APPROXIMATING THREE-DIMENSIONAL POTENTIAL PROBLEMS ON A SPHERE USING THE COMPLEX VARIABLE BOUNDARY ELEMENT METHOD: APPLICATIONS

T.V. Hromadka II[1] and R.J. Whitley[2]

Key Words: CVBEM, Complex Variable Boundary Elements, three-dimensional approximations, potential functions

Abstract

In the current research, the primary focus is to introduce an extension of the two dimensional (2D) CVBEM to solving potential problems in a three dimensional (3D) sphere geometry. This is achieved by applying the CVBEM to three coupled projections of the 3D sphere, onto orthogonal 2D planes, and then superimposing the resulting corresponding 2D CVBEM solutions. The resulting numerical approximations demonstrate the utility of using a 2D boundary element method, such as the CVBEM, towards solving 3D potential problems with Dirichlet boundary conditions.

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1 Professor of Mathematics, Environmental Studies, and Geologic Sciences, Department of Mathematics, California State University, Fullerton, California 92634

2 Professor, Department of Mathematics, University of California, Irvine, California 92616.

Application A. Three Point Heat Source/Sink

The potential field described by two sources and one sink, all located exterior of a sphere, is studied by the CVBEM. The sphere used has diameter of 5 units with center located at (2.5, 2.5, 2.5). The exact solution to the test problem is included in Figure 8, which also contains a plot of the northern hemisphere temperature isothermals. The 3D application of the CVBEM, using a total of n = 72 nodes, results in the northern hemisphere isothermals and error isothermals shown in Fig. 9. Figures 10 and 11 provide the isothermal plots for the exact solution, the approximation, and the error of approximation, for the southern hemisphere, respectively. All isothermal plots were developed by a general purpose graphics computer program using the test points of Figs. 4 to 7.

Application B. Dirichlet Problem: T=100° in Northern Hemisphere, T=0° in Southern Hemisphere

For the sphere used in Application A, and the above given boundary conditions, another 72 node CVBEM application was developed. The approximation and error isothermals are shown in Fig. 12 for the northern hemisphere, with the corresponding southern hemisphere results shown in Fig. 13. By rotating the sphere 90 degrees, the transitions between the 100-degree and 0-degree boundary conditions is seen as approximated by the same 3D CVBEM model. It is noted that in this model, the boundary conditions are model as linearly decreasing between 100 degrees and 0 degrees across two latitudes of the sphere, Figure 14.

Application C. Triple Product Potential: T=xyz

The potential field of T=xyz was modeled using the same CVBEM layout as used in applications A and B. The corresponding northern hemisphere results are shown in Figs. 15 and 16, with the southern hemisphere results shown in Figs. 17 and 18.

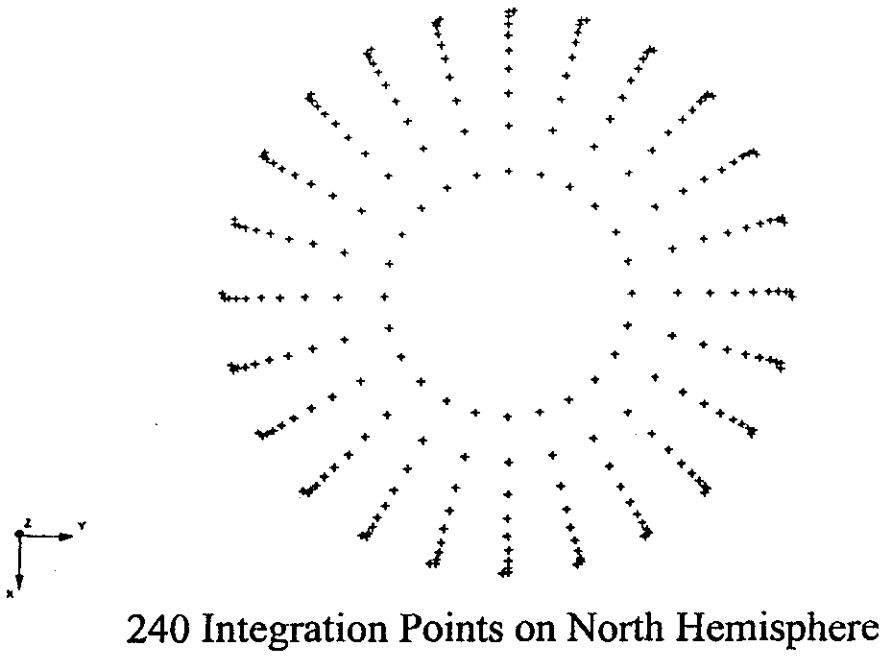


Figure 4. Location of CVBEM integration points on northern hemisphere.

