A STOCHASTIC INTEGRAL EQUATION ANALOG OF RAINFALL-RUNOFF PROCESSES

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Abstract

Issues regarding rainfall-runoff modeling complexity and the apparent lack of success in achieving significant further improvement in modeling accuracy is well documented. In this paper, a multi-linear unit hydrograph approach is used to develop subarea runoff, and is coupled to a multi-linear channel flow routing method to develop a link-node rainfall-runoff model network. The spatial and temporal rainfall distribution over the catchment is equated to a known rainfall data source located in the catchment in order to account for the random nature of rainfall with respect to the rain gauge measured data. The resulting link node model structure is a series of stochastic integral equations, one equation for each subarea. A cumulative stochastic integral equation is developed as a sum of the above series, and includes the complete spatial and temporal variabilities of the rainfall over the catchment. The resulting stochastic integral equation is seen to be an extension of the well-known single area unit hydrograph method, except that the model output of a runoff hydrograph is seen to be distribution of outcomes (or realizations) when applied to problems involving prediction (rather than reconstitution) of storm runoff.

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Introduction

Issues regarding rainfall-runoff modeling complexity and the apparent lack of success in achieving further improvement in modeling accuracy is well documented (for example, Jakeman and Hornberger, 1993; Loague and Freeze (1985); Hornberger et al, 1985; Hooper et al, 1988; Beven, 1989; Hromadka and Whitley, 1989). An apparent barrier to improvement in rainfall-runoff modeling accuracy is the unknown temporal and spatial distribution of rainfall over the catchment. Raines and Valdes (1993) state that "...the estimate of the rainfall parameters is the most subjective task and seems to be responsible for the major sources of error...".

In this paper, the unit hydrograph approach is used to generate catchment subarea runoff which is then coupled to a multi-linear channel flow routing analog to develop a link-node rainfall-runoff model network. Jakeman and Hornberger (1993), observed a "predominant linearity in the response of watersheds over a large range of catchment scales even if only a simple adjustment is made for antecedent rainfall conditions. The linearity assumption of unit hydrograph theory therefore seems applicable in temperate catchments and works just as well for slow flow as for quick flow." The spatial and temporal rainfall distribution over the catchment is equated to a known rainfall data source in the catchment (i.e., the rain gauge) in order to account for the random nature of rainfall with respect to the measured rain gauge data. The resulting link node model structure is a series of random integral equations. A summation stochastic integral equation is synthesized from the above series that includes the complete spatial and temporal variabilities of the rainfall over the catchment. The resulting stochastic integral equation is seen to be an extension of the well-known single area unit hydrograph method, except that the model output is a distribution of outcomes (or realizations) when applied to problems involving prediction (rather than reconstitution) of storm runoff. The distribution of outcomes can then be used to develop probability distributions for runoff criterion variables, such as peak flow rate, detention basin volume (among others), whereby confidence intervals may be developed.

The use of stochastic integral equations to model rainfall-runoff response is shown to be a straightforward application of stochastics to model uncertainty in runoff modeling, given uncertainty in the problem initial and boundary conditions (i.e., rainfall). In this paper, the work of Hromadka and Whitley (1989) is rederived in a constructive mathematical model development that streamlines the accounting of rainfall variations over the catchment, resulting in an easier to use set stochastic equations.

Stochastic Rainfall-Runoff Model Development

Similar to the development in Hromadka and Whitley (1989), a stochastic integral equation will be developed under the premise that the uncertainty in the spatial and temporal distribution of rainfall, with respect to a single known rainfall data source, $P_g^i(\cdot)$, for storm event i, dominates the rainfall-runoff uncertainty problem. In the following analysis, it is assumed that a quasi-linear modeling structure can be used to represent the rainfall-runoff process. (From the development, the approach applies, in general, to free draining catchments without dominating effects of storage such as due to dams or other similar effects).

The stochastic integral equation rainfall-runoff model is developed with respect to a distributed parameter link-node model setting, including nonhomogeneous loss functions, multi-linear subarea runoff response, multi-linear channel flow routing, and the random processes involved with the spatial and temporal variation of rainfall over the entire catchment. In this way, the randomness of the problem's initial and boundary conditions (i.e., the prior and current rainfall over the catchment) is combined with the integration of the various mutually dependent random components of the runoff process, resulting in a stochastic integral equation.

