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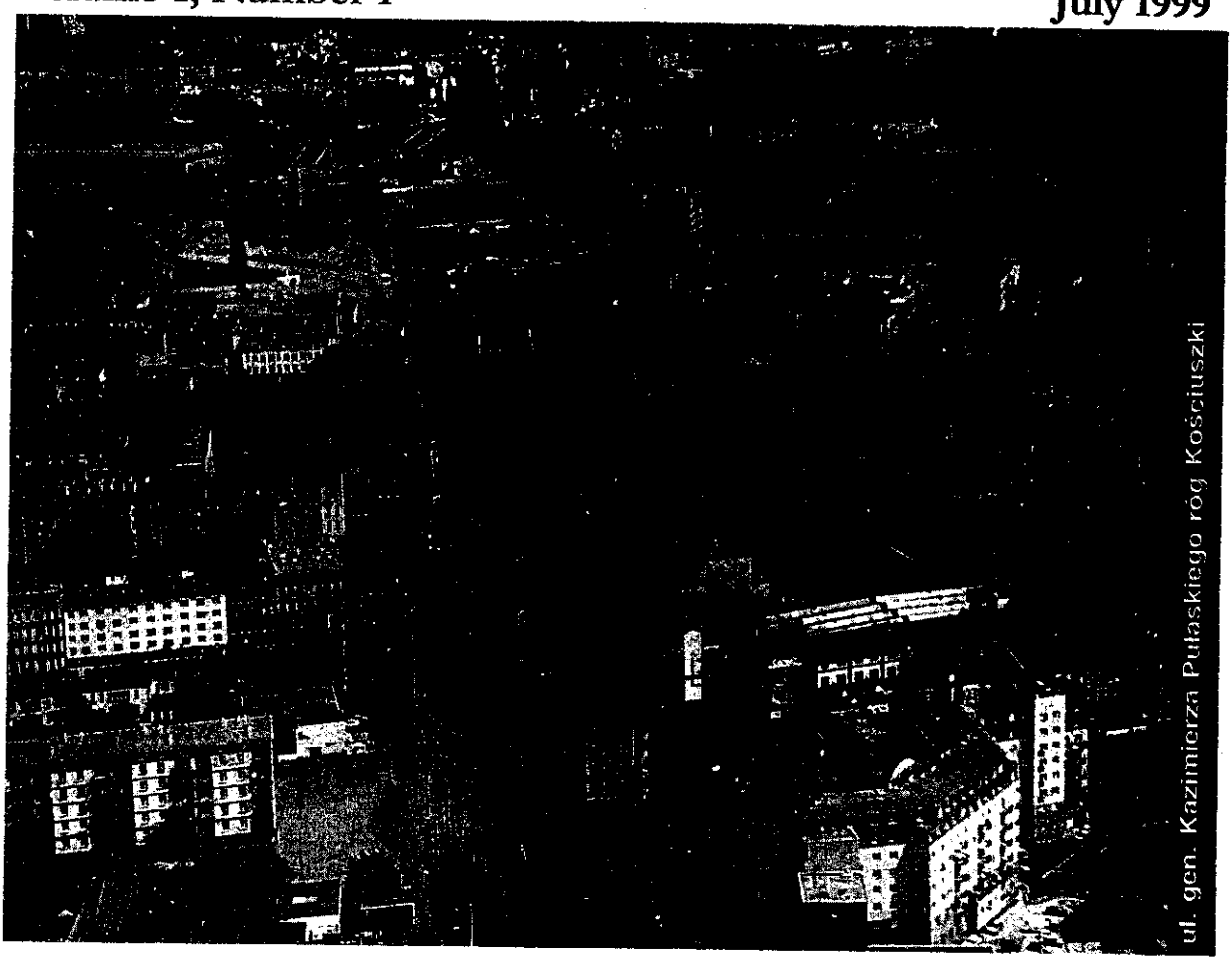

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Computing Approximate Confidence Intervals For Design Floods

Abstract

A basic practical problem in hydrology is the computation of the values of T-year floods for use in design. This calculation uses estimations of the mean, standard deviation, and skew from yearly maximal discharge data. The value of the T-year flood, which is the flood value having the probability 1/T of being exceeded in any given year, so computed is uncertain because the parameters are estimated, in contrast to being known exactly. It is important to quantify this uncertainty to indicate the likelihood that mitigation measures will actually provide the level of protection required.