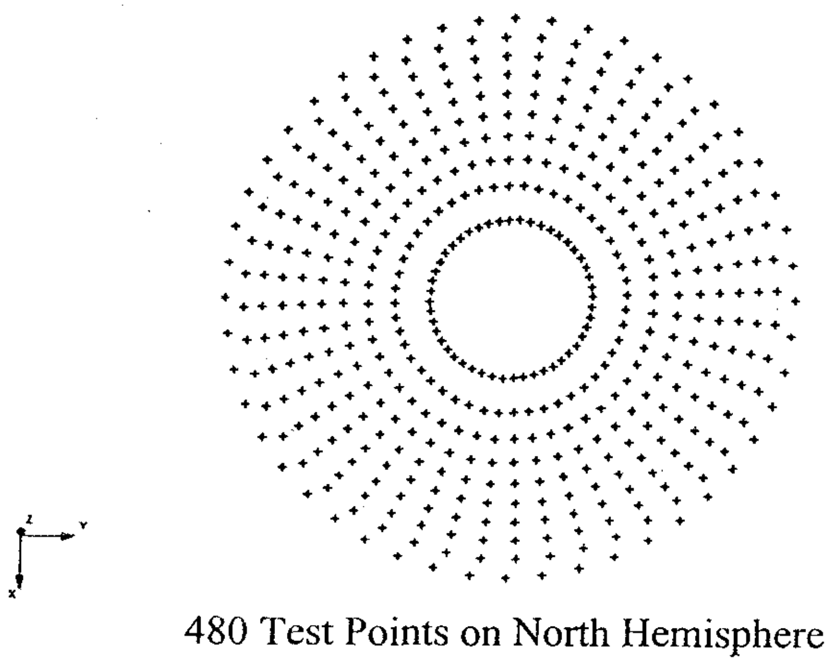


Figure 5. Location of test points on northern hemisphere.

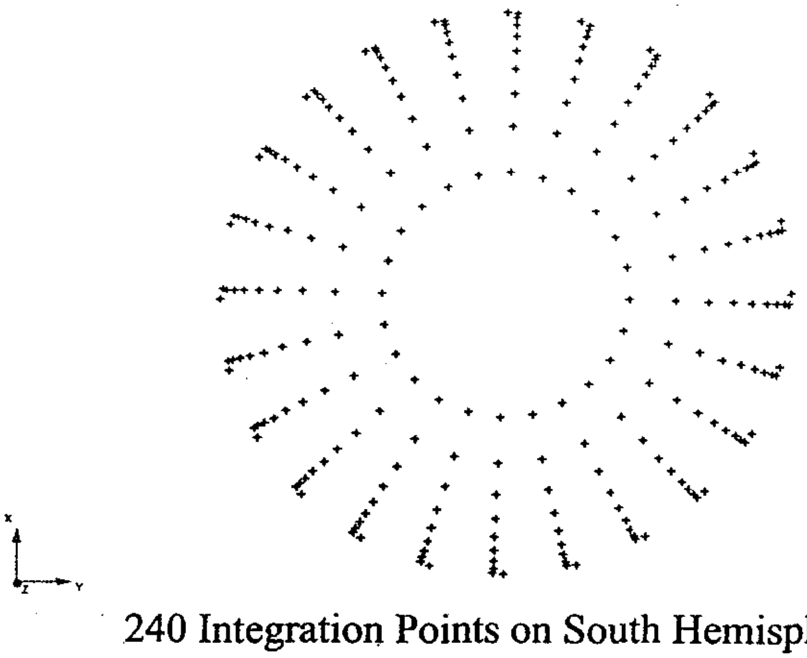


Figure 6. Location of CVBEM integration points on southern hemisphere.

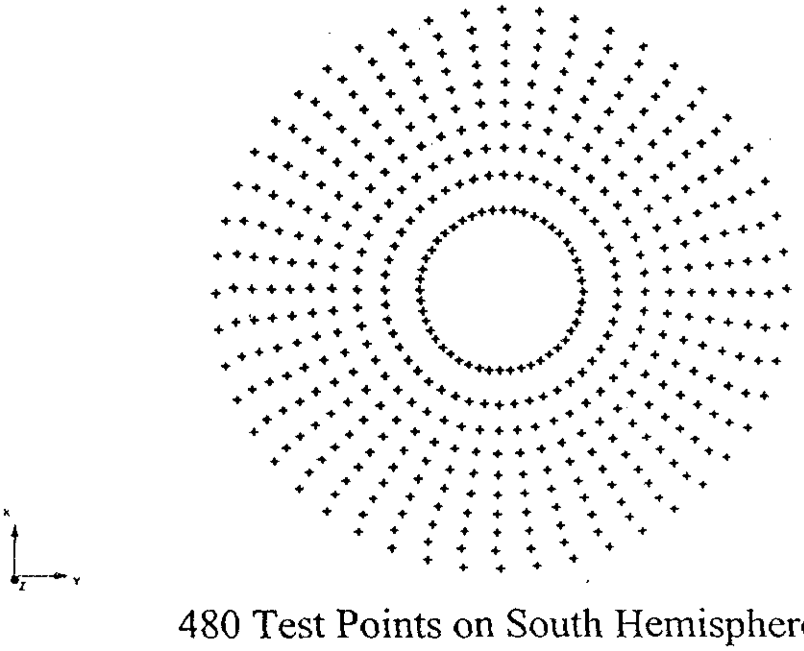
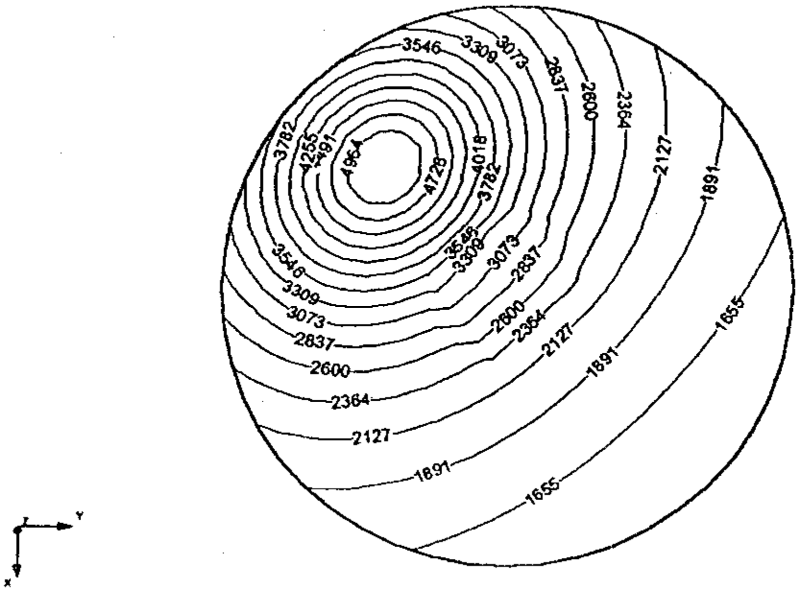


