A GENERALIZED 3D COMPLEX VARIABLE BOUNDARY ELEMENT METHOD (CVBEM): APPLICATIONS

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Key words: CVBEM, boundary elements, complex variables, complex variable boundary elements, numerical methods

Abstract

The Complex Variable Boundary Element Method, or CVBEM, is extended into a three-dimensional (3D) problem solving capability. The new 3D CVBEM is easy to apply, although the mathematical underpinnings of this new numerical technique, for solving 3D Dirichlet problems, is rather involved. In this paper, a mathematical construction is reviewed, and the corresponding numerical analog is developed. Several 3D demonstration problems are considered that evaluate various 3D geometries.

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APPLICATIONS OF THE 3D CVBEM

In all of the application problems considered, a set of five projection planes were used, all with the same 3D orientation about the problem domain's enclosing sphere. Additionally, although several Dirichlet problems were examined, only a single common Dirichlet problem is presented in this paper for brevity. The problem considered is a 2 source and 1 sink temperature problem where the sources and sink are located closely to the 3D problem boundary in order to more vigorously test the 3D CVBEM. The exact solution to the test problem is T(x,y,z) where

$$\begin{aligned} T(x,y,z) &= -500/[x\text{-}5)^2 + (y-0)^2 + (z+2)^2]^{1/2} \\ &+ 10000/[(x-1)^2 + (y-0.1)^2 + (z-5)^2]^{1/2} \qquad (17) \\ &+ 100/[(x-0)^2 + (y-5)^2 + (z-6)^2]^{1/2} \end{aligned}$$

and (x,y,z) are the 3D coordinates. The problem boundary conditions are simply the above T(x,y,z) evaluated at the problem boundary's integration points. For each problem geometry, the exact solution results, from (17), and the 3D CVBEM results, are plotted along selected 2D surfaces of the 3D domains or boundaries. For each problem considered, only four CVBEM nodes are used, evenly spaced, on each projection plane; with five projection planes used in the approximation, a total of 20 CVBEM nodes are employed.

Solid T-Shaped Stand

The geometry of this problem is shown in Figure 7a. Figures 7b, 7c, and 7d depict the exact solution, 3D CVBEM approximation, and the approximation error, respectively, at a 2D slice taken at $x = 1.5$.

Solid Sphere

For a sphere, of radius 2.5 units, the examination of results can be plotted on the sphere's surface for the "north hemisphere" and the "south hemisphere". The exact solution, the CVBEM approximation, and the approximation error, respectively, are depicted on Figures 8a,b,c, for the north hemisphere. Similar depictions are provided on Figures 8d,e,f, for the south hemisphere.

Solid Door Knob

The geometry of this problem is shown in Figure 9a. Figures 9b, 9c, and 9d depict the exact solution, 3D CVBEM approximation, and the approximation error, respectively, at a 2D slice taken at x = 2.0. A plot of relative error is provided in Figure 9e.

Figures 9f,g,h,i and 9j,k,l,m show similar depictions of results for the 2D slices oriented at z = 0 and y = 0, respectively, as measured in Figure 9a (i.e., before reorientation to the first octant).

Solid Box with Lid Off

Figure 10a depicts the box geometry. Figure 10b shows the integration points used on the problem boundary. Figures 10c through 10n depict various results and error plots, for slices through the box bottom and sides, as measured from the original geometric orientation of Figure 10a.