A statistical way of quantifying such uncertainty is by means of confidence levels. This note shows how to easily compute confidence levels for the 100-year flood values when using the standard approach following the recommendation of U.S. Water Resource council's Bulletin 17B Advisory Council on Water Data (1982).

Introduction

The notion of statistical uncertainty is pervasive in our technological society, involving political issues as in whether or not to use sampling in the census, issues of life and death as in the testing of prescription drugs, and issue of public interest as in newspaper public opinion polls. A common thread in these matters is the acknowledgement that certainty, however desirable, is impossible to achieve.

The notion of a confidence interval allows a quantification of some of the uncertainties in question. An example is that of testing the effectiveness of a drug. Suppose for simplicity there are only two outcomes, a cure and not a cure, and this drug is tested on some number m of patients resulting in Y cures and N non-cures, Y+N=m. The two most common ways of applying confidence intervals use the probability distribution of the proportion of cures, $p = \frac{Y}{m}$, and some required level of significance, say 99%, to construct intervals discussed below.

In the first type of interval a number d is computed so as to have the interval $[p - d, p + d]$ be a 99% confidence interval for the true (unknown) proportion of cures that the drug will have on a very large group of patients. This is often described by saying that the true proportion of cures will lie in this interval "99% of the time"; a statement correct only if interpreted properly. As a matter of fact once the interval is computed the true unknown proportion of cures either lies in the interval or does not lie in the

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interval, we know not which because the problem to begin with is that the proportion is not known. The proper interpretation of a 99% confidence interval is that it is a procedure for computing an interval that has the property that if this procedure is followed over and over again, each time on a set of values with the same hypothesized probability distribution, then in the long run 99% of these intervals will contain the true proportion of cures. Therefore, when such an interval is computed if we suppose that the true proportion lies in the interval we will be in error only 1% of the time if we repeatedly do this.

The second type of confidence interval is a one-sided interval, in which a number d^l is computed so that, with the meaning above, 99% of the time the true value of the proportion of cures will be larger than $p - d^l$. Consequently the drug will be shown to have some beneficial effect, at the 99% level, if the number $p - d^l$, which is referred to as the 99% confidence level, is greater than $\frac{1}{2}$.

The first type of confidence interval emphasizes how accurately the cure rate p is known, the second type emphasizes the confidence one has that the cure rate p is at least as large as the computed confidence level. In either case, the calculation of the numbers d and d^l is possible only if the probability distribution of the sampled values is known, and therefore a great deal of attention is paid to randomizing patient samples and in general making the experiment conform to a model with a probability distribution for which confidence levels can be computed.

Many of the reasons that confidence intervals and levels are important for the testing of experimental drugs, also apply to 100-year flood estimates, where it is necessary to address the uncertainty of the estimate so that the agency responsible for supplying flood protection indicates not only that the protection is for the 100-year flood but also the confidence that the desired 100-year protection is actually being provided; this is provided by a one-sided confidence interval, indicating that the protection provided is, with the specified confidence level, for at least a T-year flood.

Computing 100-year Confidence Levels

The use of a log Pearson III distribution for yearly maximal discharges in order to estimate the T-year flood, with its parameters estimated by use of the sample mean $\hat{\mu}$, sample standard deviation $\hat{\sigma}$, and sample skew $\hat{\gamma}$, is recommended in U.S. Water Resource council's *Bulletin 17B Advisory Council on Water Data* (1982). In practice the log Pearson III distribution is used extensively because of the authority of the U.S. Water Resource Council. One clear benefit of this has been the consistency so obtained, and thereby the ability to compare values since they have been computed in the same way.

The Water Resources Council's procedure requires computing three parameters of the logarithms (base 10) of the m data values of maximal yearly discharge. The first two estimates, the sample mean $\hat{\mu}$ and sample standard deviation $\hat{\sigma}$ are well-known; $\hat{\mu}$ is the mean and $\hat{\sigma}$ is the sample second moment about the mean. The sample skew is the third sample moment about the mean divided by $\hat{\sigma}^3$, see formula 4a on page 10 of *Advisory Committee on Water Data* (1982), and measures the extent to which the distribution is asymmetrically tilted on the left or right. These estimators are the standard ones found in any statistic text.