Figure 7. Location of test points on southern hemisphere.

3-D Sphere

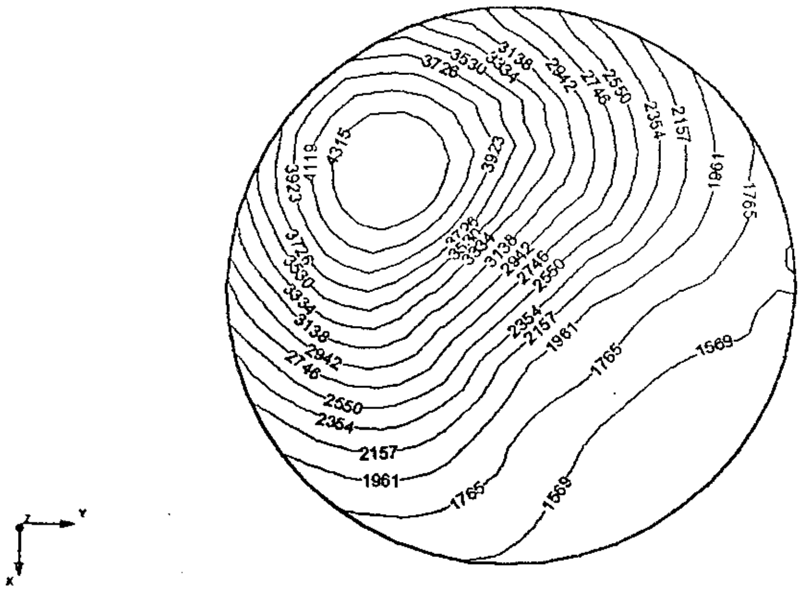
R = 2.5 @ (2.5,2.5,2.5)

$$f(x,y,z) = \frac{-500}{[(x-5)^2 + (y-0)^2 + (z+2)^2]^{1/2}} + \frac{10000}{[(x-0)^2 + (y-0.1)^2 + (z-5)^2]^{1/2}} + \frac{100}{[(x-0)^2 + (y-5)^2 + (z-6)^2]^{1/2}}$$

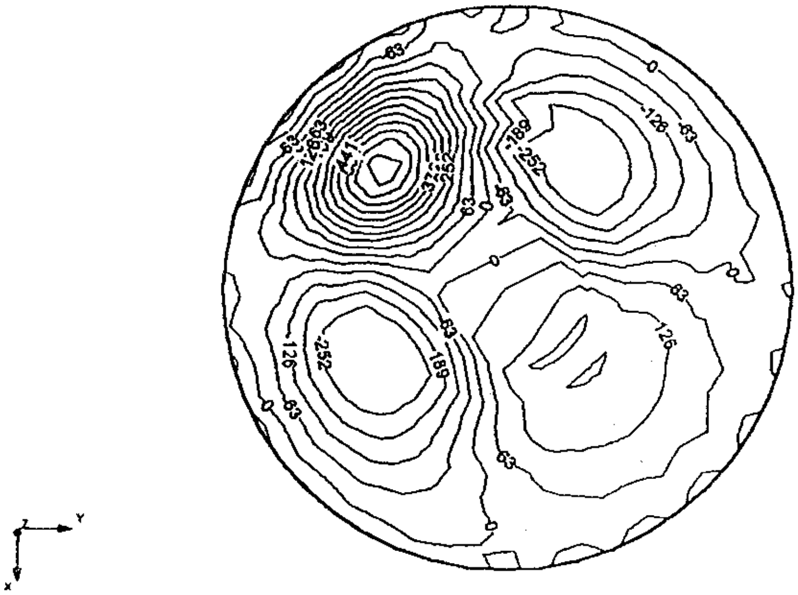


Exact Solution on North Hemisphere

Figure 8. Exact Solution, on northern hemisphere, of Application A.



