LINKING THE BALANCED DESIGN STORM UNIT HYDROGRAPH, RATIONAL, AND U.S.G.S. REGRESSION EQUATION METHODS FOR ESTIMATING RUNOFF PEAK FLOW RATES

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Abstract

In practice, runoff peak flow rates are typically estimated by the Rational Method, a design storm unit hydrograph (UH) method, or a statistical regression equation. In this paper, the balanced design storm UH procedure (HEC Training Document 15) is used to derive a Rational Method peak flow rate equation that, in turn, is used to derive a regression equation. This new mathematical linkage across these three widely used peak flow rate estimation techniques provide foundation as to how these approaches differ or agree, and may also provide an answer as to which method is "best"; specifically, the methods are essentially the same for many practical conditions, and where they differ, the underpinnings of their mathematical structures is illuminated.

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INTRODUCTION

The Rational Method continues to be a widely used runoff peak flow rate estimator for designing small drainage facilities (Hromadka, et al, 1987, Hromadka et al, 1994, among others). The unit hydrograph (UH) balanced T-year design storm method, as described in the U.S. Army Corps of Engineers Training Document 15 (or TD-15, 1982), is another widely used technique for estimating peak flow rates, and involves considerably more computational effort than the Rational Method. Additionally, HEC TD-15 has been adopted, with modifications, as the basis for a number of recently developed Hydrology Manuals for county flood control agencies (see Hromadka et al, 1986, 1987, 1992, among others). Peak flow rates are also estimated by statistical regression equations (e.g., the USGS equations) that are calibrated to local runoff data.

In this paper, the balanced design storm UH approach is used to mathematically derive a Rational Method equation for the two cases of (1) catchment areas less than 1 square mile (also see the derivation contained in Hromadka, 1995), and (2) catchment areas greater than 1 square mile. It is shown that peak flow rates developed from the well-known TD-15 (1982) balanced design storm UH method are equal to Rational Method peak flow estimates, except that the underlying normalized UH (or S-Graph) results in a new constant to be multiplied to the usual Rational Method mean rainfall intensity. The linkage developed herein between the Rational Method and the balanced design storm UH method also depends upon the loss function used. The widely used phi-index (constant loss function) approach and the constant proportion loss functions are considered. The mathematical development results in a simple modification of the standard Rational Method equation structure, with the introduction of a fixed constant (multiplied to mean rainfall intensity) that corresponds to the parent normalized UH (or S-Graph type) and also the rainfall depth-duration log-log exponent. For areas greater than 1 square mile, the effects of depth-area adjustments are included, resulting in a peak flow rate estimator that corresponds to the typical regression equation structure. Although it is oftentimes conjectured that there exists a linkage between the three considered peak flow rate estimators, it appears that a constructive mathematical derivation across these different peak flow rate estimators has heretofore not been presented in the open literature.

MATHEMATICAL DEVELOPMENT

In the following, a Rational Method peak flow rate estimator is derived from the balanced design storm UH method (of HEC TD-15). This derivation is presented in detail in Hromadka (1995). Only the key steps are presented herein for the reader's convenience, so that the subsequent extension to regression equations can be better seen.

1. Unit Hydrographs

Unit hydrographs (UH) for a catchment may be developed from normalized S-graphs (Hromadka and Whitley, 1989; HEC TD-15, 1982). Generally, S-graphs can be developed that apply across large regions; for example, several county-wide hydrology manuals use S-graphs that apply to mountain, desert, foothill, or valley area catchments (see Hromadka, 1986, 1987, 1992). The S-graph is typically expressed by $S(\ell)$ where ℓ is a proportion (percent) of catchment lag, where catchment lag can be related to the catchment time of concentration, T_c, by (Hromadka et al, 1987)

$$\text{lag} = \gamma T_c \tag{1}$$

where γ is a calibration constant. Then $S(\ell) = S\left(\frac{t100}{\gamma T_c}\right)$, where the UH is expressed as a function of T_c.