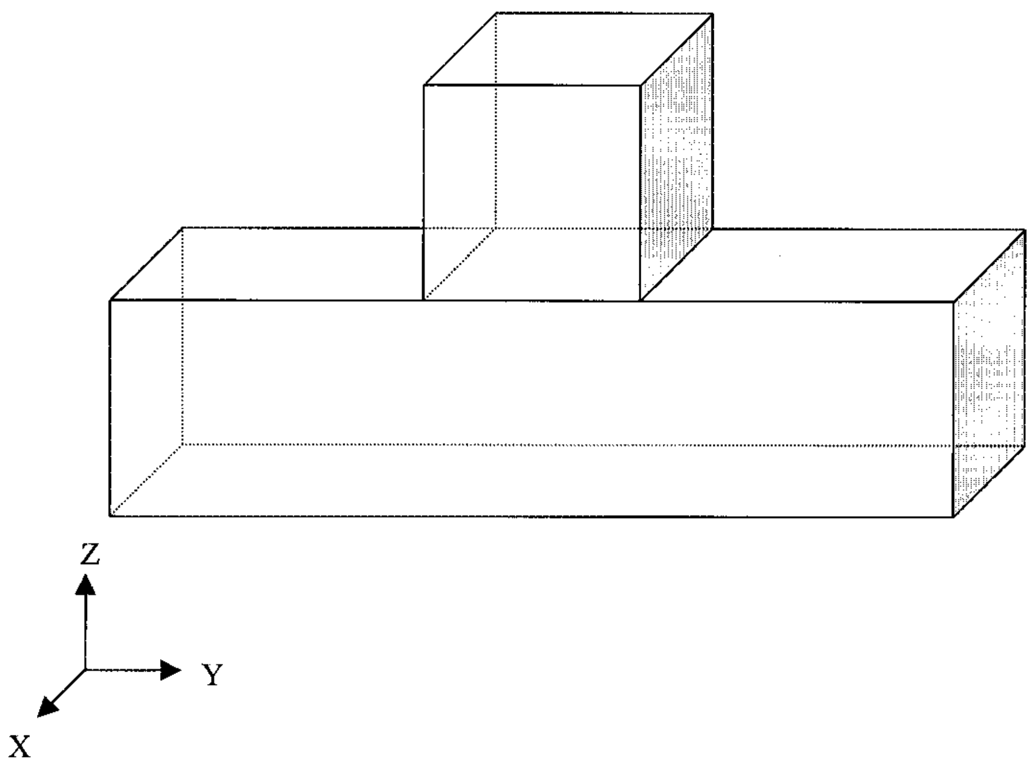

Figure 1. **Example 3-D Domain**

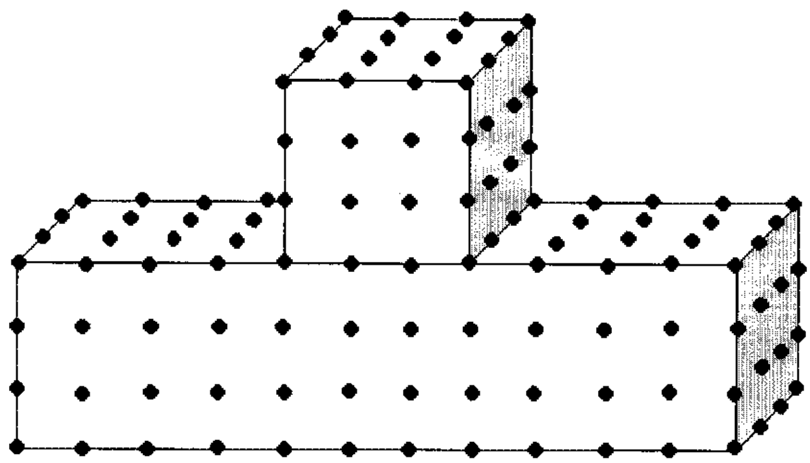

Figure 2. **3-D Domain and Integration Points**

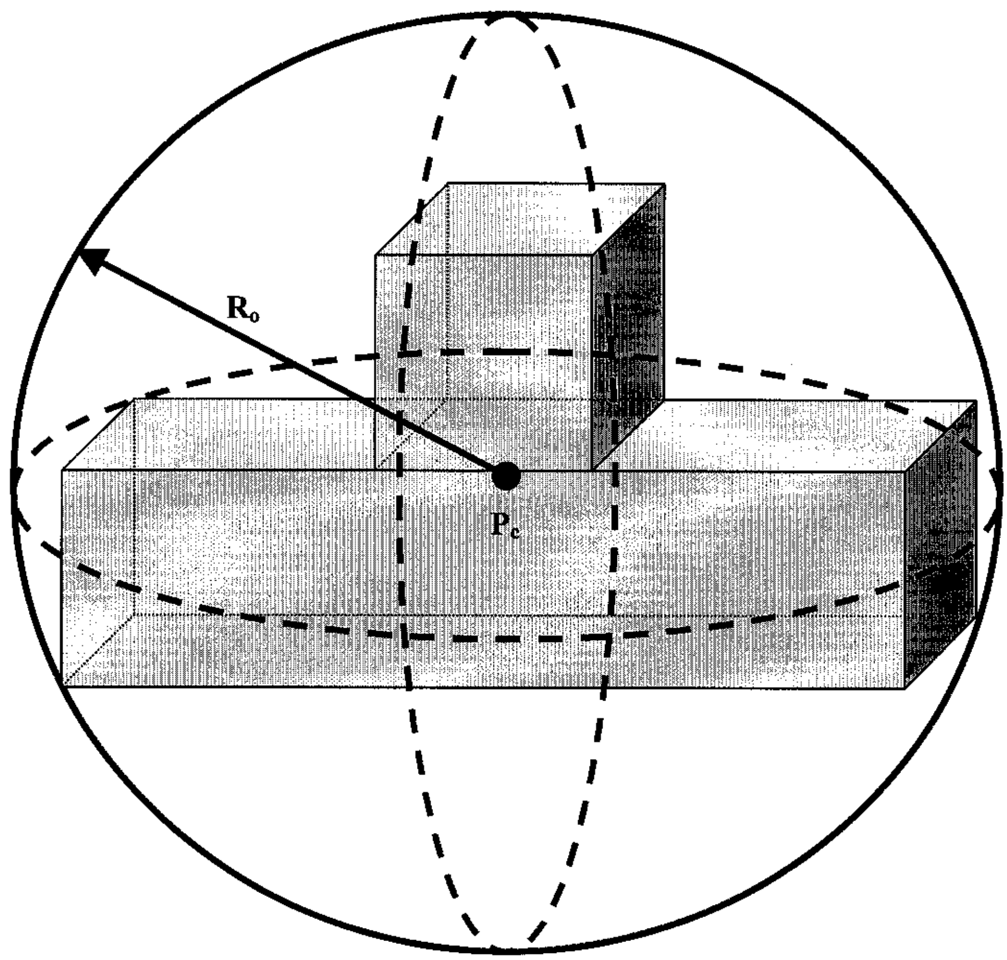

Figure 3. **3-D Domain and Enclosing Sphere**

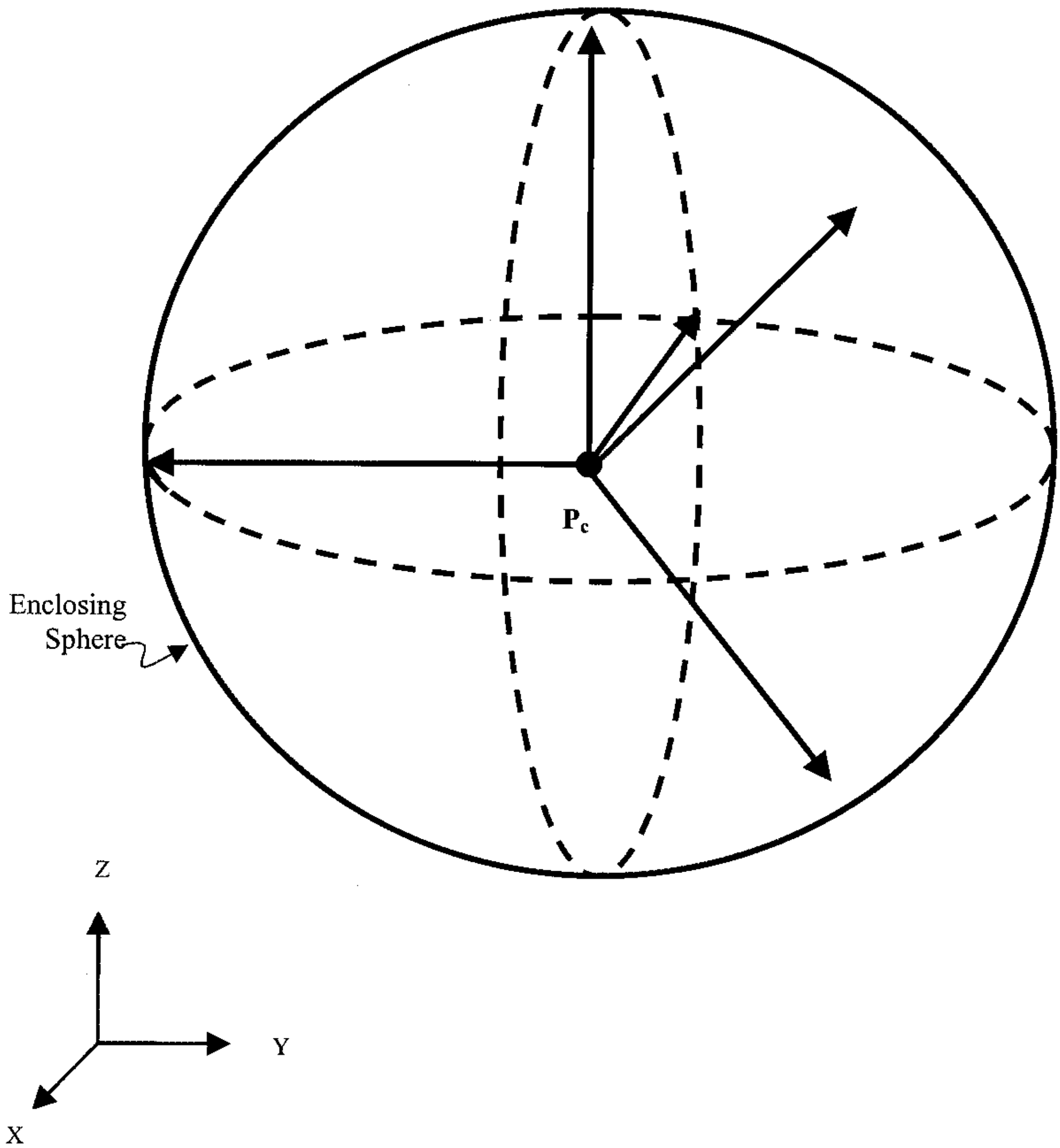

Figure 4. **5-Projection Vectors**

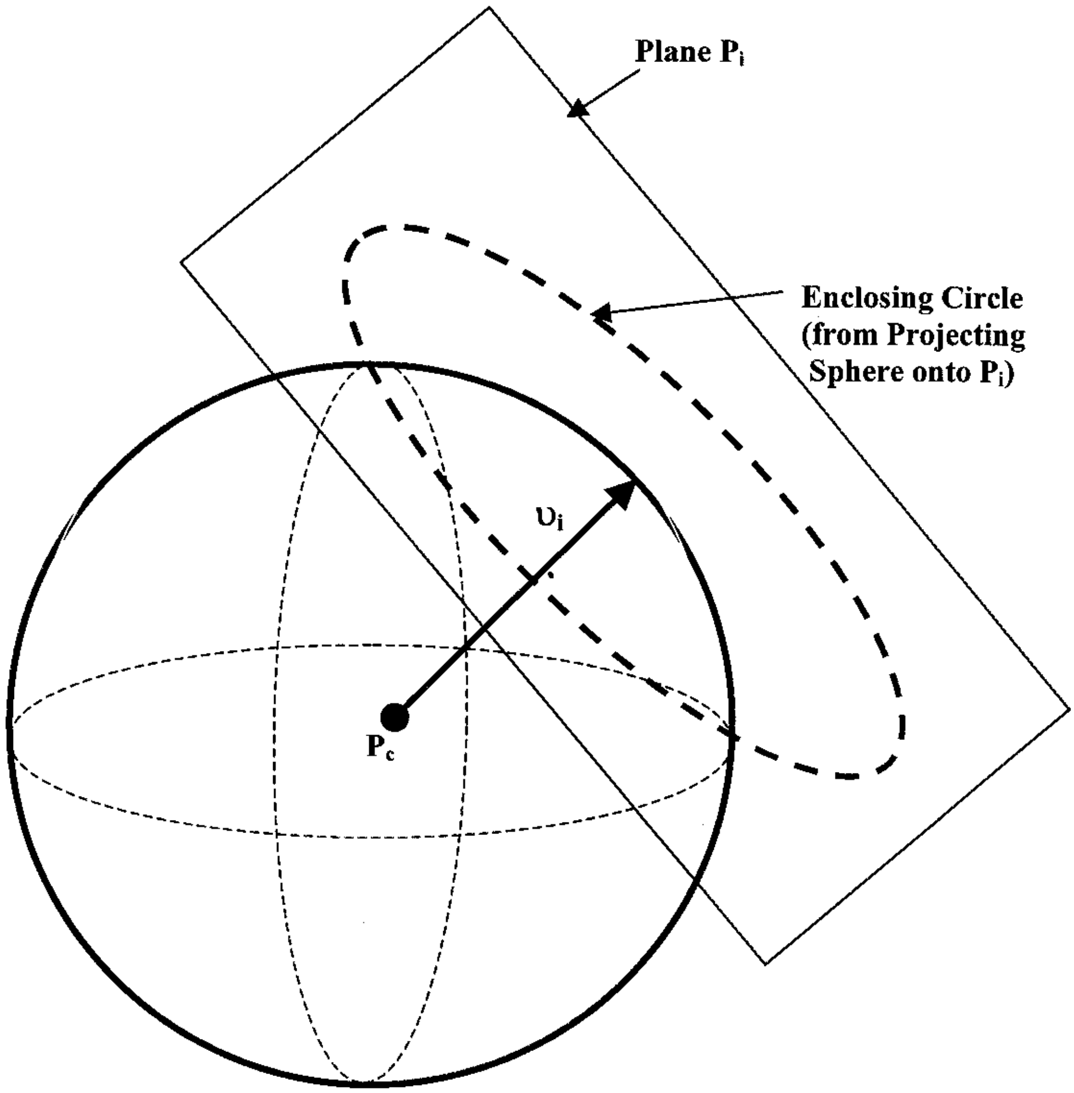

Figure 5. Example Projection Plane, P_i

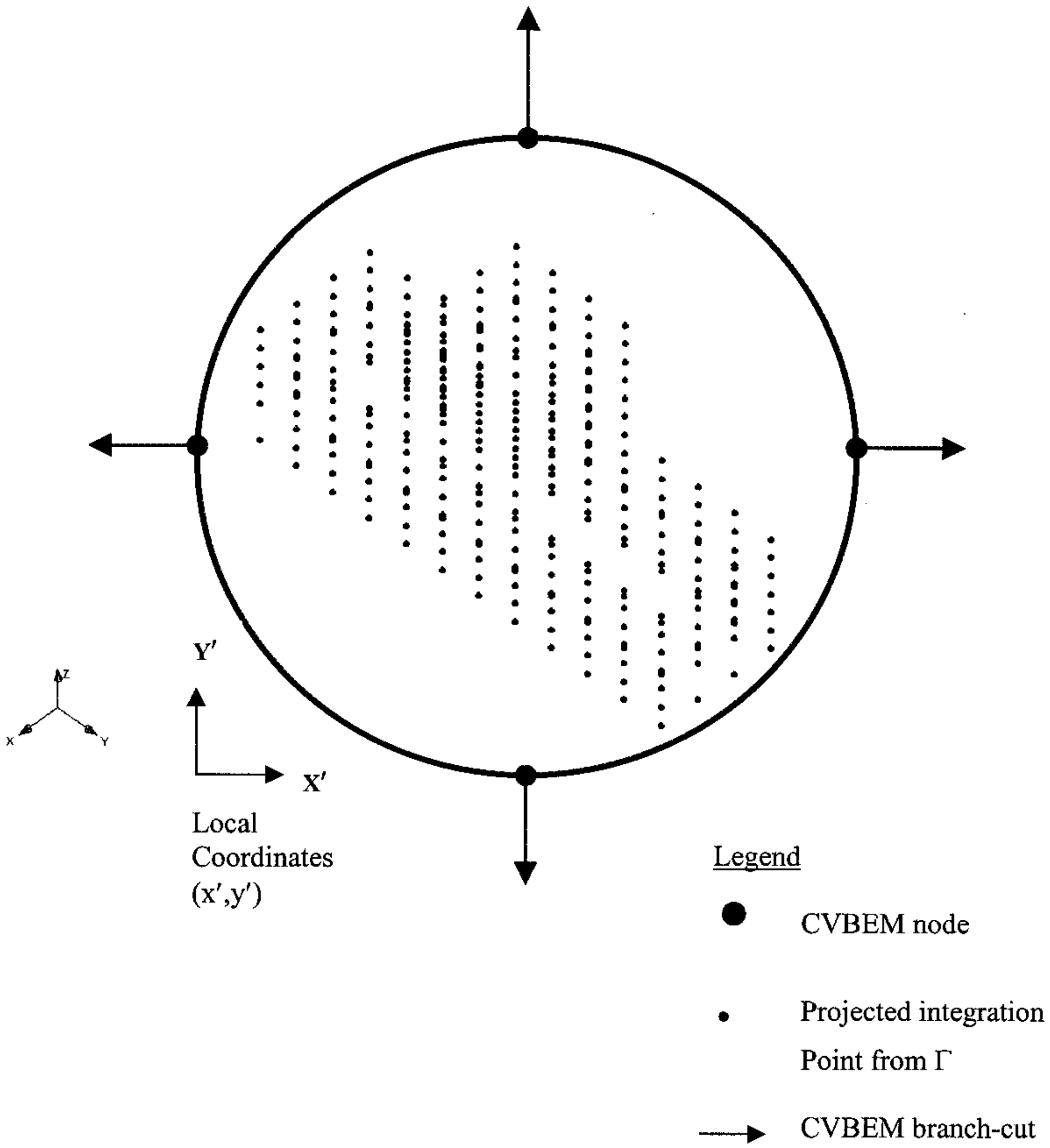

Figure 6. Projected Integration Points and Enclosing Circle
Note Local Coordinates (x′,y′)