Let the catchment be divided into hydrologic subareas, R_j, such as discussed in Hromadka et al (1987). Each R_j is homogeneous in that a single loss function transform, $F_j(\cdot)$, operates on the subarea point rainfall uniformly. The effective rainfall (or rainfall less losses) is given by $e_j^i(\cdot)$, for storm event i, where

$$e_j^i(t) = \int_{R_j} \int F_j(P^i(x,y,t))\, dxdy \,/\, A_j \tag{1}$$

where A_j is the area of subarea, R_j. The point rainfall is written as a sum of proportions of the available rain gauge data by

$$P^i(x,y,t) = \sum_{k=1}^{n_p} \lambda^i_{xyk}\, P_g{}^i(t+\theta^i_{xyk}) \tag{2}$$

where λ^i_{xyk} is a proportion factor at coordinates (x,y) for event i, and θ^i_{xyk} is a timing offset at (x,y) for event i. Combining (1) and (2),

$$A_j\, e_j^i(t) = \int_{R_j} F_j \left[\sum_{k=1}^{n_p} \lambda^i_{xyk}\, P_g{}^i(t+\theta^i_{xyk}) \right] dR_j \tag{3}$$

Let F_j satisfy the property that

$$F_j \left[\sum_{k=1}^{n_p} \lambda^i_{xyk}\, P_g{}^i(t+\theta^i_{xyk}) \right] = \sum_{k=1}^{n_p} \lambda^i_{xyk}\, F_j\left(P_g{}^i(t+\theta^i_{xyk})\right) \tag{4}$$

(An example of such a loss transform is $F_j(\cdot) = C_j(\cdot)$, where C_j is a constant for R_j).

<u>Subarea Runoff Contribution for Event i</u>

The runoff contribution for subarea j is given by

$$q_j^i(t) = \int_{s=0}^{t} e_j^i(t-s)\, \phi_j(s)\, ds = \int_{s=0}^{t} \int_{R_j} \sum_{k=1}^{n_p} \lambda^i_{xyk}\, F_j\left(P_g{}^i(t+\theta^i_{xyk}-s)\right) \phi_j(s)\, dR_j\, ds \tag{5}$$

$$= \int_{s=0}^{t} F_j(P_g{}^i(t-s)) \int_{R_j} \sum_{k=1}^{n_p} \lambda^i_{xyk}\, \phi_j(\theta^i_{xyk}+s)\, dR_j\, ds \tag{6}$$

$$= \int_{s=0}^{t} F_j(P_g{}^i(t-s))\, \psi^i{}_j(s)\, ds \tag{7}$$

where

$$\psi^i_j(s) = \int_{R_j} \sum_{k=1}^{n_p} \lambda^i_{xyk}\, \phi_j(\theta^i_{xyk}+s)\, dR_j \tag{8}$$

We can introduce nonlinearity with the $\phi_j(\cdot)$ based upon the magnitude of $e_j^i(\cdot)$, such as $\phi'_j(\cdot) = (\phi_j(\cdot) \,|\, e_j^i(\cdot))$. One method is to define a set of subarea transfer functions according to the severity of storm; i.e., by storm class (e.g., mild, moderate, severe, flooding, etc.).

From (7) and (8), randomness is inherent in the λ^i_{xyk} and θ^i_{xyk} values, for each storm event i. That is, for prediction of runoff, the λ^i_{xyk} and θ^i_{xyk} values are samples of random variables distributed as $[\lambda_{xyk}]$ and $[\theta_{xyk}]$, respectively. In this paper, the notation $[\cdot]$ refers to both the random process and its distribution.

Channel Flow Routing

In the link-node network model, we have accumulating runoff contributions at nodes, with flow routing along each link.

Using a multilinear flow routing analog, without channel losses, (e.g., see Doyle et al (1983), Becker and Kundzewicz (1987)),

$$Q_{j+1}^i(t) = q_{j+1}^i(t) + \sum_{k=1}^{n_r} \alpha_k\ Q_j^i(t+\beta_k) \tag{9}$$

where the link is known given nodes j, j+1; node j+1 is downstream of node j, n_r is the number of flow routing translates used in the analog; and the α_k and β_k are constants. The Convex, Muskingum, and many other flow routing techniques are given by (9). The parameters α_k, β_k are link dependent (upon $Q_j^i(\cdot)$), by $\alpha'_k = (\alpha_k \,|\, Q_j^i(\cdot))$ and $= \beta'_k = (\beta_k \,|\, Q_j^i(\cdot))$ and may be defined, in prediction, according to the storm class system used for the $\phi'_j(\cdot)$ realizations. The realization $\phi^i_j(\cdot)$ can be resolved into characteristic statistics, such as the maximum, or upper percentile values of $Q^i_j(\cdot)$, which can then be used to select the storm class.

Thus we can correlate the α_k and β_k storm class values to the measured rainfall, $P_g^i(\cdot)$, by $\alpha'_k = (\alpha_k | P_g^i(\cdot))$ and $\beta'_k = (\beta_k | P_g^i(\cdot))$.