For $T_c = 1$ and catchment area $A = 1$, a normalized UH results, $U(t)$. For $T_c \neq 1$ or $A \neq 1$, the catchment UH, $u(t, T_c, A)$, is given by

$$u(t) = u(t,T_c,A) = \frac{A}{T_c} U\left(\frac{t}{T_c}\right) \tag{2}$$

where by definition,

$$\int_0^\infty u(t, T_c, A)\, dt = A\int_0^\infty U\left(\frac{t}{T_c}\right)\frac{dt}{T_c} = AU_o \tag{3}$$

where U_o is a constant; and where $u(t, T_c, A)$, may be written as $u(t)$.

2. Rainfall Depth-Duration Relationships

Precipitation depth-duration relationships, for a given return frequency, is generally given by the power law (Hromadka and Whitley, 1996),

$$D(\tau) = a\tau^{b} \tag{4}$$

where a>0 is a function of return frequency, and is assumed constant for a selected design storm return frequency; "b" is typically a constant for large regions (e.g., entire counties); $D(\tau)$ is the rainfall depth corresponding to peak duration τ; and τ is the selected duration of time of peak rainfall depth.

Mean rainfall intensity, I(t), is

$$I(\tau) = \frac{1}{\tau} D(\tau) = a\tau^{b-1} \tag{5}$$

and instantaneous rainfall intensity, i(t), is

$$i(\tau) = \frac{d}{d\tau} D(\tau) = ab\tau^{b-1} = bI(\tau). \tag{6}$$

It is noted that $I(\tau)$ is the usual mean rainfall intensity used in the Rational Method for a T_c value of τ.

The balanced design storm effective rainfall pattern (i.e., rainfall less losses, or rainfall excess), e(t), is a function of the instantaneous rainfall which is formulated into a nested storm pattern as described in HEC Training Document 15 (or TD-15 (1982)). Figure 1 illustrates an extension of the TD-15 balanced design storm pattern that is defined to have a peak at storm hour 16 (rather than at hour 12) and where rainfall is uniformly distributed with 2/3 of its mass preceding the peak (rather than being symmetrical about the peak).

With respect to Fig. 2, the nested design storm rainfall intensity can be resolved into components $i^{+}(t^{+})$ and $i^{-}(t^{-})$, respectively.

For a proportioning of rainfall quantities by allocation of a θ-proportion (for all durations) prior to time $t^{\pm} = 0$ (see Fig. 2 for the case of $\theta = 2/3$), instantaneous rainfall intensities are given by

$$i^-(t^-) = i^-(\theta t) = i(t) \tag{7}$$

or

$$i^-(t^-) = i\left(\frac{t^-}{\theta}\right) = \left(\frac{1}{\theta}\right)^{b-1} i(t^-) \tag{8}$$

Similarly,

$$i^+(t^+) = \left(\frac{1}{1-\theta}\right)^{b-1} i(t^+) \tag{9}$$

In the above, the HEC TD-15 balanced design storm instantaneous rainfall intensities, given a power law relationship of (6), is obtained by $\theta = 1/2$.

3. Peak Flow Rate Estimates from the Balanced Design Storm Unit Hydrograph Procedure and the Rational Method

Let $v(t) = v(\eta T_c - t)$ where $v(t)$ is a time-reversed plot of the UH, $u(t)$, and $X_o = \eta T_c$ is the total duration of the UH where η is a constant for a given S-graph. From Fig. 2 and, in order to obtain a peak flow estimate, aligning the UH peak to occur at time $t^{\pm} = 0$ (see Fig. 1),

$$v^+(t^+) = u(T_p - t^+),\ 0 \le t^+ \le T_p \tag{10}$$

$$v^-(t^-) = u(T_p + t^-),\ 0 \le t^- \le X_o - T_p = \eta T_c - T_p \tag{11}$$

where T_p is the time-to-peak of the UH. Then the peak flow rate from the balanced design storm UH procedure (in this case, for a constant loss rate "phi-index" model) is given by

$$Q_p = \int_{t^+=0}^{T_p} e^+(t^+)\, v^+(t^+)\, dt^+ + \int_{t^-=0}^{\eta T_c - T_p} e^-(t^-)\, v^-(t^-)\, dt^- \tag{12}$$

or

$$Q_p = \int_0^{T_p} i^+(t^+)\, v^+(t^+)\, dt^+ + \int_0^{\eta T_c - T_p} i^-(t^-)\, v^-(t^-)\, dt^-$$

$$- \phi\left[\int_0^{T_p} v^+(t^+)\, dt^+ + \int_0^{\eta T_c - T_p} v^-(t^-)\, dt^-\right] \tag{13}$$

where in (13), a "phi index" (or constant) loss function is used to compute rainfall excess; also, a necessary constraint imposed is that $i(\eta T_c) \geq \phi$.