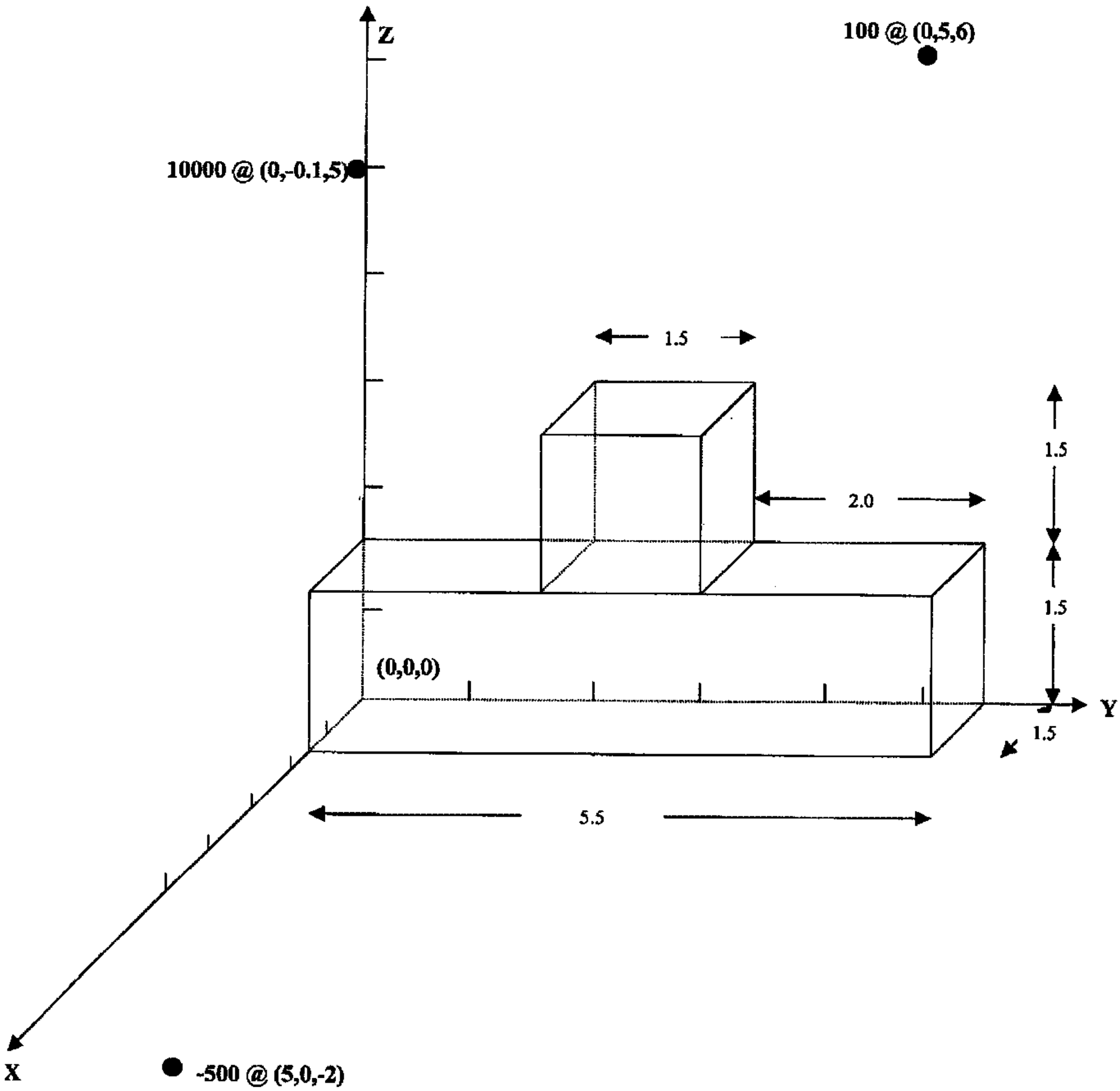

Figure 7a. 3-D Solid T-Shaped Stand Geometry showing Sink and Sources

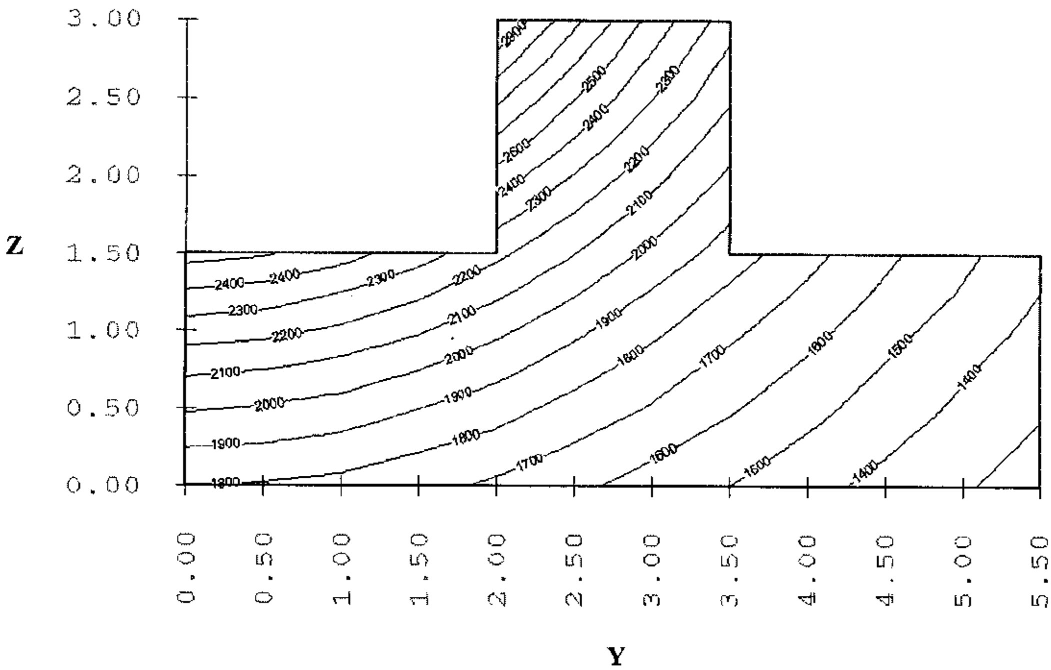

Figure 7b. **Exact Solution on X=1.5**

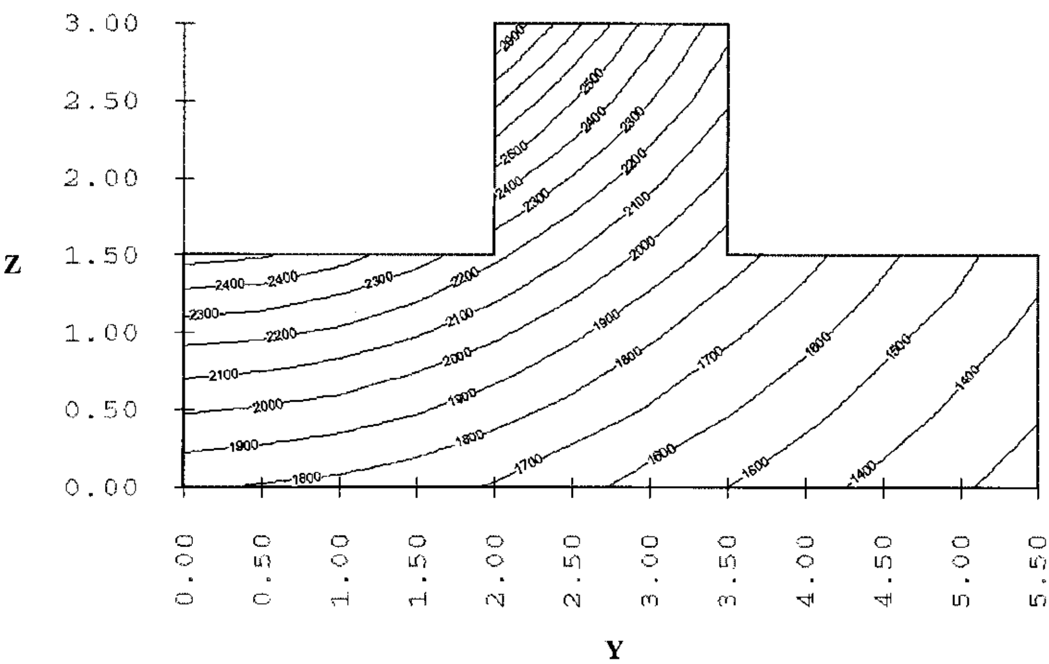

Figure 7c. **Approximation Solution on X=1.5**

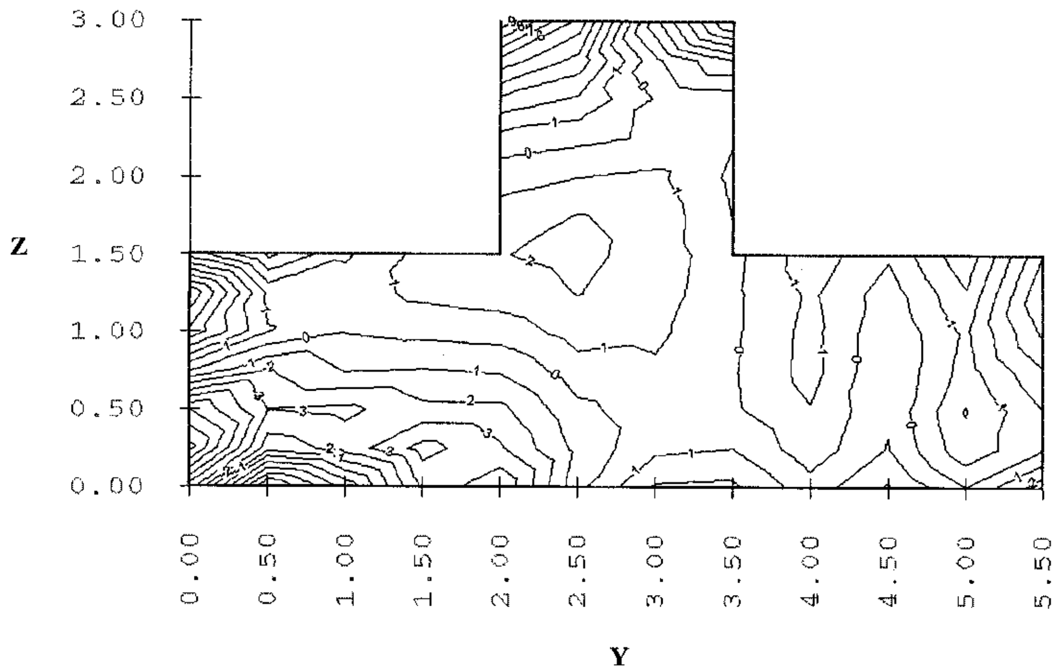

Figure 7d. **Approximation Error on X=1.5**

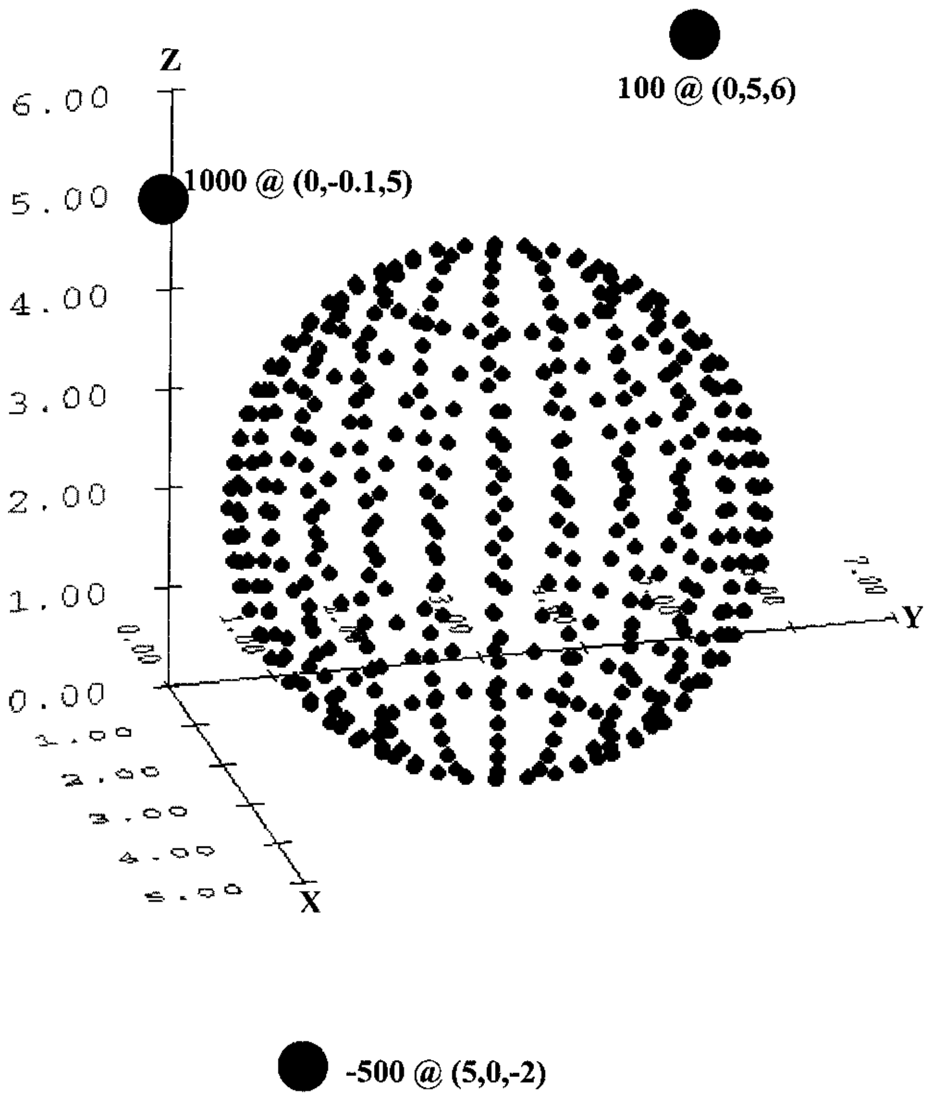

Figure 8a. Sphere Geometry showing Integration Points, and Sink and Sources

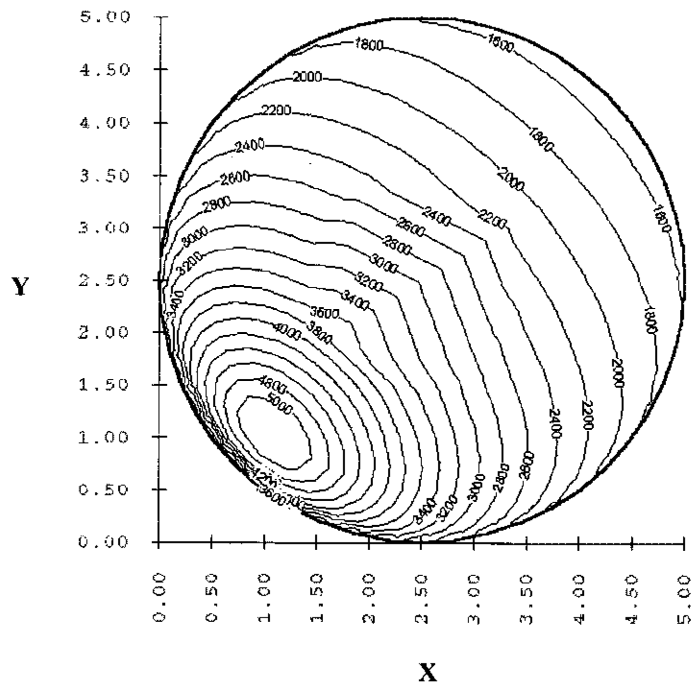

Figure 8b. Exact Solution on North Hemisphere

With Five Projection Planes, Vector displacements from Center of Sphere: (0.,0.,-2.5), (0.,-2.5,0.), (-2.5,0.,0.), (2.5,2.5,2.5) and (-2.5,2.5,-2.5)