Approximation on North Hemisphere



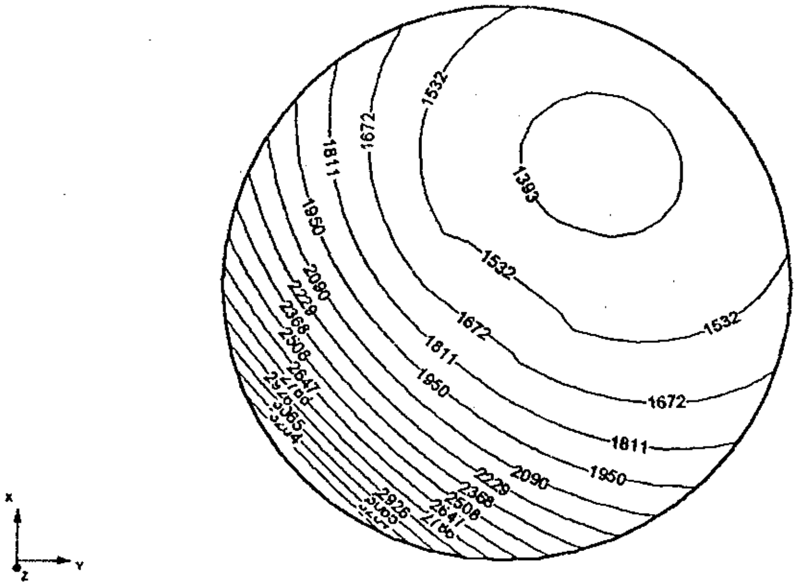
Approximation Error on North Hemisphere

Figure 9. Application A - approximation results on northern hemisphere.

3-D Sphere

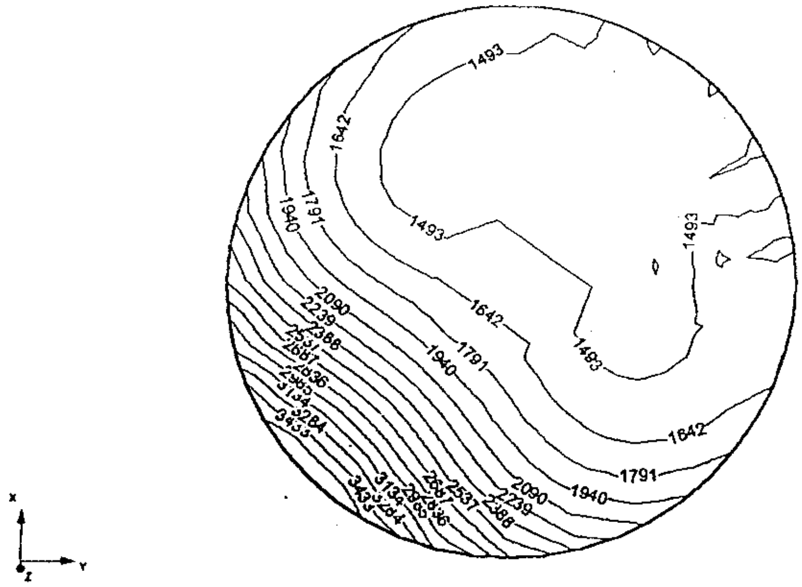
R = 2.5 @ (2.5,2.5,2.5)

$$f(x,y,z) = \frac{-500}{[(x-5)^2+(y-0)^2+(z+2)^2]^{1/2}} + \frac{10000}{[(x-0)^2+(y-0.1)^2+(z-5)^2]^{1/2}} + \frac{100}{[(x-0)^2+(y-5)^2+(z-6)^2]^{1/2}}$$

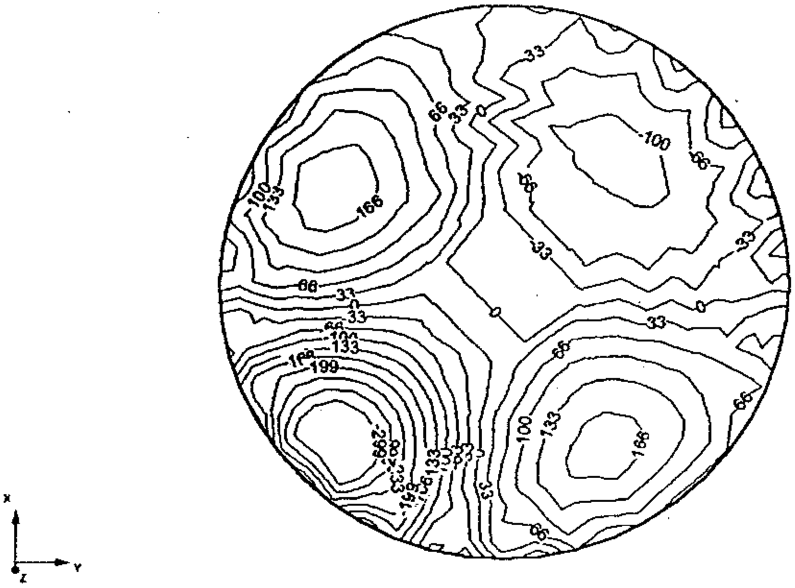


Exact Solution on South Hemisphere

Figure 10. Exact solution, on southern hemisphere, of Application A.

