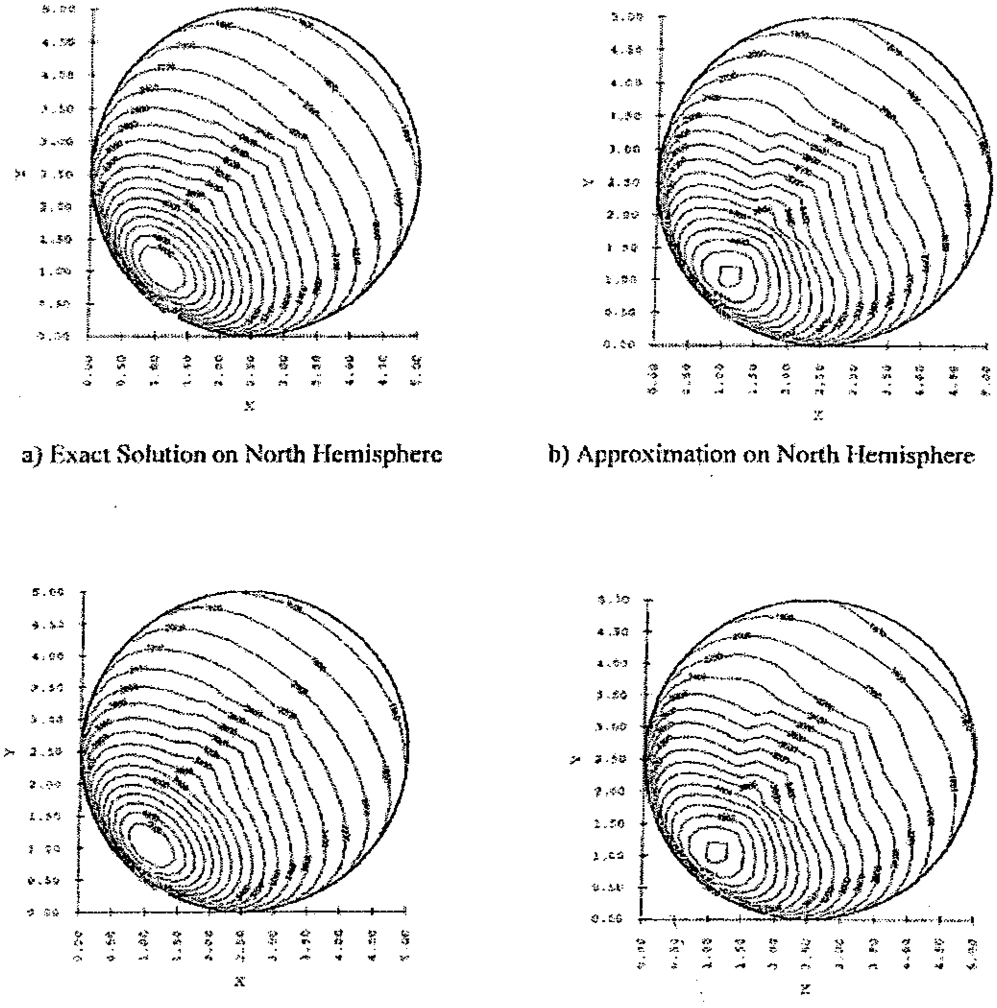


Figure 2: 5-Projection vectors



a) Exact Solution on North Hemisphere

b) Approximation on North Hemisphere

d) Exact Solution on South Hemisphere

e) Approximation on South Hemisphere

Figure 5: Sphere problem with five projection planes, vector displacements from center of sphere: (0., 0., -2.5), (0., -2.5, 0.), (-2.5, 0., 0.), (2.5, 2.5, 2.5) and (-2.5, 2.5, -2.5)