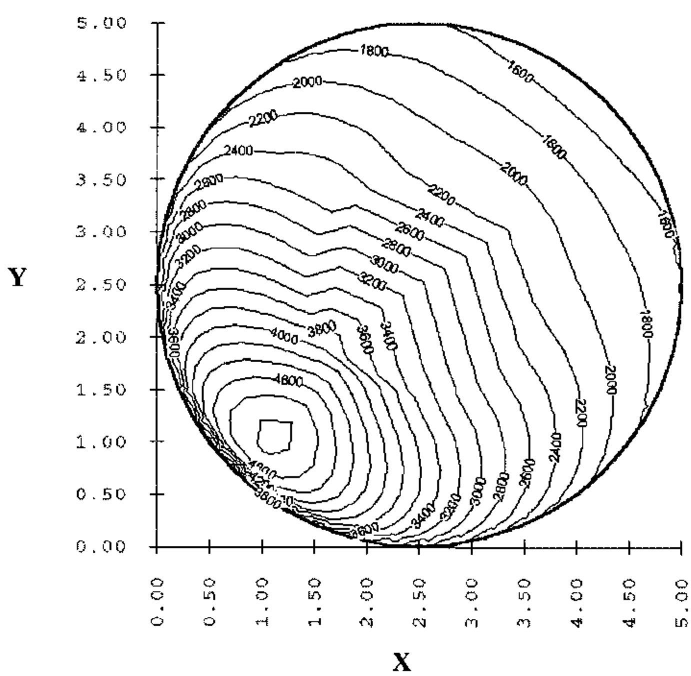

Figure 8c. Approximation on North Hemisphere

With Five Projection Planes, Vector displacements from Center of Sphere: (0.,0.,-2.5), (0.,-2.5,0.), (-2.5,0.,0.), (2.5,2.5,2.5) and (-2.5,2.5,-2.5)

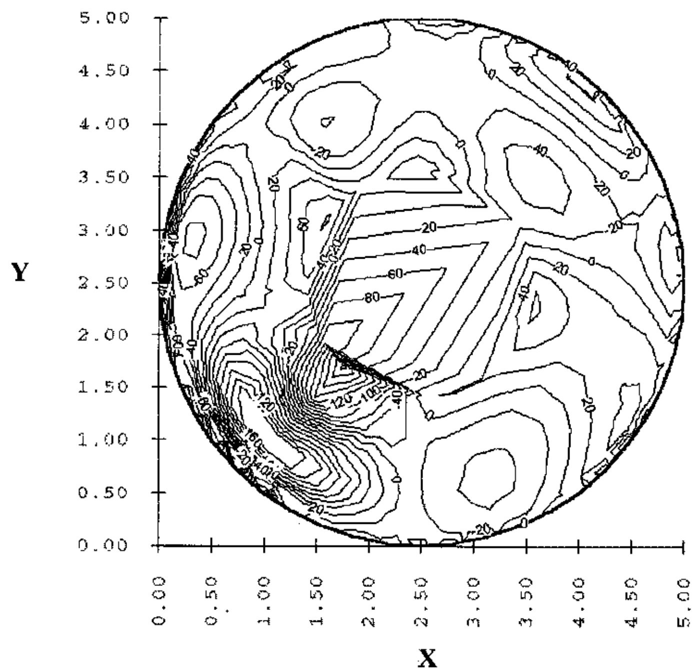

Figure 8d. Approximation Error on North Hemisphere

With Five Projection Planes, Vector displacements from Center of Sphere: (0.,0.,-2.5), (0.,-2.5,0.), (-2.5,0.,0.), (2.5,2.5,2.5) and (-2.5,2.5,-2.5)

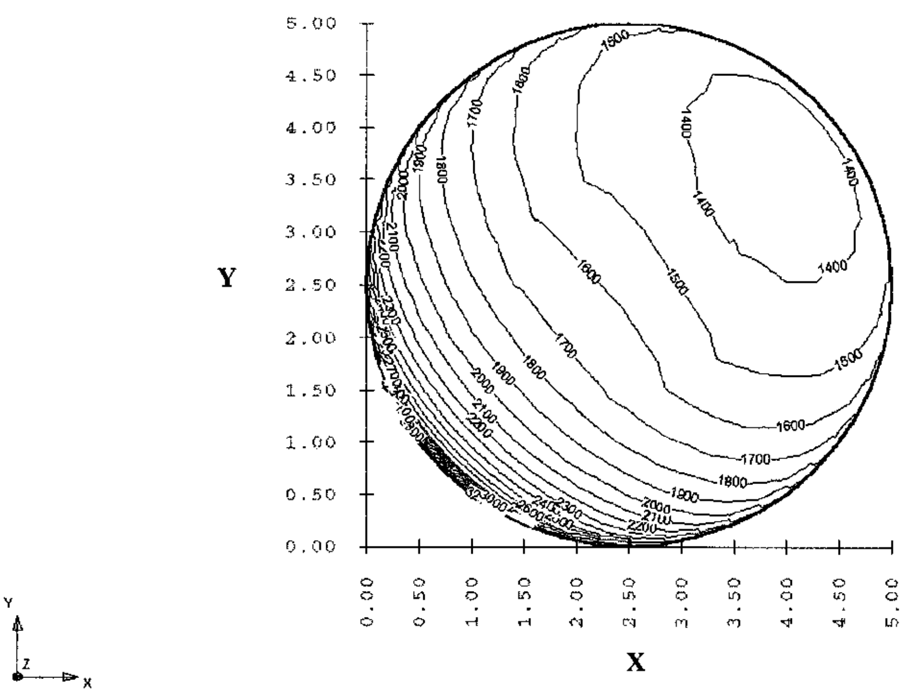

Figure 8e. Exact Solution on South Hemisphere

With Five Projection Planes, Vector displacements from Center of Sphere: (0.,0.,-2.5), (0.,-2.5,0.), (-2.5,0.,0.), (2.5,2.5,2.5) and (-2.5,2.5,-2.5)

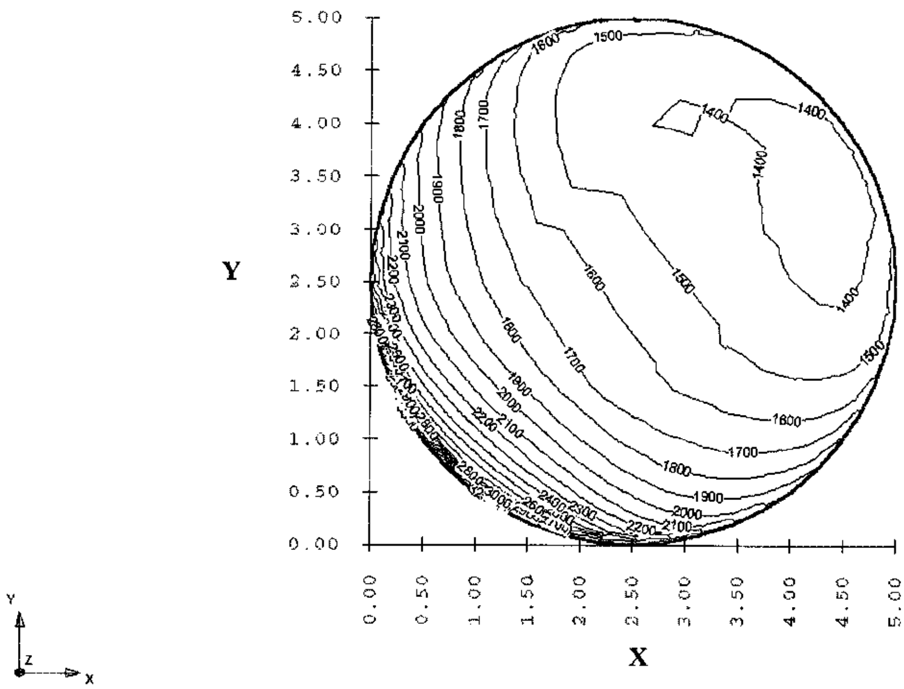

Figure 8f. Approximation on South Hemisphere

With Five Projection Planes, Vector displacements from Center of Sphere:
(0.,0.,-2.5), (0.,-2.5,0.), (-2.5,0.,0.), (2.5,2.5,2.5) and (-2.5,2.5,-2.5)

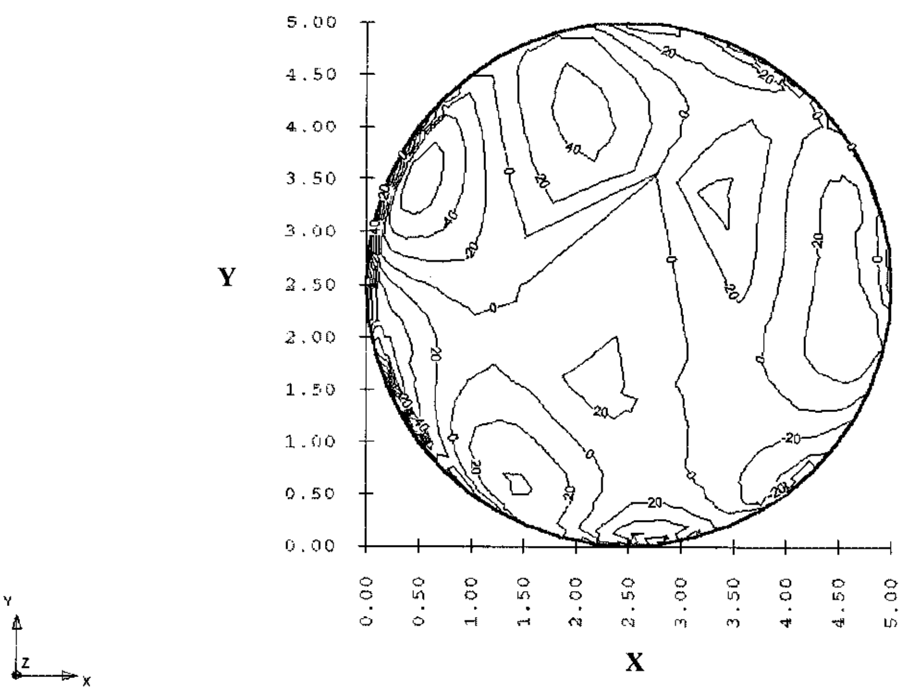

Figure 8g. Approximation Error on South Hemisphere

With Five Projection Planes, Vector displacements from Center of Sphere:
(0.,0.,-2.5), (0.,-2.5,0.), (-2.5,0.,0.), (2.5,2.5,2.5) and (-2.5,2.5,-2.5)