Introducing a local time coordinate s defined by

$$s = \frac{t}{T_c} \tag{14}$$

then $t = sT_c$, $dt = T_c\, ds$.

The balanced design storm instantaneous rainfall intensities, $i^{\pm}(t^{\pm})$, can be rewritten in terms of $s^{\pm}$ (analogous to $t^{\pm}$) where $s^{\pm} = t^{\pm}/T_c$ by

$$i^+(t^+) = \left(\frac{1}{1-\theta}\right)^{b-1} ab(s^+T_c)^{b-1} = \left(\frac{T_c}{1-\theta}\right)^{b-1} i(s^+) \tag{15}$$

and

$$i^-(t^-) = \left(\frac{T_c}{\theta}\right)^{b-1} i(s^-) \tag{16}$$

For a given S-graph, t_p and η are constants. For a given precipitation region, log-log exponent "b" is a constant. Following the derivation presented in presented in Hromadka (1995), Eq. (13) can be simplified by including (5) as

$$Q_p = [\alpha I(T_c) - \phi U_o]A \tag{17}$$

where α is a derived constant for the given S-graph and precipitation region.

In English units, $U_o = 1$ and Q_p $[\alpha I(T_c) - \phi]A$, which is the usual form of this type of Rational Method peak flow rate estimator.

Another popular loss function is a constant proportion loss rate given by

$$e(t) = ki(t) \tag{18}$$

where k is a constant dependent upon catchment land use and soil cover.

Using (18) and (13) and repeating the above mathematical derivation results in the balanced design storm UH procedure peak flow rate estimator, Q_p, given by

$$Q_p = k\alpha I(T_c)\, A \tag{19}$$

where in (19), α is the same constant (and same values) used in (17). The corresponding Rational Method peak flow rate estimator, Q_R, is $Q_R = kI(T_c)\,A$. Note that in (17) and (19), the shape factor, θ, used to define the balanced design storm shape in (8) and (9), is absorbed into the single constant α.

It is noted that the derived constant, α, is a function of only the S-graph type (e.g., mountain, valley, desert, etc.) and the regional rainfall log-log equation exponent (which typically is constant for large regions). The reader is referred to Hromadka (1995) regarding application of (17) and (19), and the calibration of the constant α to the balanced design storm UH method.

4. Including Rainfall Depth-Area Effects

The balanced design storm UH procedure includes rainfall depth-area effects for catchment areas greater than 1 square mile (see HEC TD-15). Depth-area adjustment reduces area-averaged T-year point rainfall values according to catchment area. Several California flood control agencies (see refs. 4, 5, 6, 7) use depth-area curves derived from a major regional storm called the Sierra-Madre storm event (California) of 1943. The 1- and 3- hour depth-area curves are plotted in Fig. 3 and demonstrate a strong logarithmic relationship

$$\Delta(A) = eA^f \tag{20}$$

where e and f are constants, A is the catchment area, and $\Delta(A)$ is the depth-area adjustment factor for a given peak storm duration. Such a logarithmic relationship is typically found in most depth-area curve sets. The influence of either curve (shown in Fig. 3) upon the balanced design storm UH method peak flow rate strongly depends on the catchment area and the time of concentration, T_c. For T_c values less than about 2 hours, the 1-hour depth-area curve provides the dominant influence. For T_c values greater than 2 hours (and less than 5 hours), the 3-hour depth-area curve provides the dominant influence. For simplicity, we will focus on T_c values less than 2 hours (and where the 1-hour depth-area curve is dominant); this case applies for the majority of runoff studies in California that use the Sierra-Madre depth-area curves (obviously, the 3-hour depth-area curve, or other duration, can be used accordingly in the following development). Depth-area adjustment is accomplished by multiplying the depth-area factor with the rainfall, and then using the modified rainfall values for loss rate calculations.

