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A Computer Model to Evaluate the Effect of Land Development on Sediment Transport

Abstract

A procedure is described which allows the computation of the total sediment transport for each of several T-year hydrographs. An approximate probability distribution for the total transported sediment, can be obtained. Applications are given where the mean and standard deviation of this distribution are computed.

Comparing the results for the catchment in its present state with a future developed state, using a selection of new channel parameters, indicates how to improve the channel to control changes in sedimentation due to development. Because changes in channel morphology, as caused by changes in sediment transport, affects downstream floodplains, it is important to consider the impacts of land development in sediment transport. The computed information provides a basis for estimating a new channel configuration such that the new flow conditions retain, as best as possible, the existing condition sedimentation effects.

Theoretical Development

A regression equation [1] which approximately relates the instantaneous sediment transport dS/dt and discharge Q can be synthesized into the form:

$$\frac{dS}{dt} = aQ^b \quad (1)$$

Given this equation, the parameter values of a and b , and a T -year hydrograph [3] of values $Q_T(t)$ of Q , the total sediment transported, S_T , is given by

$$S_T = \int_0^{\infty} aQ_T(t)^b dt \quad (2)$$

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Generally speaking, increasingly larger storms have the capacity to move increasingly larger amounts of sediment: the T-year storm having probability $1-1/T$ of being exceeded. This argument, which is only approximate as storms are not linearly ordered, nevertheless indicates that the number S_T is a reasonable estimate for the $1-1/T$ values for the random variable S_{tot} which gives the total amount of sediment transported by a storm:

$$\text{Prob}(S_{tot} \leq S_T) = 1-1/T \quad (3)$$

See [3, page 271-275]. Writing F for the probability distribution function for S_{tot} , the above equation is the same as

$$F(S_T) = 1-1/T \quad (4)$$

Once F is determined, the expectation $E(S_{tot})$ of S_{tot} can be obtained by an integration by parts

$$E(S_{tot}) = \int_0^{\infty} t dF(t) = \int_0^{\infty} (1-F(t)) dt \quad (5)$$

and, similarly, the second moment is

$$E(S_{tot}^2) = \int_0^{\infty} t^2 dF(t) = \int_0^{\infty} 2t(1-F(t)) dt \quad (6)$$

from which the standard deviation for S_{tot} can also be computed.

These equations will be used to calculate the distribution function F and its mean and standard deviation.

To see the effects of future land development on the distribution of S_{tot} , an indirect approach is needed because the parameters a and b of equation (1) which would apply to the developed area cannot be obtained since there are no regression data.

The basic assumption we make is that, as a function of the velocity of flow V , $[dS/dt]/Q$ will remain approximately the same when the area is developed; i.e. the instantaneous sediment transport capacity per unit cfs of discharge for a given velocity of flow will remain the same for the channel. Consequently, if the flow in the channel increases while transporting the same quantity of sediment per unit of flow (cfs), then erosion may occur.

Under a wide range of conditions for open channel flow the flow velocity V , due to a discharge Q , is approximated by the equation [2]

$$V = \alpha Q^\beta \quad (7)$$

where, intuitively, α is related to the slope and friction in the channel and β is related to the channel geometry, factors of wetted perimeter, flow area, and flow top-width. Knowing the values of α and β , and the values of a and b in the first equation, it is immediate that

$$\frac{dS}{dt} / Q = kV^l \quad (8)$$

where $l = (b-1)/\beta$ and $k = a\alpha^{-l}$. The total sediment transport for a storm Q can be written

$$\int_0^{\infty} [kV(t)]^l Q(t) dt \quad (9)$$

Under present conditions, equation (9), applied to a T -year discharge Q_T , reduces to equation (2).

For a T -year hydrograph reflecting future developed conditions, equations (3-6) and (9) allow the computation of the distribution of S_{tot} for different values of α and β . When α and β are chosen to have the values for the existing channel this computation shows how the volumes of sediment transported would change, because of the increased discharge caused by development, if the channel were not improved.