Confidence levels can be simply computed from the three estimates $\hat{\mu}$, $\hat{\sigma}$, and $\hat{\gamma}$ using the method of Whitley and Hromadka (1999) by the use of curves which were derived by means of a neural network function. In brief, to compute a specific confidence level for a 100-year flood for a site with a given number of data points requires a curve for that confidence level and that number of data points. Given this curve, call it f, the required confidence level is given by

$$10^{\hat{\mu}+\hat{\sigma}f(\hat{\gamma})} \tag{1}$$

Typical curves for this function *f*, which we call a confidence factor, are given below. Because the confidence factor increases with confidence level, the lowest curve is for the 50% level, the next highest for the 75% level, and the highest for the 90% level (see Graphs 1-3).

The numbers of points 10, 20 and 30 considered in the examples give quite variable skew estimates and are smaller than the numbers considered desirable in professional practice for a skew analysis, however the neural network curves take this variability into account and provide a large confidence value corresponding to the variability of the parameter estimates.

To get an idea of the effect of the uncertainty it helps to compute some values using these curves for a typical storm. A medium size storm will be represented by values for logarithms to the base 10 of yearly maximal discharge in m^3s^{-1} of:

$$\text{medium storm: } \hat{\mu} = 3, \hat{\sigma} = 0.3 \qquad (2)$$

The examples calculated will be for sample skews $\hat{\gamma}$ of $-\frac{1}{2}$, 0, and $\frac{1}{2}$. To better see the effect of the variability in the parameter estimation, first suppose that the parameters for the medium storm are known *exactly*. Then with no variability in the estimates, $\hat{\mu} = 3$, $\hat{\sigma} = 0.3$ and $\hat{\gamma}$:

Medium Storm: 100-year flood values for *known* parameters.

$$\begin{aligned} &4000\ m^3s^{-1} \text{ for } \gamma = -\tfrac{1}{2} \\ &5000\ m^3s^{-1} \text{ for } \gamma = 0 \\ &6500\ m^3s^{-1} \text{ for } \gamma = \tfrac{1}{2} \end{aligned} \qquad (3)$$

values given to the nearest 500 cfs.

As a partial solution to the problem of computing confidence intervals using the computation described in Whitley and Hromadka (1986), it is possible to compute confidence intervals when the mean and standard deviation are estimated but the skew is assumed to be known exactly. These results are given in Table 1.

The values in Table 2 below are computed using formula (1), the parameters values of (2), and the appropriate graph. For example, for $m = 10$ and $\hat{\gamma} = 0$, the curve in Graph 1 for the 75% confidence factor f(0) gives 3.6, formula (1) gives $3 + 0.3(3.6) = 4.08$, and $10^{4.08} = 12{,}023$ gives the rounded value in parentheses in the middle of the first row of Table 2.

The most striking features of Table 2 is the sizable increases necessary in going from a 50% confidence level to a 90% confidence level. The variation of confidence levels with $\hat{\gamma}$ is also quite significant, increasing substantially from $\hat{\gamma} = 0$ to $\hat{\gamma} = 0.5$, and decreasing somewhat in dropping from $\hat{\gamma} = 0$ to $\hat{\gamma} = -0.5$. A comparison of the values in (3) for known parameters shows that for positive $\hat{\gamma}$, the most common situation, even the 50% confidence level can be much larger than the estimates of (3) which ignore the uncertainty in estimating the parameters. As expected, increasing the number of data points makes the estimates more certain and decreases the confidence level values.

To get an idea of the effect for smaller and larger discharges, taking a small storm as one with $\hat{\mu} = 2$ and e same $\hat{\sigma} = 0.3$ just divides every entry in Table 1 by 10. Similarly, taking a large storm as one with $\hat{\mu} = 3.7$, so that $10^{3.7} = 5000$ approximately, and the same $\hat{\sigma} = 0.3$ just multiples every entry in Table 1 by 5.

Conclusion

Using the results of Whitley and Hromadka (1999) it is easy to compute confidence levels for 100-year flood values. The confidence levels obtained can be significantly larger than estimates which do not take in account the uncertainty in the estimate. It is important in practice to compute such confidence levels so as to clarify exactly how certain it is that a given level of protective is provided by proposed and present mitigation measures. From this prospective, educating the public and the courts as to the impossibility of supplying certain protection is a necessary and challenging part of flood control.