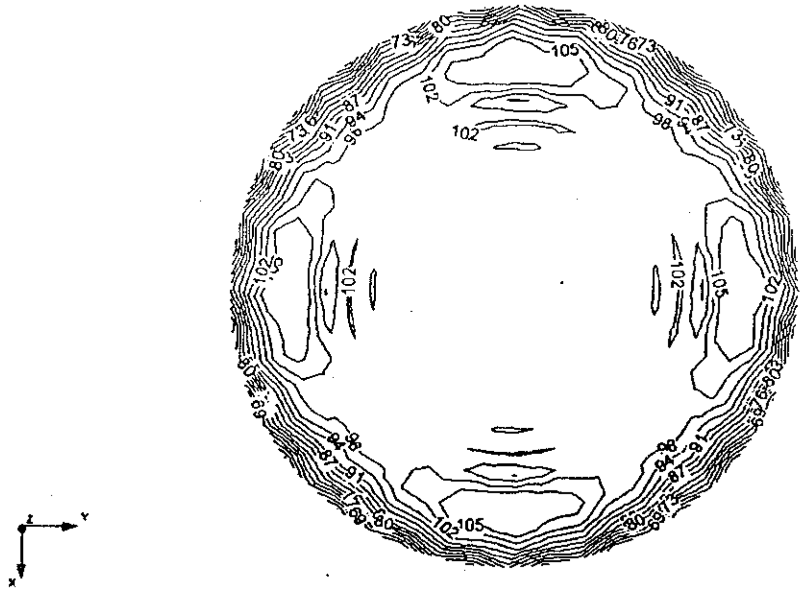


Approximation on South Hemisphere

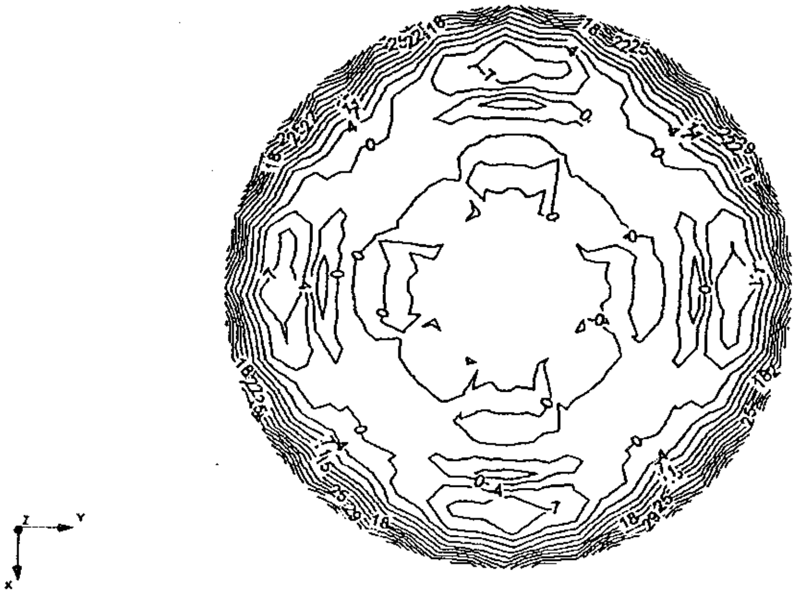


Approximation Error on South Hemisphere

Figure 11. Application A - approximation results on southern hemisphere.

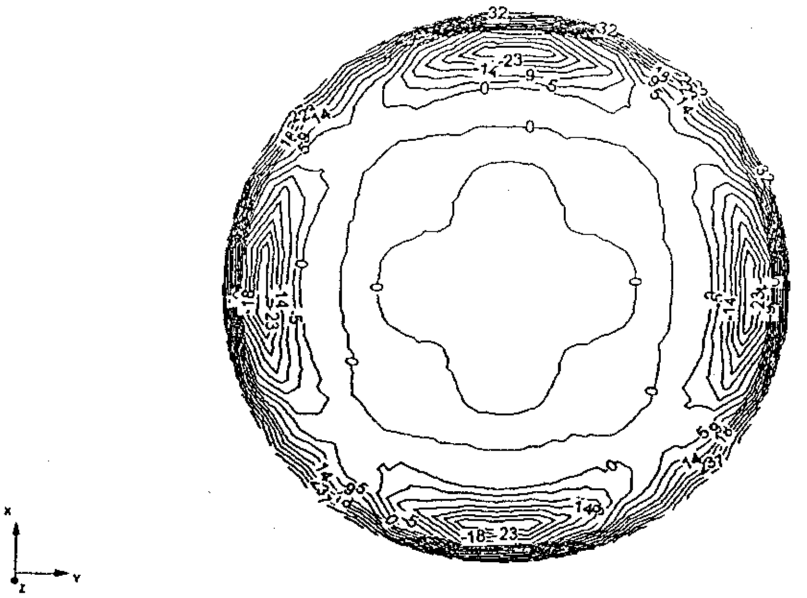


Approximation of F(x,y,z) = 100 on North Hemisphere
(F(x,y,z)=0 on South Hemisphere)