By combining Eqs. (17) and (20), a peak flow rate estimator is (for catchments greater than 1 square mile, and T_c less than 2 hours):

$$Q_p = [\alpha e A^f I(T_c) - \phi]A \qquad (21)$$

Similarly, combining (19) and (20) gives

$$Q_p = ek\alpha I(T_c)\, A^{1+f} \qquad (22)$$

Equations (21) and (22) provide an extension of the Rational Method to larger catchment sizes, and is mathematically derived from the extended HEC TD-15 balanced design storm UH method peak flow rate estimator.

5. Linkage to Peak Flow Rate Regression Equations

By substituting (5) into (21) and (22), respectively,

$$Q_p = [\alpha e a A^f (T_c)^{b-1} - \phi]A \qquad (23)$$

or

$$Q_p = aek\alpha (T_c)^{b-1}\, A^{1+f} \qquad (24)$$

The U.S. Army Corps of Engineers use an estimator for catchment lag of the form (see refs. 1 through 9)

$$\text{lag} = 24\bar{n}\left(\frac{L \cdot L_c}{\sqrt{S}}\right)^{\beta} \qquad (25)$$

where

$\bar{n}$ = basin factor, representative of system's hydraulic response (selected from a calibrated set of values);

L = length of longest watercourse;

L_c = length along longest watercourse to catchment centroid;

S = slope of longest watercourse;

β = calibration exponent (constant).

Use of (25) is usually appropriate for larger catchments where depth-area effects are also important. From (1) and (25), an estimator for T_c is

$$T_c = \frac{24\bar{n}}{\gamma}\left(\frac{L \cdot L_c}{\sqrt{s}}\right)^{\beta} \tag{26}$$

where s = H/L, where H is the drop in elevation along the longest watercourse. Then,

$$T_c = \frac{24\bar{n}}{\gamma} L^{3\beta/2} L_c^{\beta} H^{-\beta/2} \tag{27}$$

Equations (24) and (27) can be combined as

$$Q_p = aek\alpha\left(\frac{24\bar{n}}{\gamma}\right)^{(b-1)} L^{3\beta(b-1)/2} L_c^{\beta(b-1)} H^{\beta(1-b)/2} A^{1+f} \tag{28}$$

A similar extension for Eq. (23) follows directly.

In Eq. (28), the several parameters are included for rainfall (a,b), depth-area effects (e,f), loss rate (k), normalized unit hydrograph type (α), balanced design storm shape (θ), catchment timing via a lag estimation ($\bar{n}$, L, L_c, H, β, γ), and catchment area (A).

A power law regression equation corresponding to (28) is

$$Q_{reg} = C_0 L^{P1} L_c^{P2} H^{P3} A^{P4} \tag{29}$$

Assuming that the ratio L_c/L is approximately constant (true for watershed having similar shapes), and recalling that catchment slope S = H/L, (29) may be rewritten as

$$Q_{reg} = C_0 L^p S^q A^r \tag{30}$$

which is of the form of many peak flow rate regression equations in use today.

Equation (30) completes the constructive mathematical linkage between the Rational Method , the balanced design storm UH method as presented in HEC Training Document TD-15 (1982), and peak flow rate regression equations for both small and large catchments. Although many regression equations use a daily or annual precipitation value, such a variable can be included directly in (30).

CONCLUSIONS

Runoff peak flow rates are typically estimated by the Rational Method, a design storm unit hydrograph (UH) method, or a regression equation. In this paper, the balanced design storm UH procedure is used to derive a Rational Method peak flow rate equation that, in turn, is used to derive a regression equation. This new linkage across these three widely used peak flow rate estimation techniques provide foundation as to how these approaches differ or agree, and may also provide an answer as to which method is "best"; specifically, the methods are identical for most practical conditions, and where they differ, the underpinnings of their mathematical structures is illuminated. (From the practitioner's viewpoint the "best" method may be based on the availability of hydrologic data; scope and level of detail called for by a study; or time and funds available.) The fact that all of the above three cited techniques continue to be widely used for peak flow rate estimation by flood control public agencies demonstrates the utility of the three methods in practice. It is anticipated that the derived mathematical linkage will initiate research into improving all three modeling approaches by inverse methods in parameter estimation (i.e., having calibrated one of the three techniques, the other two techniques can be calibrated), among other topics.