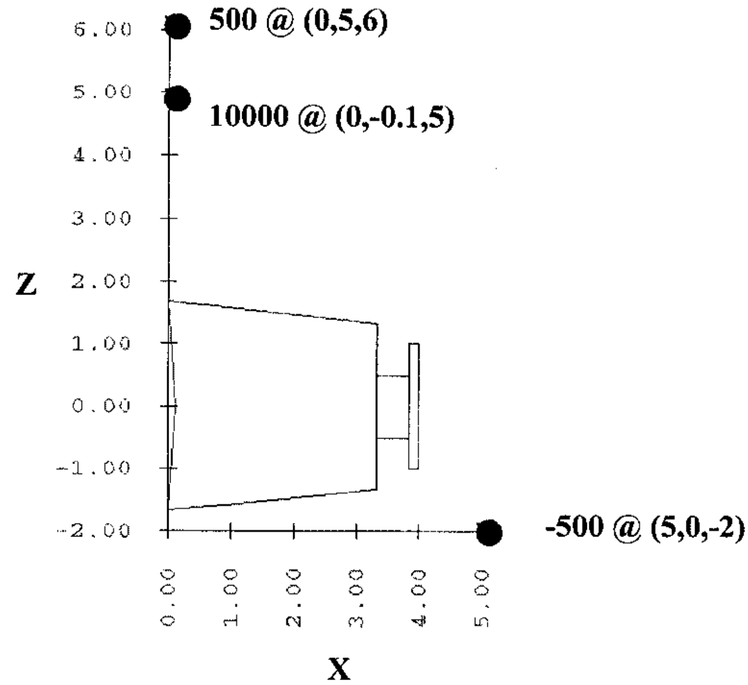

Figure 9a. Solid Door Knob Geometry showing Sink and Sources

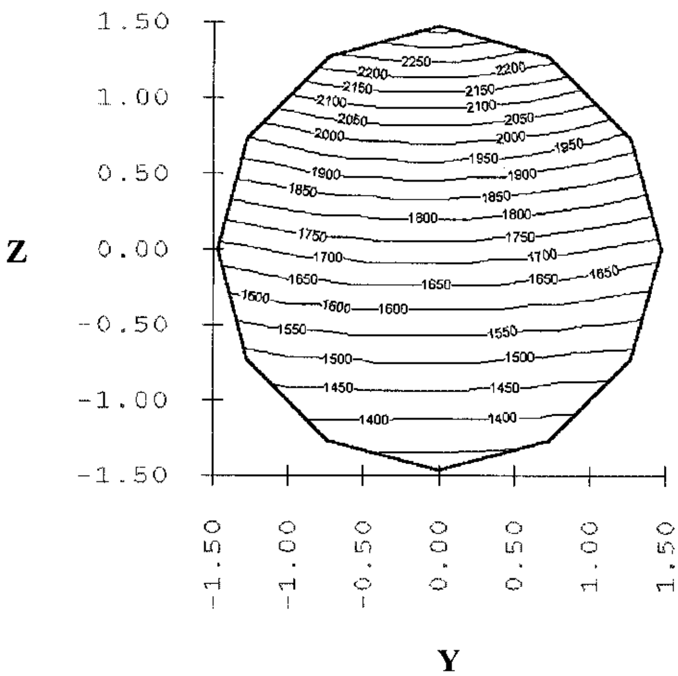

Figure 9b. **Exact Solution (slice at x = 2.0)**

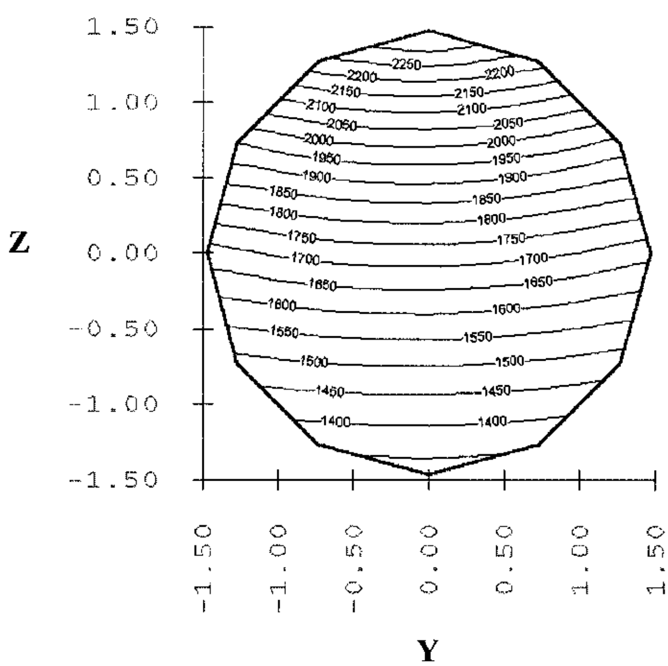

Figure 9c. **Approximation Solution (slice at x = 2.0)**

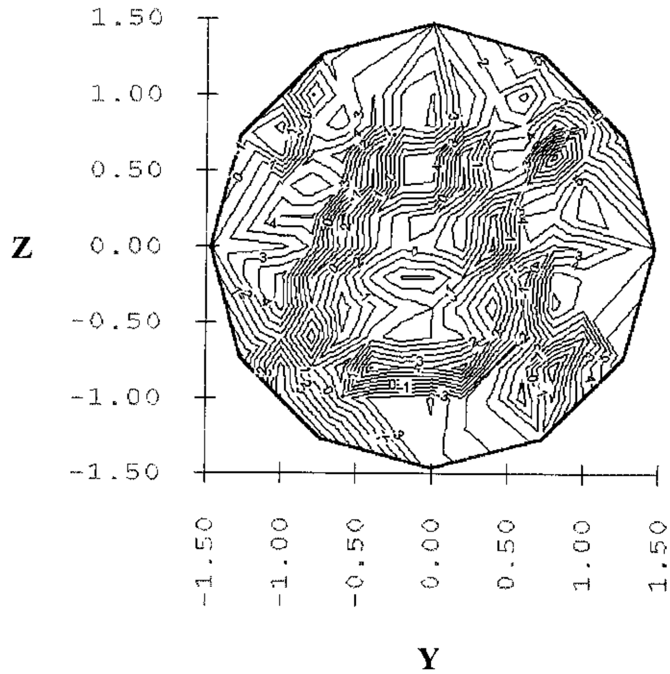

Figure 9d. **Approximation Error (slice at x = 2.0)**

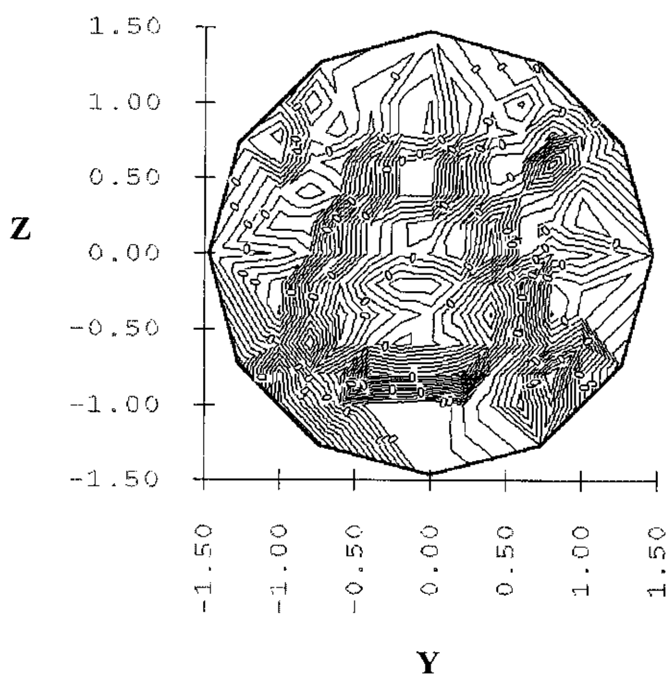

Figure 9e. **Relative Error (slice at x = 2.0)**

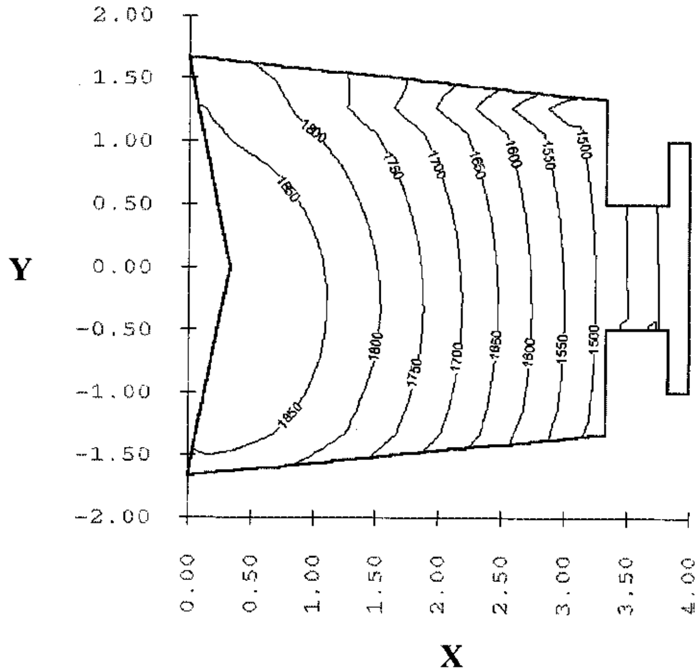

Figure 9f. **Exact Solution (slice at z = 0.)**

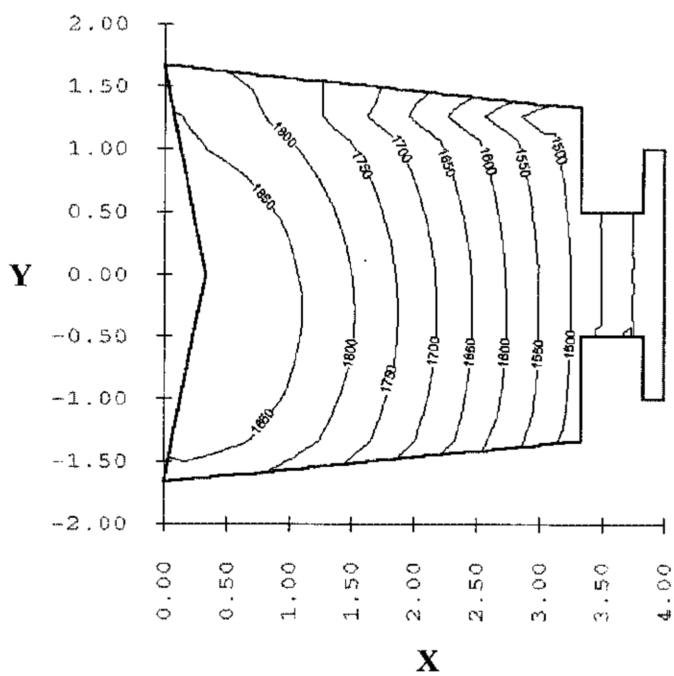

Figure 9g. **Approximation Solution (slice at z = 0.)**