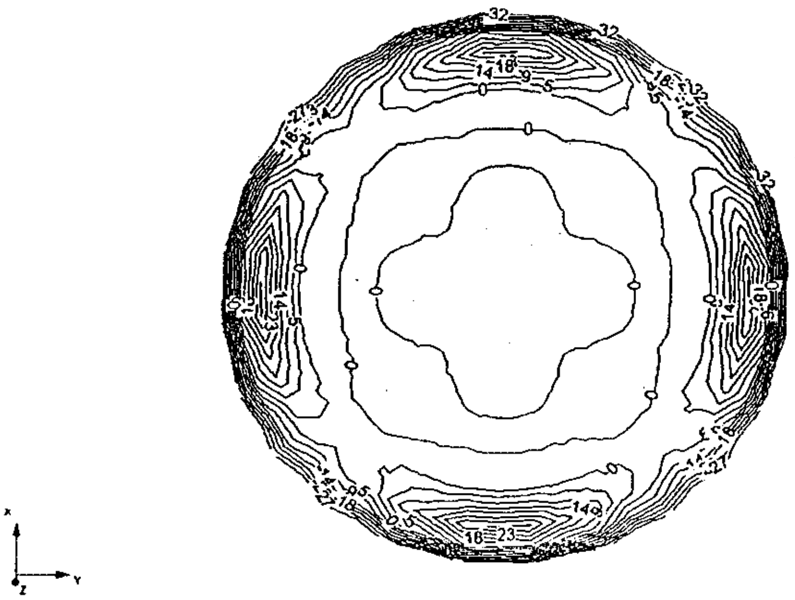


Approximation Error on North Hemisphere

Figure 12. Application B - approximation results on northern hemisphere.

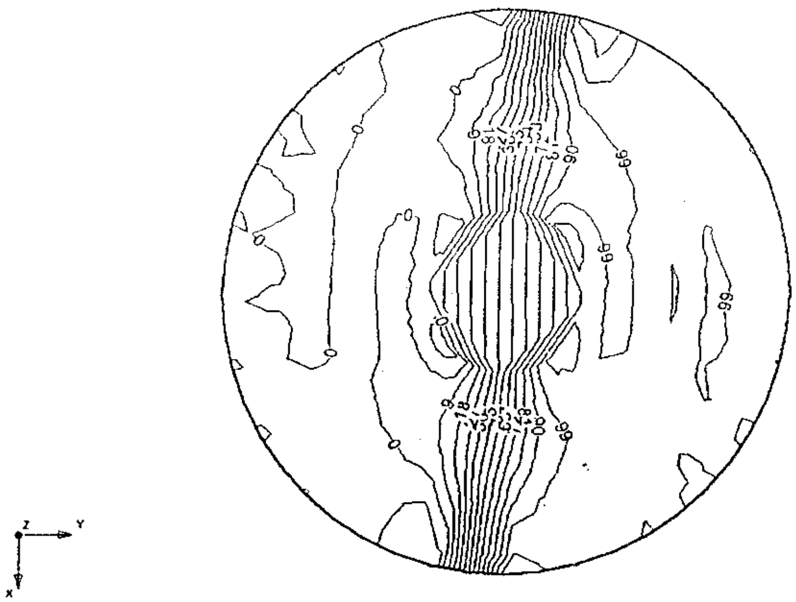


Approximation of F(x,y,z) = 0 on South Hemisphere
(F(x,y,z)=100 on North Hemisphere)

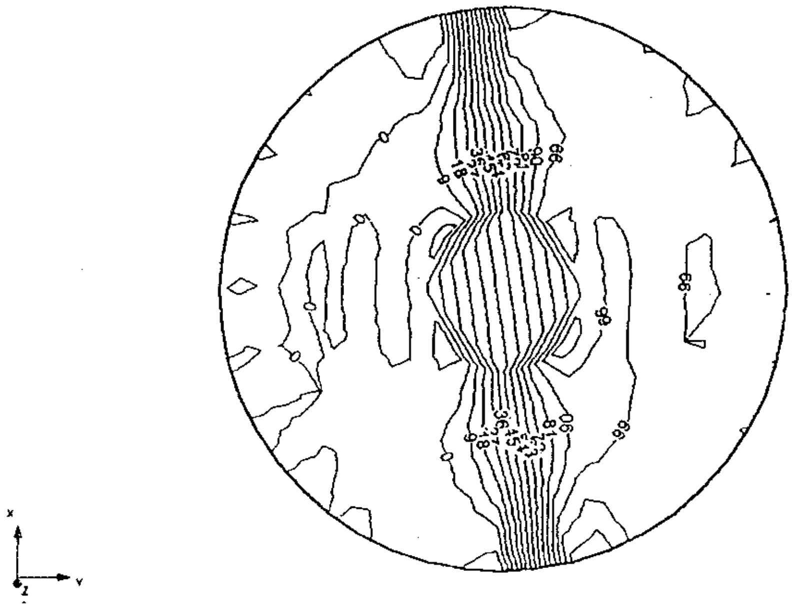


Approximation Error on South Hemisphere

Figure 13. Application B - approximation results on southern hemisphere.