For design purposes, α and β can be varied to show the effect on S_{tot} of improvements in the channel. For example, to mitigate the effects of development, one design criterion could be to keep the mean of the total sediment S_{tot} the same, so that in the long run the channel is neither filled or eroded. Other criteria could take into account in the standard deviation of S_{tot} , which give some idea of the expected amounts of variation in sediment transport, resulting in filling or scouring in the channel.

Applications

A. Conceptual System

For discussion purposes, a hypothetical flow and sediment transport regime is examined that is qualitatively similar to arid conditions in southern California. An actual case study is considered in Application B.

The runoff hydrographs are each taken to have a 6-hour duration, rising in a straight line from 0 to a maximum M at 3 hours and decreasing from M in a straight line to 0 at 6 hours. For existing conditions, the peak discharge M_T was taken to have 10-year value of $M_{10} = 2000$, and $M_{100} = 5000$. The 2 and 25-year values of 1050 and 2850 were obtained by taking $\log(T)$ and $\log(M_T)$ to be linearly related. The hydrograph for developed conditions was obtained in the same way by setting $M_{10} = 3500$ and $M_{100} = 6000$, and finding $M_2 = 2400$ and $M_{25} = 4350$.

The values of $\alpha = 1.9$ and $\beta = .40$ for equation (7) represent a typical channel, and the values $a = .70$ and $b = 1.28$ for equation (1) are taken from one of the channels in the actual case study below.

Table 1. Total Sediment Transport In Kilotons

	T=2	T=10	T=25	T=100	mean	sd
<i>existing hydrology</i> old channel ($\alpha = 1.9, \beta = .4$)	24.1	55.0	88.1	180.0	31.5	28.7
<i>future hydrology</i> old channel ($\alpha = 1.9, \beta = .4$)	71.3	114.9	150.6	227.3	70.6	43.7
<i>future hydrology</i> new channel ($\alpha = 0.6, \beta = .4$)	31.8	51.3	67.2	101.4	31.5	19.5
<i>future hydrology</i> new channel ($\alpha = \epsilon, \beta = 1.8$)	23.7	55.1	88.8	183.7	31.5	29.1

Notes:

$a = 0.700, b = 1.280, \epsilon = 1.2 \times 10^{-5}$, one kiloton = 2×10^6 lbs, sd = standard deviation.

The main points to note from Table 1 are:

- 1) The second row shows the effect of development on the sediment transported if the channel is not improved.
- 2) The third row shows that a new channel with $\alpha = 0.6$ and the same β will transport the same average total sediment, with a similar variation about this mean. But note also that the 2-year value is higher, and the 10-year value not too different, so that in the short term there would be scouring in the channel; equilibrium with the undeveloped long term average would only occur because very large (and infrequent) storms would move less sediment.
- 3) The last row shows that it is possible to closely duplicate the current sediment transportation after development, but only with a channel with essentially zero slope.

B. The second application is an actual case study for the City of Yucaipa, California, Master Plan of Drainage study.

Three channels were studied: Gateway Creek, Oak Glen Creek, and Wilson Creek. Using cross-section data obtained from topographic maps, a typical cross-section was derived for each channel and α, β parameters developed for (7). The Brownlie equation fitted parameters of a and b were developed from the application [4]. Using a calibrated unit hydrograph method, runoff hydrographs for several return frequency events, for both undeveloped and developed conditions, were also prepared in [4]. Results from the computer analysis of the three creeks are summarized in Tables 2-4 in a fashion analogous to Table 1 and Application A.