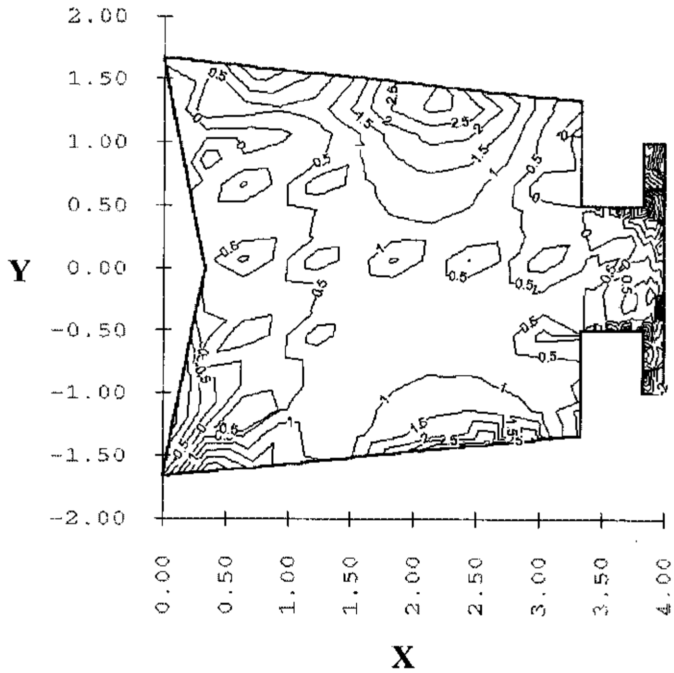

Figure 9h. **Approximation Error (slice at z = 0.)**

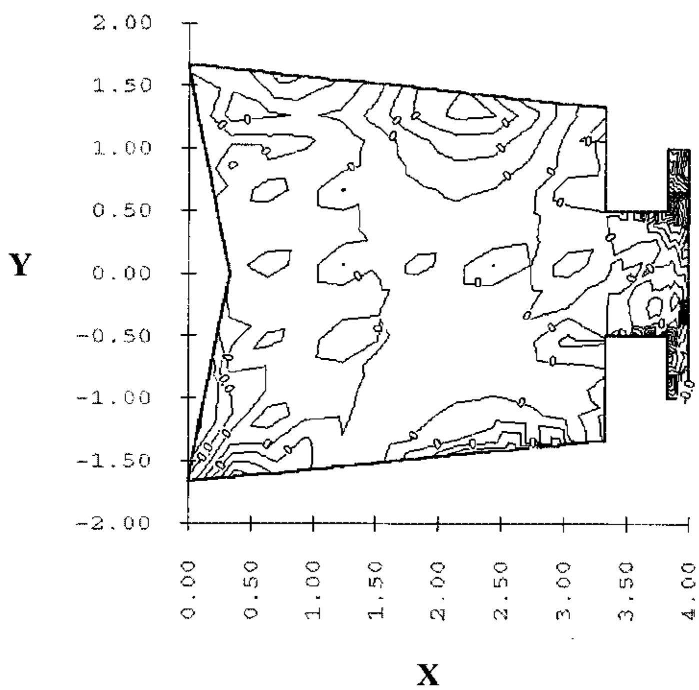

Figure 9i. **Relative Error (slice at z = 0.)**

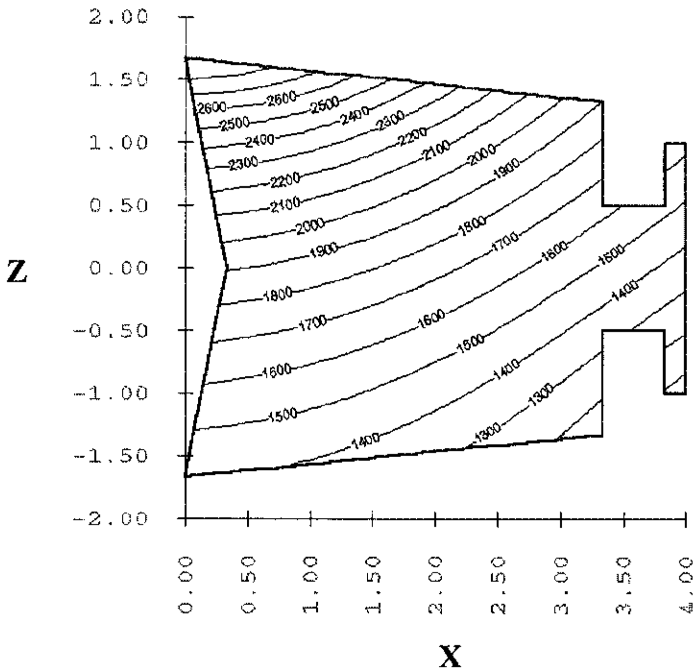

Figure 9j. **Exact Solution (slice at y = 0.)**

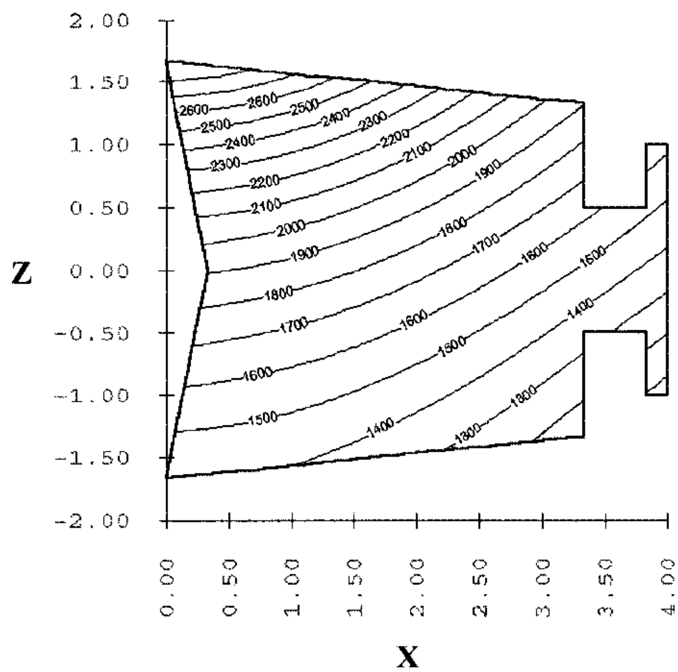

Figure 9k. **Approximation Solution (slice at y = 0.)**

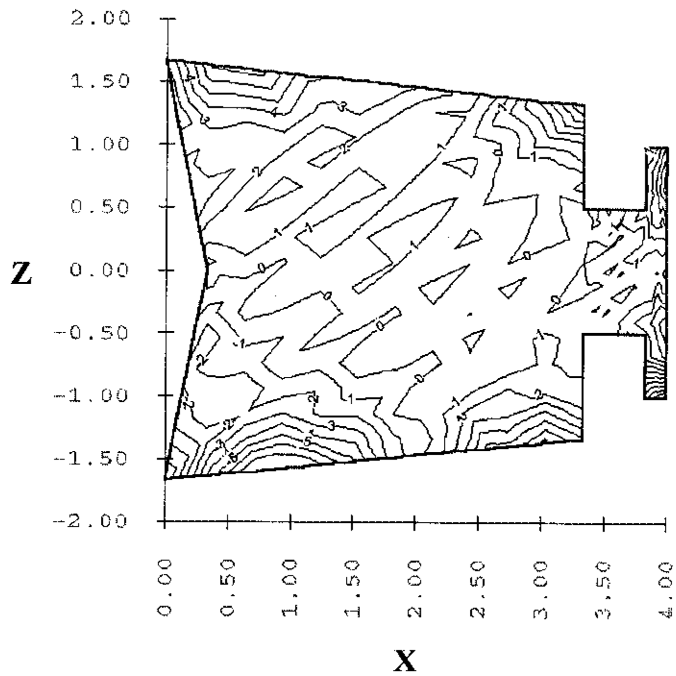

Figure 9l. **Approximation Error (slice at y = 0.)**

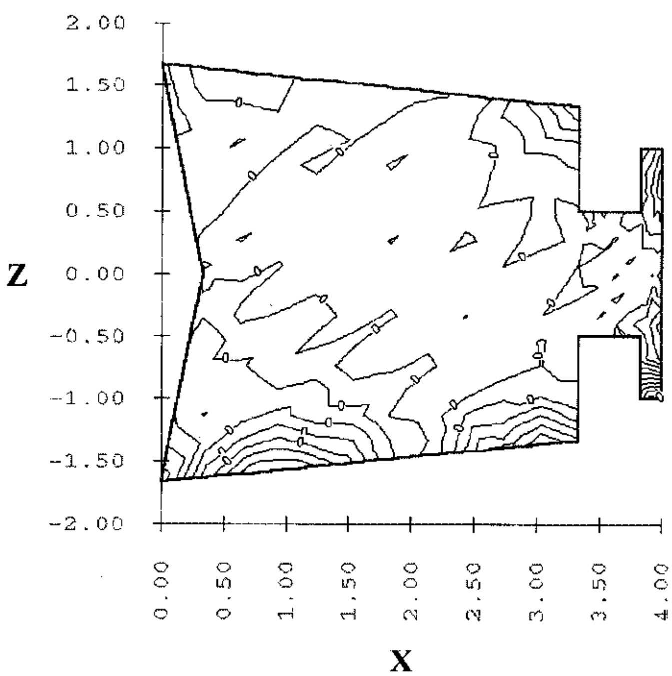

Figure 9m. **Relative Error (slice at y = 0.)**

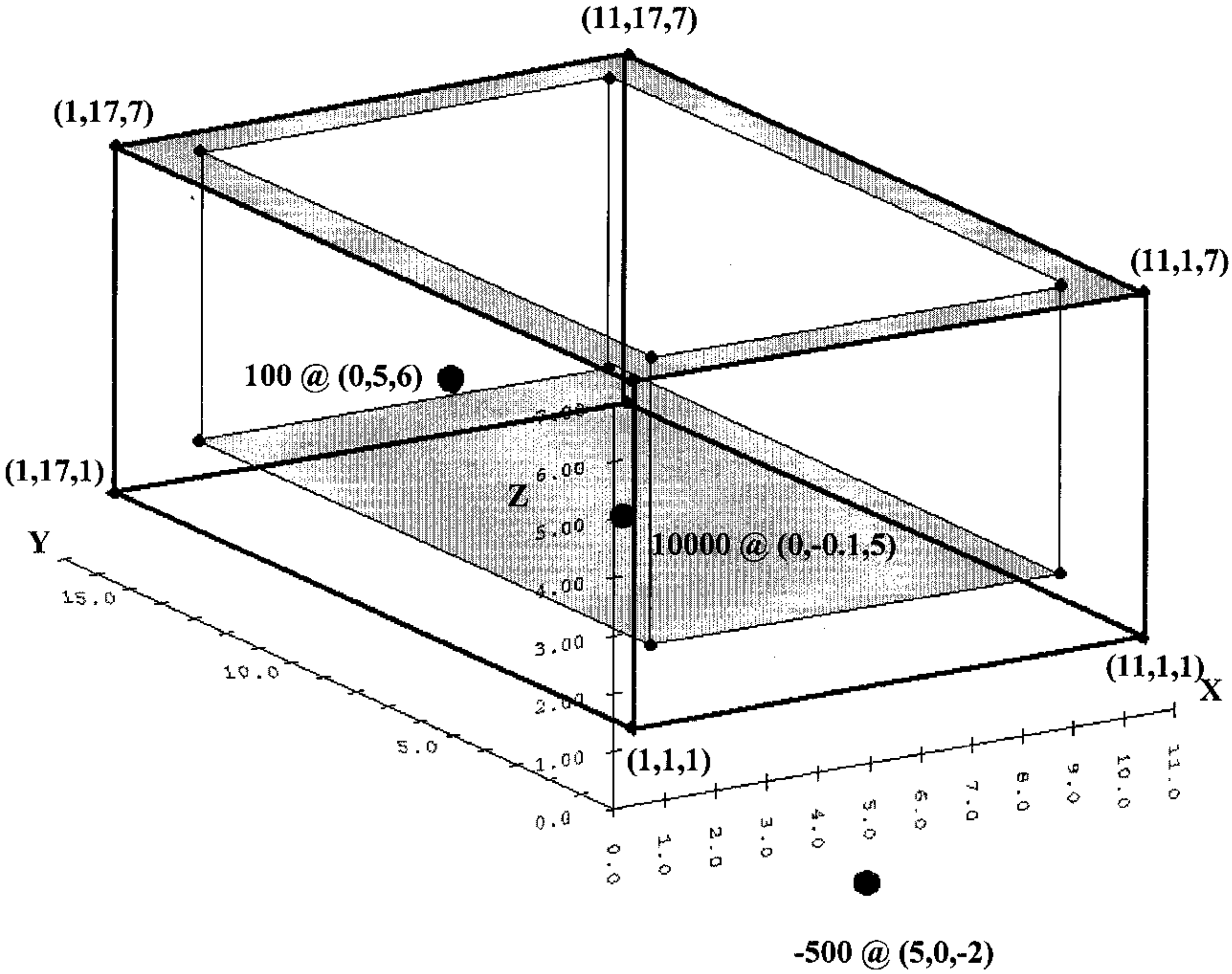

Figure 10a. 3-D Geometry of Open Box (with Thickness = 1) showing Sink and Sources