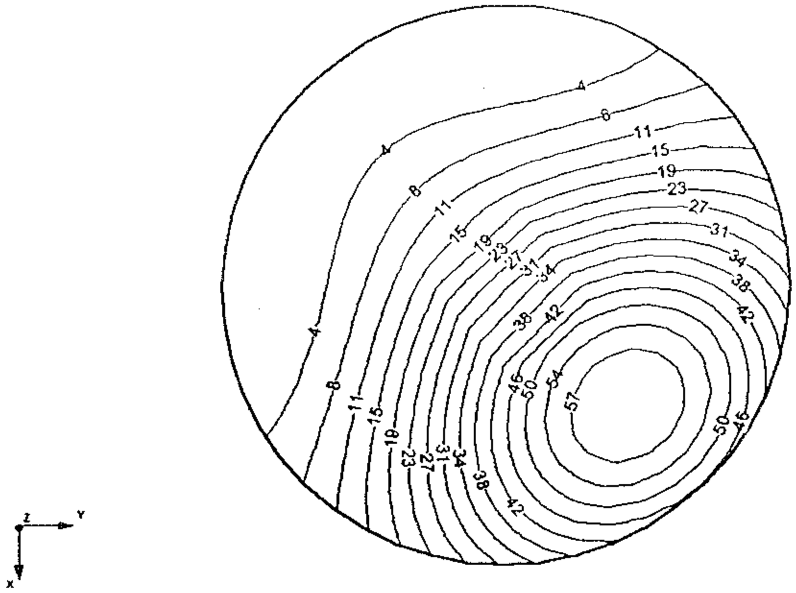


North Hemisphere Approximation of
F(x,y,z) = 100 on East Hemisphere
F(x,y,z) = 0 on West Hemisphere

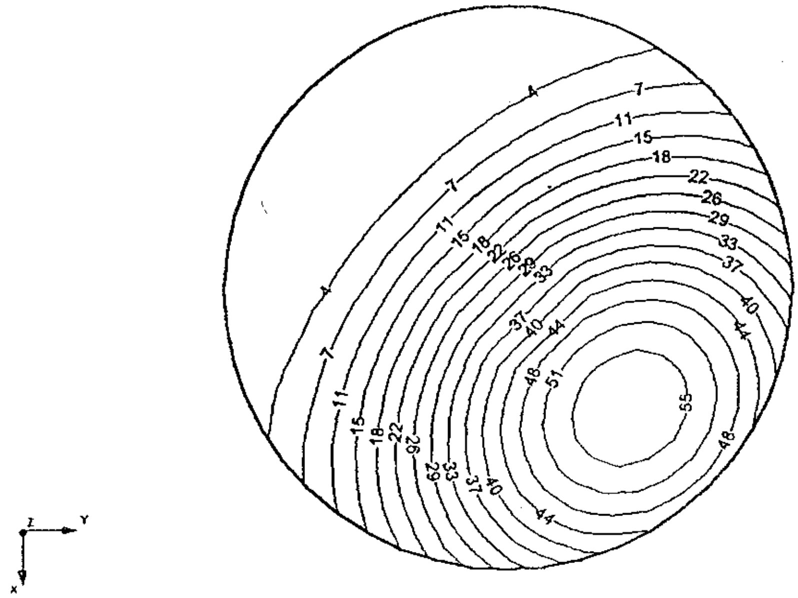


South Hemisphere Approximation of
F(x,y,z) = 100 on East Hemisphere
F(x,y,z) = 0 on West Hemisphere

Figure 14. Application B - approximation results on sphere rotated 90 degrees.