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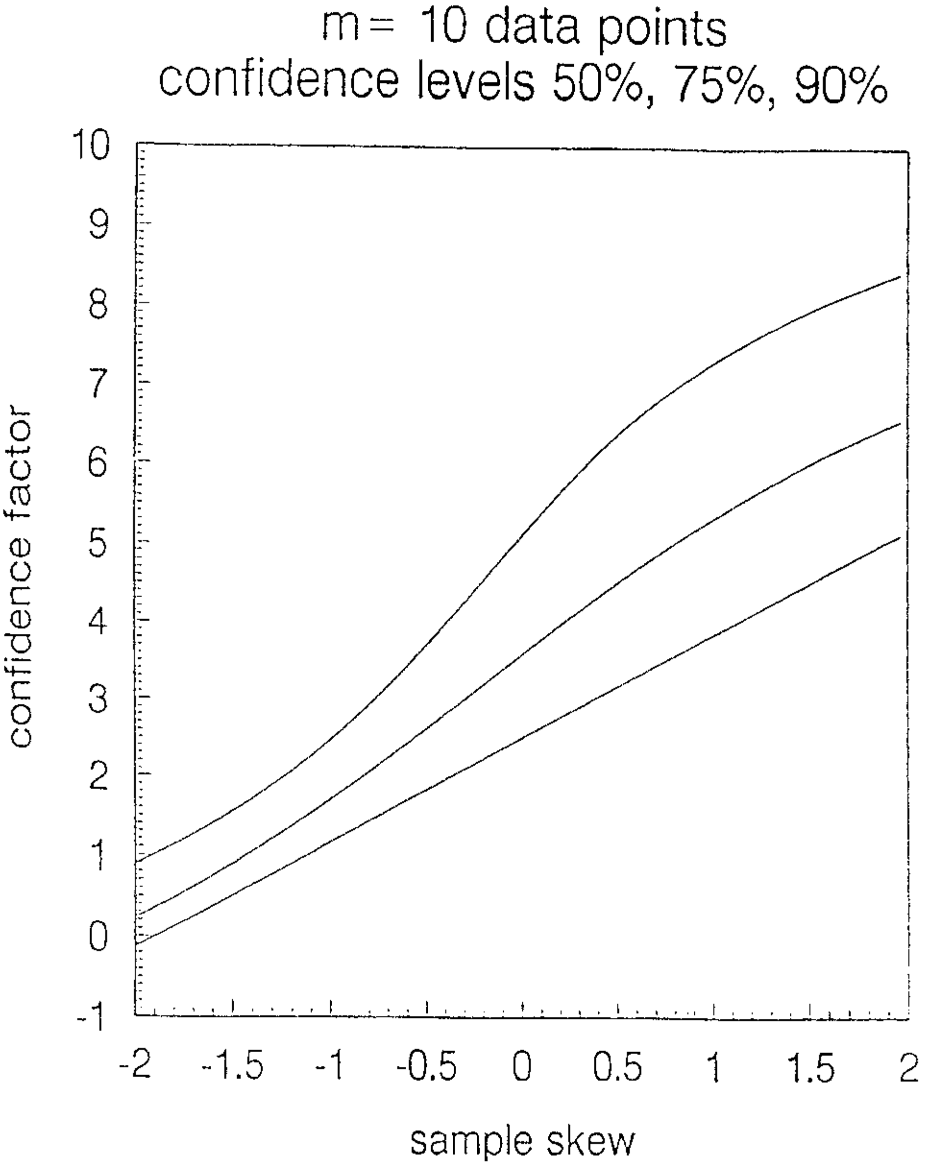
Table 1: Confidence Levels: 50% (75%) 90% (discharge : m^3s^{-1})			
	$\gamma = -0.5$	$\gamma = 0.0$	$\gamma = 0.5$
m=10	4,000 (5,500) 7,500	5,500 (7,500) 11,500	7,000 (11,500) 19,000
m=20	4,000 (4,700) 5,700	5,000 (6,500) 8,500	6,500 (9,000) 12,500
m=30	4,000 (4,500) 5,000	5,000 (6,000) 7,500	6,500 (8,500) 11,000