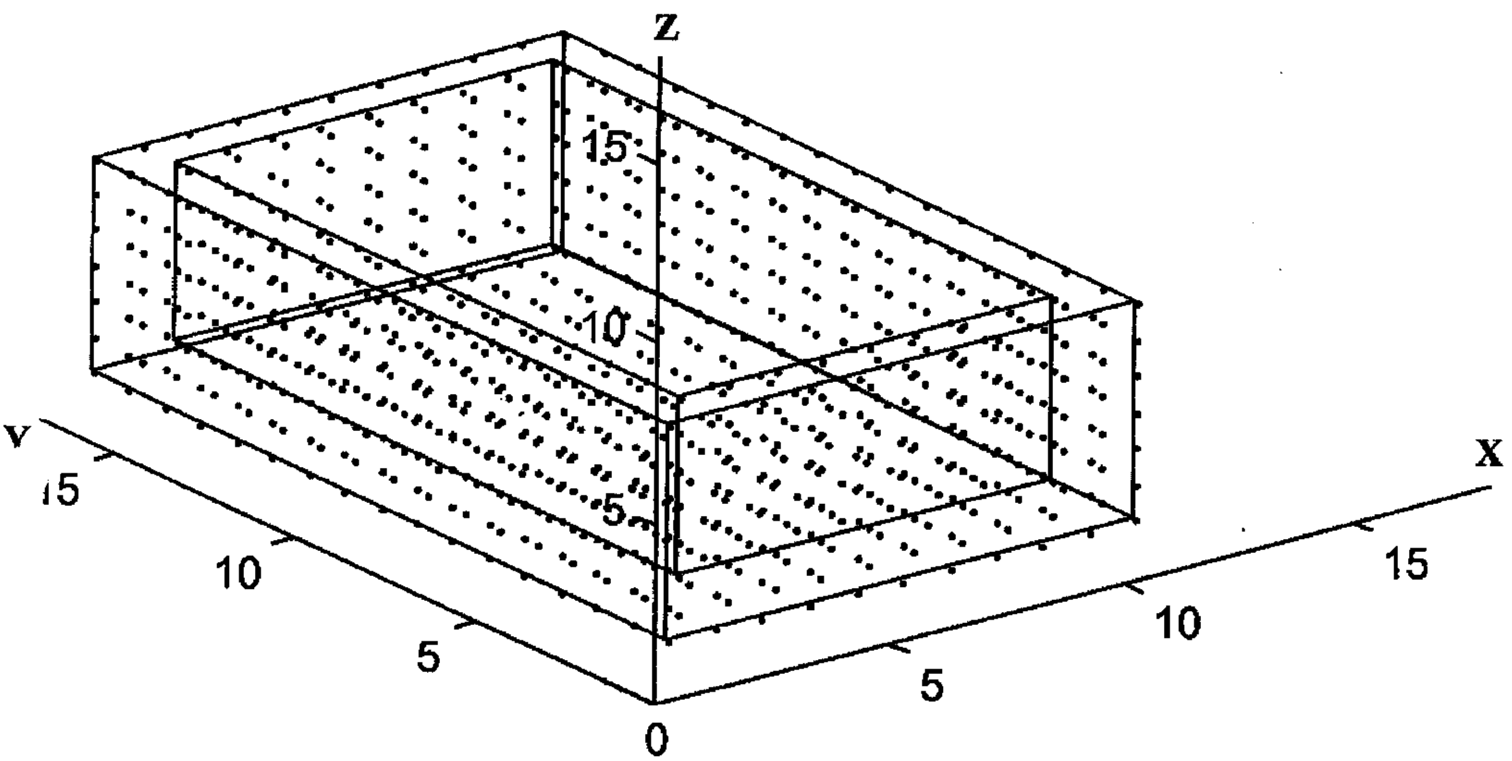

Figure 10b. Integration Point Distribution

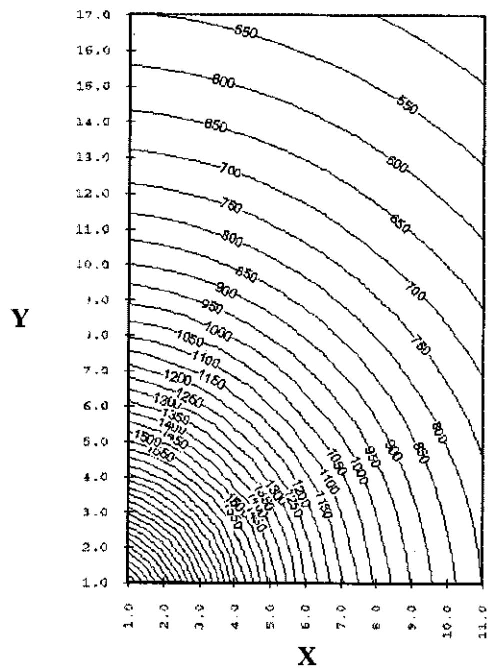

Figure 10c. **Exact Solution (slice at z = 1.5)**

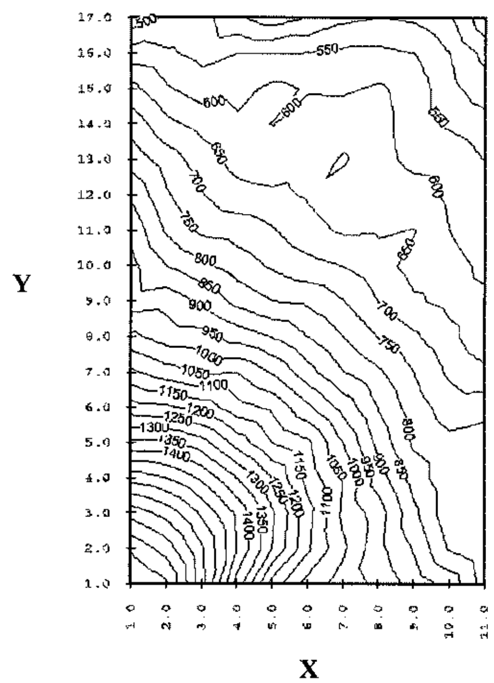

Figure 10d. **Approximation Solution (slice at z = 1.5)**

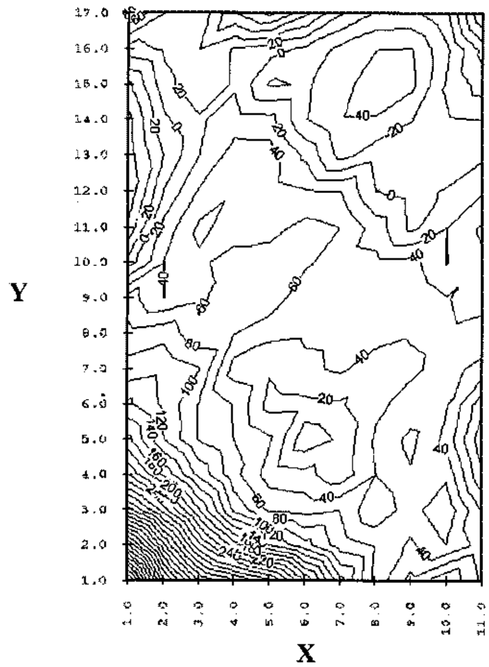

Figure 10e. **Approximation Error (slice at z = 1.5)**

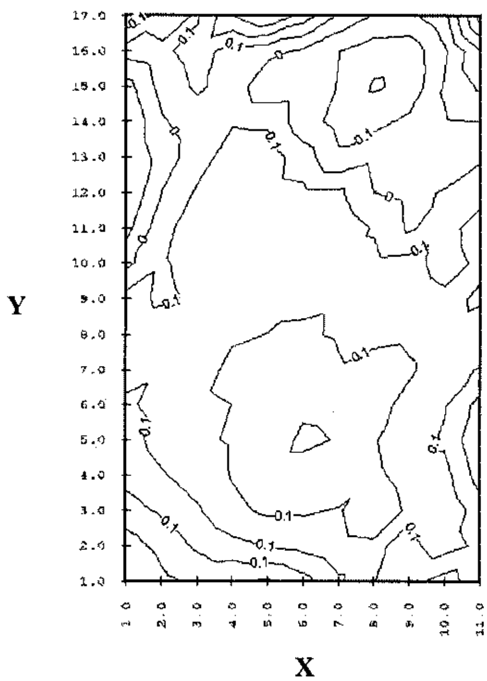

Figure 10f. **Relative Error (slice at z = 1.5)**

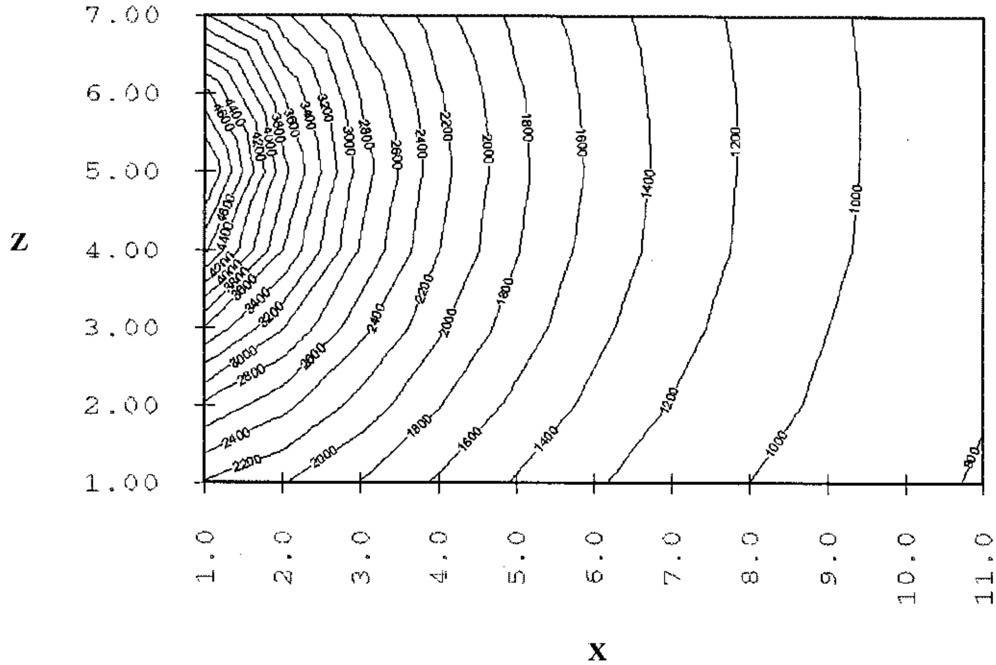

Figure 10g. **Exact Solution (slice at y = 1.5)**

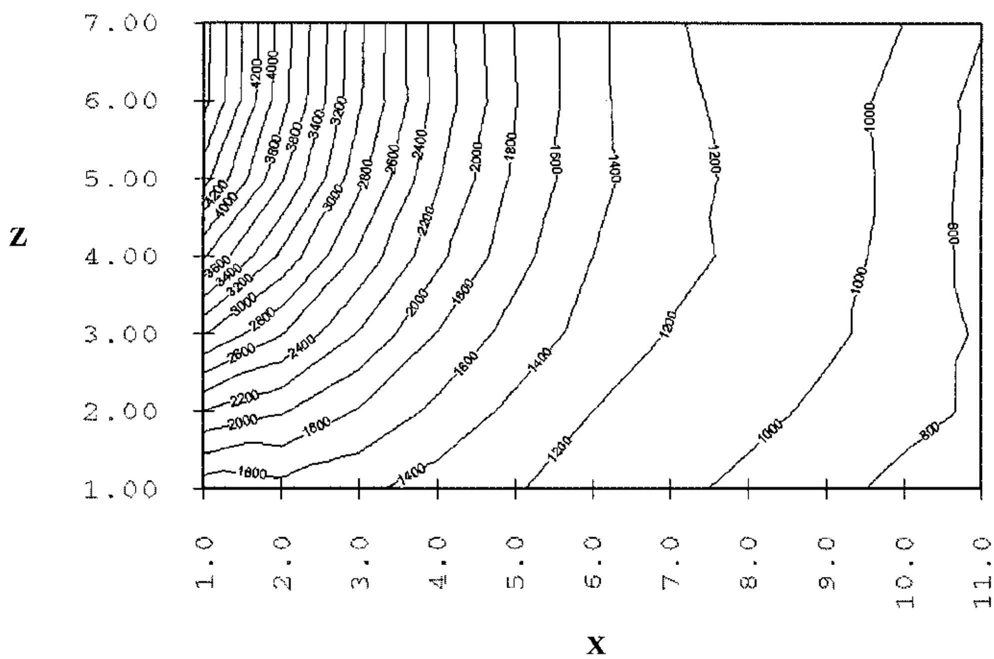