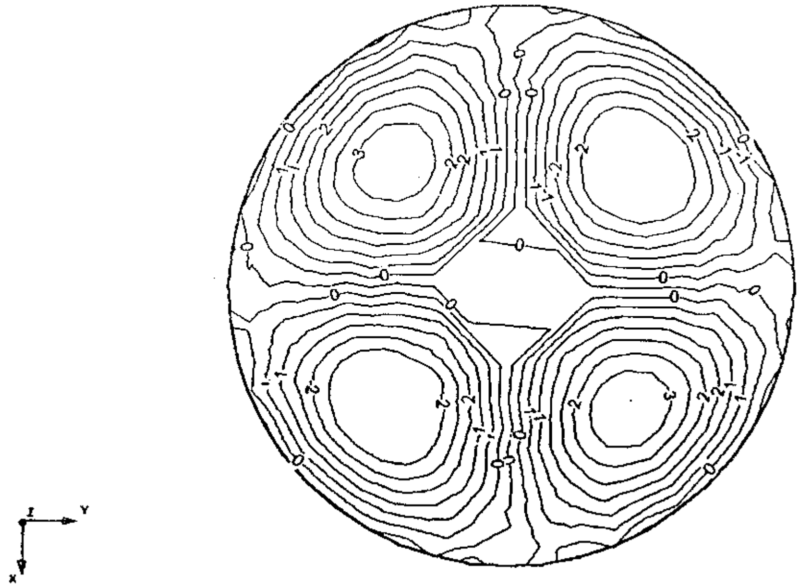


Exact Solution of F(x,y,z)=xyz on North Hemisphere



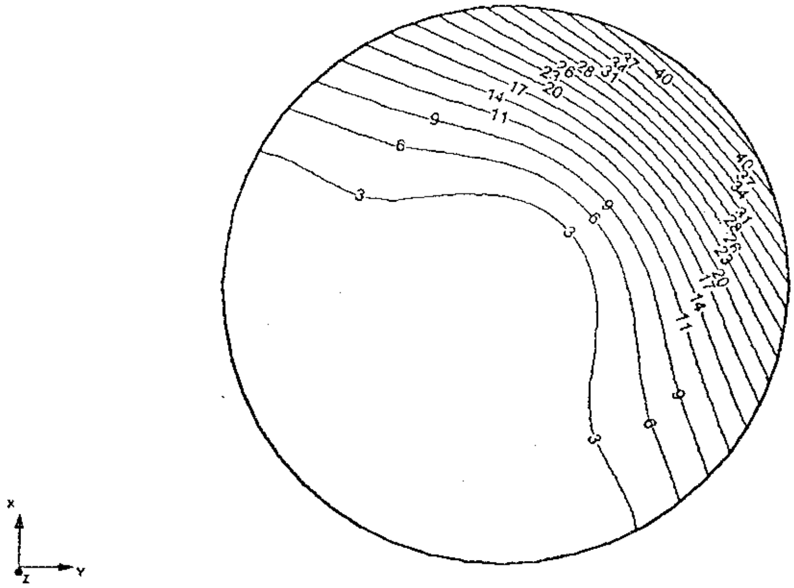
Approximation of F(x,y,z)=xyz on North Hemisphere

Figure 15. Application C - exact and approximation isothermals on northern hemisphere.

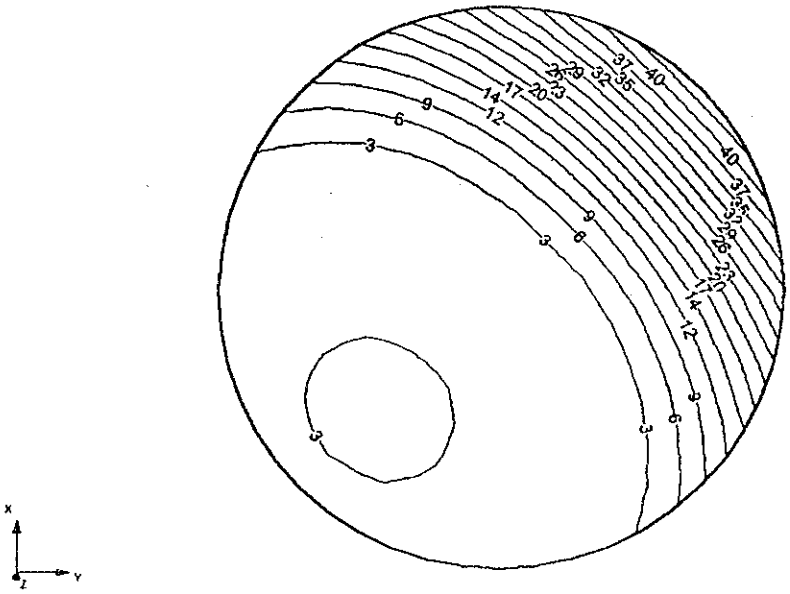


Approximation Error of F(x,y,z)=xyz on North Hemisphere

Figure 16. Application C - error isocontours on northern hemisphere.

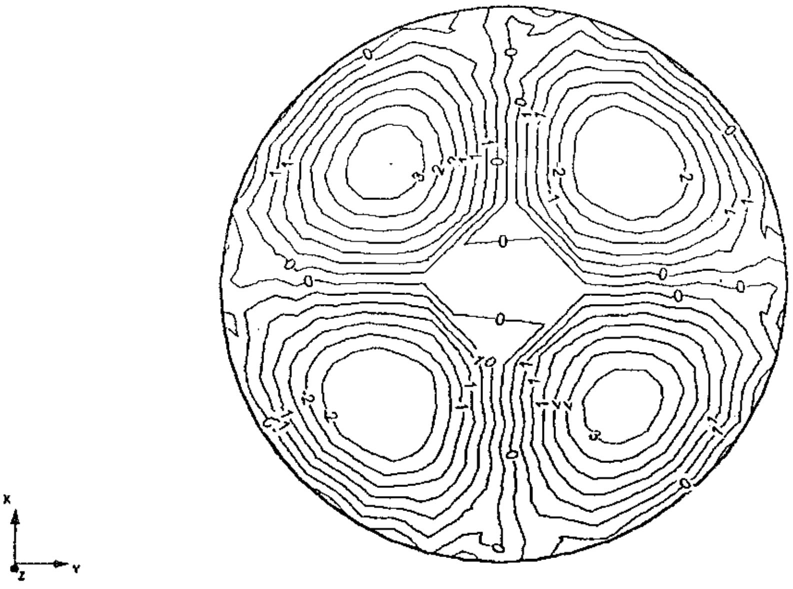


Exact Solution of F(x,y,z)=xyz on South Hemisphere



Approximation of F(x,y,z)=xyz on South Hemisphere

Figure 17. Application C - exact and approximation isothermals on southern hemisphere.



Approximation Error of F(x,y,z)=xyz on South Hemisphere

Figure 18. Application C - error isocontours on southern hemisphere.