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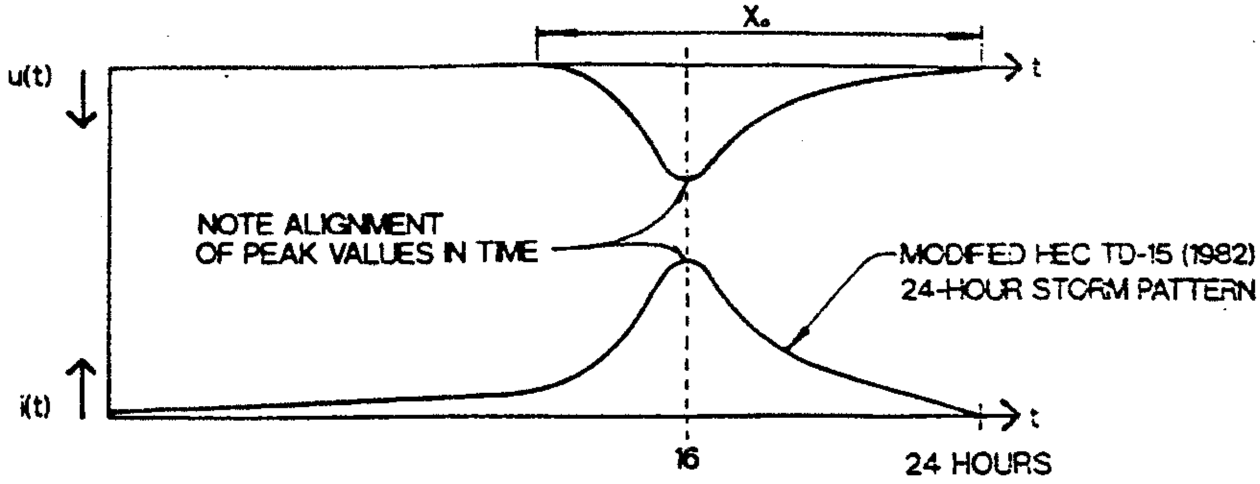

FIGURE 1

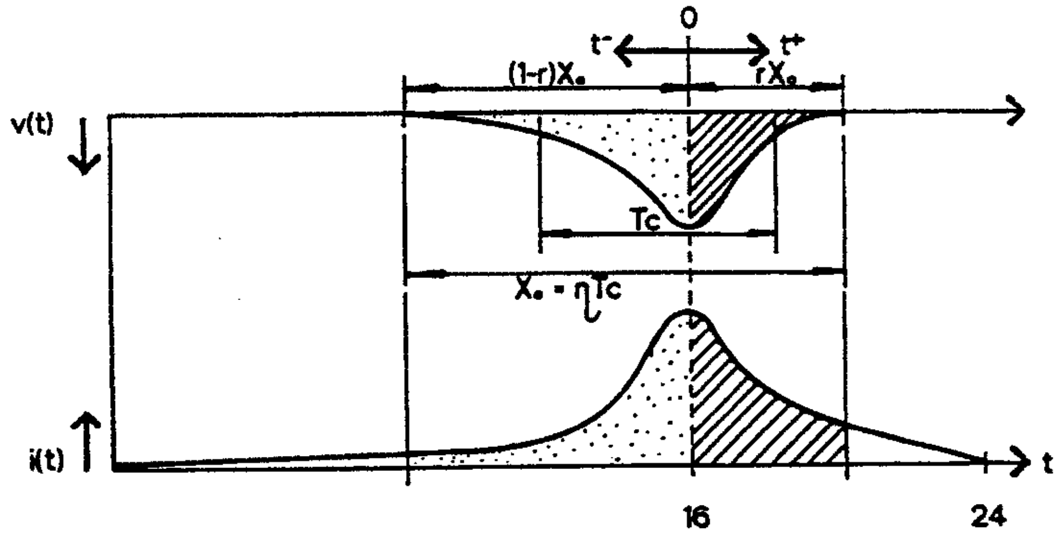

FIGURE 2