Figure 10h. **Approximation Solution (slice at y = 1.5)**

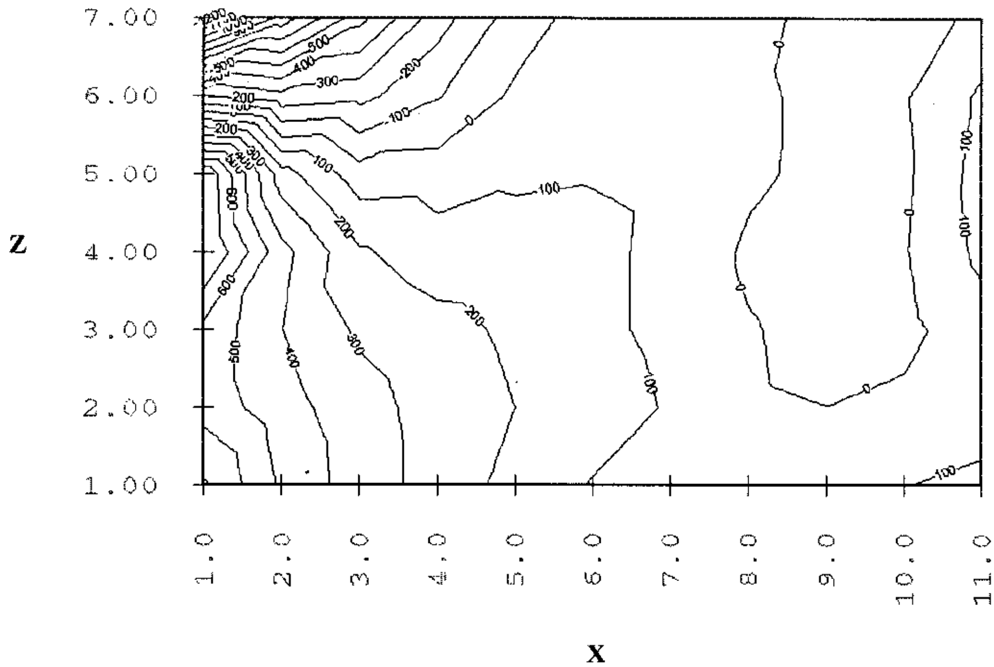

Figure 10i. **Approximation Error (slice at y = 1.5)**

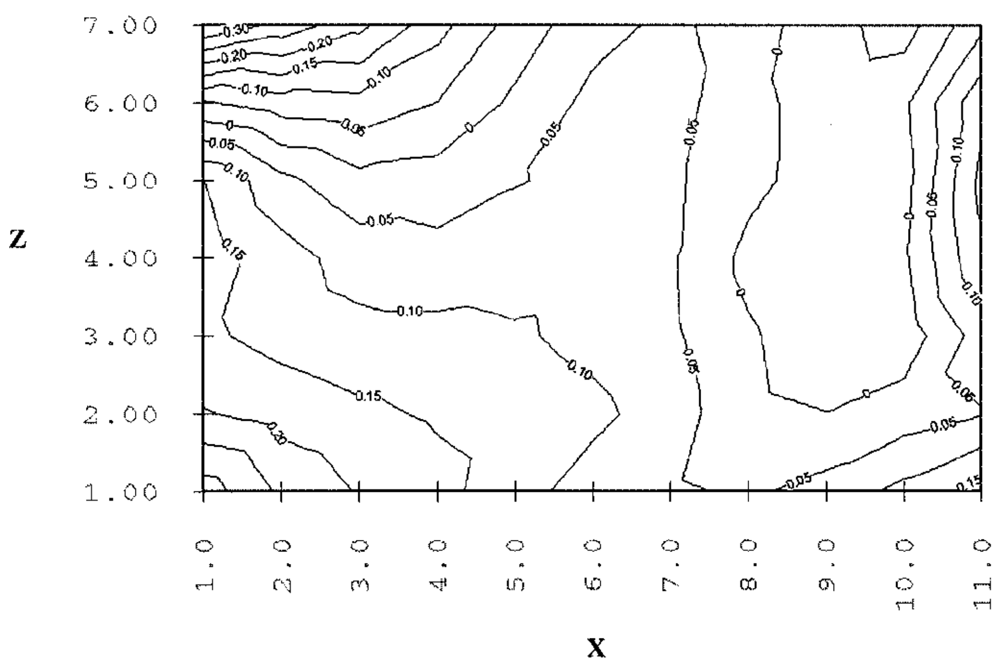

Figure 10j. **Relative Error (slice at y = 1.5)**

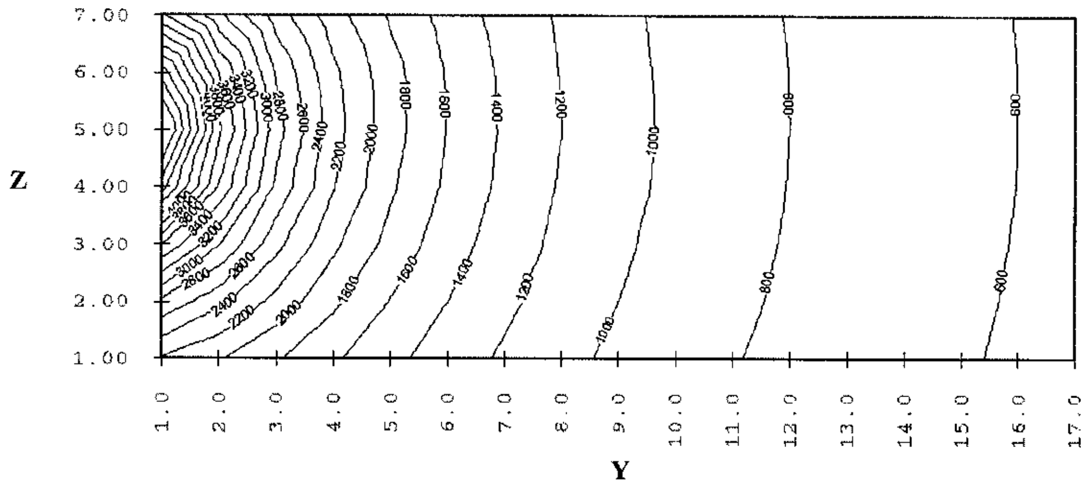

Figure 10k. **Exact Solution (slice at x = 1.5)**

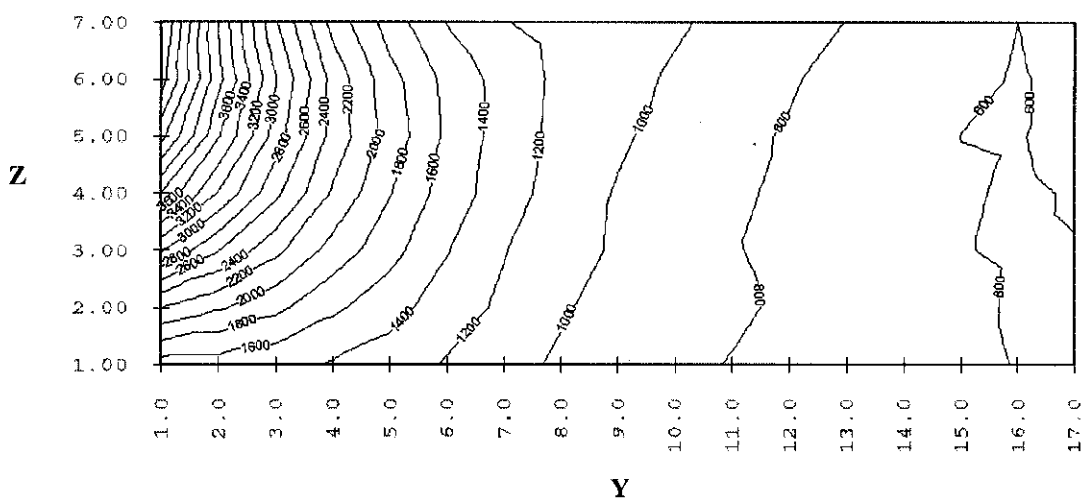

Figure 10l. **Approximation Solution (slice at x = 1.5)**

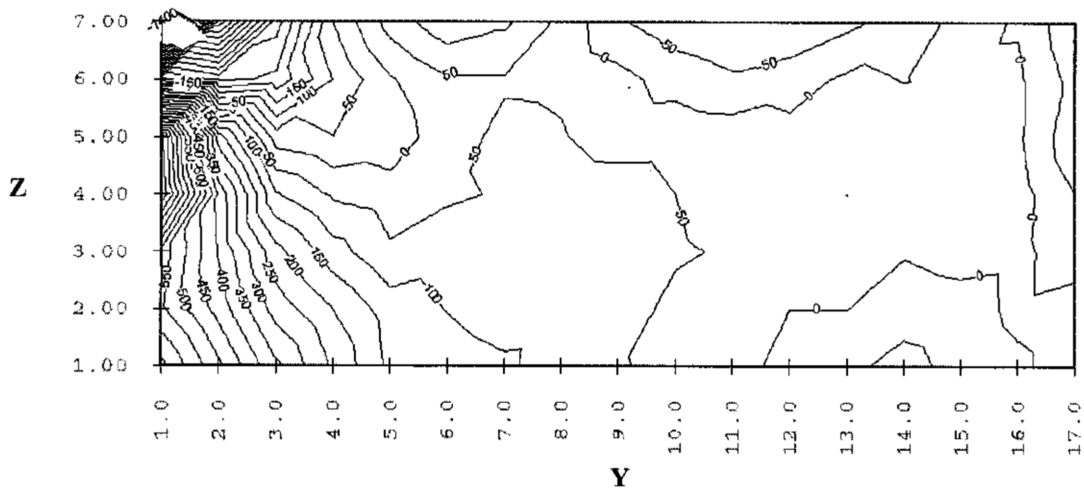

Figure 10m. **Approximation Error (slice at x = 1.5)**

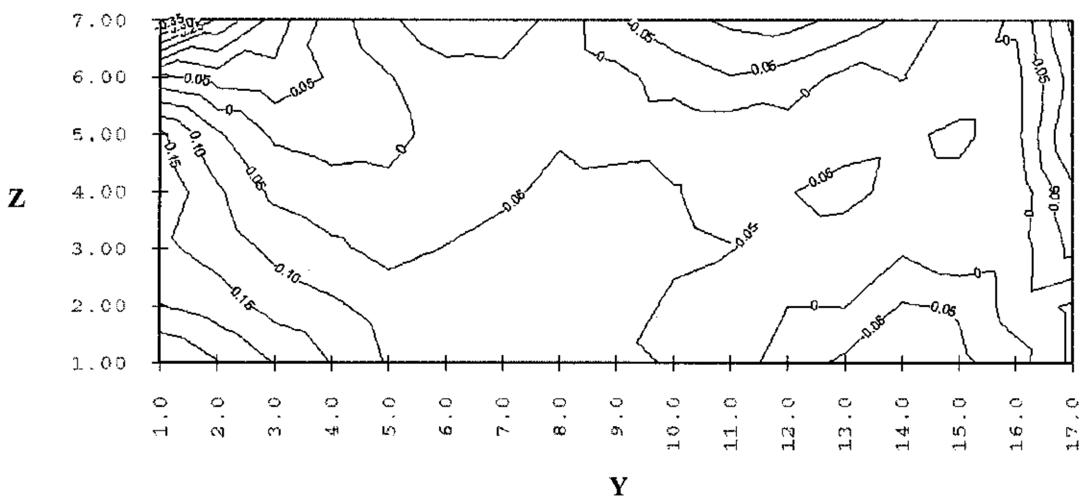

Figure 10n. **Relative Error (slice at x = 1.5)**