Table 1. Confidence Levels, Known Skew: $\hat{\mu} = 3$ and $\hat{\sigma} = 0.3$

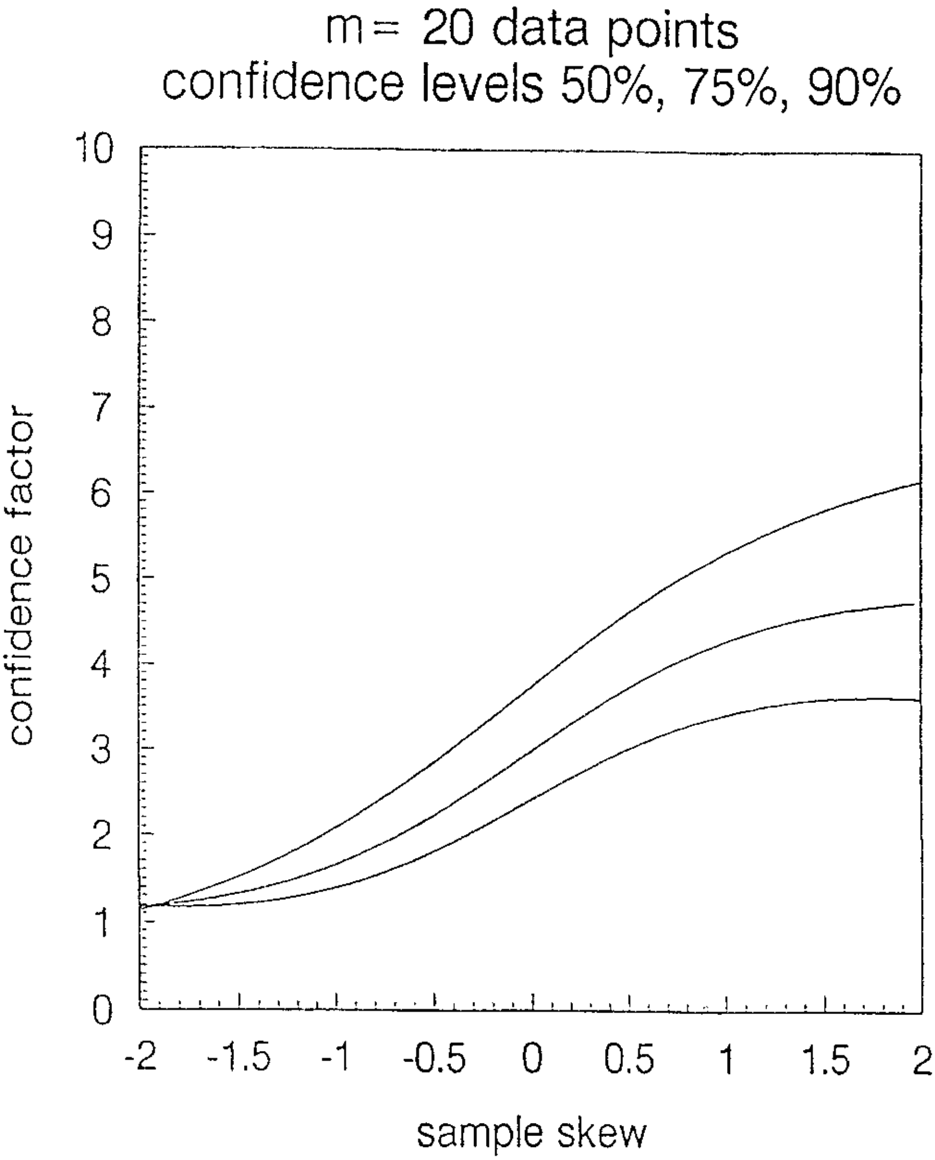
Table 2: Confidence Levels: 50% (75%) 90% (discharge : m^3s^{-1})			
	$\hat{\gamma} = -0.5$	$\hat{\gamma} = 0.0$	$\hat{\gamma} = 0.5$
m=10	3,500 (6,000) 13,000	5,500 (12,000) 34,500	9,000 (22,500) 82,000
m=20	3,500 (5,000) 7,000	5,500 (8,000) 13,500	8,000 (13,500) 24,500
m=30	3,000 (4,500) 6,000	4,000 (7,000) 10,500	5,500 (11,500) 18,000

Table 2. Confidence Levels, Estimated Skew: $\hat{\mu} = 3$ and $\hat{\sigma} = 0.3$

Graph 1: Confidence Levels (m=10 data points)



Graph 2: Confidence Levels (m=20 data points)



Graph 3: Confidence Levels (m=20 data points)