**TABLE 2. Oak Glen Creek
Total Sediment Transport in Kilotons**

	T=2	T=10	T=25	T=100	mean	sd
<i>existing hydrology</i>						
old channel ($\alpha = 2.48, \beta = .22$)	25.6	118.7	171.5	253.8	52.5	53.3
<i>future hydrology</i>						
old channel ($\alpha = 2.48, \beta = .22$)	39.7	188.8	274.4	528.8	85.6	93.9
<i>future hydrology</i>						
new channel ($\alpha = 1.7, \beta = .22$)	24.3	115.5	167.8	323.5	52.4	57.5
a = 0.70	b = 1.28					

The main points to note from Table 2 are:

- 1) The second row shows that the effect of development on the existing channel is erosive, in general.
- 2) The third row shows that a new channel with $\alpha = 1.7$ and the same $\beta = 0.22$ will approximately duplicate existing conditions.

**Table 3. Wilson Creek
Total Sediment Transport In Kilotons**

	T=2	T=10	T=25	T=100	mean	sd
<i>existing hydrology</i>						
old channel ($\alpha = 1.12, \beta = .30$)	6.9	39.4	61.3	96.7	17.1	19.3
<i>future hydrology</i>						
old channel ($\alpha = 1.12, \beta = .30$)	17.9	84.6	128.1	258.0	39.1	44.3
<i>future hydrology</i>						
new channel ($\alpha = 0.7, \beta = .30$)	5.2	41.0	62.0	124.8	17.4	22.0
a = 0.15	b = 1.47					

**Table 4. Gateway Creek
Total Sediment Transport In Kilotons**

	T=2	T=10	T=25	T=100	mean	sd
<i>existing hydrology</i>						
old channel ($\alpha = .69, \beta = .32$)	0.6	5.3	8.4	14.5	2.2	2.8
<i>future hydrology</i>						
old channel ($\alpha = .69, \beta = .32$)	1.2	8.7	16.7	49.7	4.3	7.1
<i>future hydrology</i>						
new channel ($\alpha = .44, \beta = .32$)	0.6	4.4	8.5	24.8	2.2	3.6
$a = 0.49$		$b = 1.484$				

In summary, the conceptual example and in all the three case studies, the effect of development was to approximately double the total sediment transport at all return frequencies. The effects of development could be mitigated in all the case studies by changing only the parameter α of the channel, while in the conceptual study if one wanted to preserve the probability distribution of the total sediment discharge, and not just the mean sediment transported, it was necessary to change both α and β .

For Application B, the α parameter reduction factor is 0.69, 0.63, and 0.64 for Oak Glen, Wilson, and Gateway Creeks, respectively. Assuming friction factors to remain nearly constant, such a reduction in α is associated with approximately 58-percent reduction in bed slope, using Manning's equation. That is, the channel slope must be more than halved, requiring extensive drop structure installation.

Computer Program Discussion

A program was written to calculate the quantities given in tables 1-4.

The input required is, first, the parameters α and β of equation (7) for the existing channel, and the sediment transport parameters a and b of equation (1). Second, files are required of T-year hydrographs for natural conditions for T having the values {5, 10, 25, 100}. The program then calculates the S_T values and the probability distribution F for S_T is approximated by using the S_T values obtained and the extrapolated value $S_0=D$ ($S_1=0$ is an alternate possibility, with little change in the results); F is defined to consist of straight lines joining the known points ($S_{T,1-1/T}$), with the exception that the point with the largest T value $\max T$ is taken to be ($S_{\max T,1}$), rather than ($S_{\max T,1-1/\max T}$) and extrapolating to some larger hypothetical T value. The mean and standard deviation are calculated using F described as above and equations (5) and (6).

The treatment of the conditions is similar, using new hydrographs. New values for alpha and beta, are used and the results of the corresponding channel changes are computed. This amounts to doing the integration of equation (9), with k and l given in terms of the alpha and beta values for the present channel, with $V = \text{newalpha } Q^{\text{newbeta}}$, and with Q from the T-year hydrograph for

the developed conditions. The ST values, mean, and standard deviation are used to find acceptable values for new alpha and new beta.

Summary

A synthetic study of the sediment transport response of a channel due to the increased discharge caused by development can be used to find design criteria for channel improvements to mitigate the effects of development.

References

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