Link-Node Model

For subarea 1, contributing runoff at node 1,

$$Q_1^i(t) = q_1^i(t) = \int_{s=0}^{t} F_1(P_g^i(t-s))\, \psi_1^i(s)\, ds \tag{10}$$

Routing $Q_1^i(\cdot)$ to node 2, using (9), and adding $q_2^i(\cdot)$:

$$Q_2^i(t) = q_2^i(t) + \sum_{k_1=1}^{n_r} \alpha'_{k_1}\, q_1^i(t+\beta'_{k_1}) \tag{11}$$

Routing $Q_2^i(\cdot)$ to node 3, and adding $q_3^i(\cdot)$:

$$Q_3^i(t) = q_3^i(t) + \sum_{k_2=1}^{n_r} \alpha'_{k_2}\, Q_2^i(t+\beta'_{k_2}) \tag{12}$$

$$= q_3^i(t) + \sum_{k_2=1}^{n_r} \alpha'_{k_2}\, q_2^i(t+\beta'_{k_2}) + \sum_{k_2=1}^{n_r} \sum_{k_1=1}^{n_r} \alpha'_{k_2}\, \alpha'_{k_1}\, q_1^i(t+\beta'_{k_1}+\beta'_{k_2}) \tag{13}$$

Similarly, runoff at node j is given by upstream contributions of runoff

$$Q_j^i(t) = \sum_{\ell=1}^{n_j} \Big(\sum_{<k>_{j\ell}} \alpha'_{<k>_{j\ell}}\, q_\ell^i(t+ \beta'_{<k>_{j\ell}}) \Big) \tag{14}$$

where n_j refers to the number of subareas tributary to node j; the $<k>_{j\ell}$ is an index notation for runoff contributions to node j as summed over index ℓ, for index k as demonstrated in (13).

But from (7) and (8),

$$Q_j^i(t) = \sum_{\ell=1}^{n_j} \Big(\sum_{<k>_{j\ell}} \alpha'_{<k>_{j\ell}} \int_{s=0}^{t} F_\ell(P_g^i(t-s))\ \ \psi_j^{i\prime}(s+\beta'_{<k>_{j\ell}})\, ds \Big) \tag{15}$$

where $\psi^i{}_j{}'(\cdot)$ is the $\psi^i{}_j(\cdot)$ of Eq. (8) conditioned by use of a storm class system based on the subarea $e^i{}_j(\cdot)$.

Rewriting (15),

$$Q_j^i(t) = \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^i(t-s)) \sum_{<k>_{j\ell}} \alpha'_{<k>_{j\ell}} \psi_j^{i'}(s+\beta'_{<k>_{j\ell}})\, ds \tag{16}$$

$$= \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^i(t-s))\, \Psi_j^{i'}(s)\, ds \tag{17}$$

where

$$\Psi_j^{i'}(s) = \sum_{<k>_{j\ell}} \alpha_j{'}_{<k>_{j\ell}} \psi_j^{i'}(s+\beta'_{<k>_{j\ell}}) \tag{18}$$

It is recalled that in (15) through (18), the $\alpha'<k>_{j\ell}$, $\beta'<k>_{j\ell}$, and $\psi_j^{i'}(\cdot)$ are given on a storm class basis according to the several upstream subarea $e_j^i(\cdot)$ realizations. Accordingly, $\Psi_j^{i'}(\cdot)$ is given on a storm class basis. Also, the random spatial and temporal variation of point rainfall for storm event i, given by $P^i(x,y,t)$, is reflected by the point probability distributions of $[\lambda_{xyk}]$ and $[\theta_{xyk}]$, respectively, according to (2).

It is noted from Eqs. (2) and (4), that due to the variation in point rainfall, $P^i(x,y,t)$, the various subarea runoff contributions do not directly correlate to the measured rainfall data, $P_g^i(t)$. Consequently, the various flow routing parameters and subarea transfer functions all depend upon the cumulative effects of the upstream λ^i_{xyk} and θ^i_{xyk} values. Indeed, some subareas may have zero runoff due to incidental rainfall, and the flow routing and subarea transfer functions will be inaccurate without knowledge that only incidental rainfall occurred at these locations given that the known rainfall data $P_g^i(\cdot)$ is measured as being severe.

Applications

1. A simplification of Eq. (17) is to neglect the temporal and spatial variation of point rainfall, $P^i(x,y,t)$, in the choice of storm classes for determination of the $\alpha'_{<k>j\ell}$, $\beta'_{<k>j\ell}$, and $\psi_j^{i'}(\cdot)$. This is reasonable due to the variation of $P^i(x,y,t)$ being obviously unknown. A suitable choice for determining parameter storm classes is to simply use the rainfall data itself, $P_g^i(\cdot)$. Thus, if $P_g^i(\cdot)$ is severe, all parameters are based on a severe storm class, and if $P_g^i(\cdot)$ is mild, all parameters are based on mild storm class values. Using $P_g^i(\cdot)$ for determining storm class values, (17) and (18) are reduced to

$$Q_j^{i''}(t) = \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^i(t-s))\, \Psi_j^{i''}(s)\, ds \qquad (19)$$

where

$$\Psi_j^{i''}(s) = \sum_{<k>j\ell} \alpha''_{<k>j\ell} \Psi_j^{i''}(s+\beta''_{<k>j\ell}) \qquad (20)$$

where double primes indicate that storm class designations are based only on the $P_g^i(\cdot)$ data.

The model of (19) is still multi-linear, due to the use of the storm classes, but differs from the model of (17), in that the effects of sampling the various distributions of $[\lambda_{xyk}]$ and $[\theta_{xyk}]$ are essentially ignored.

2. A further simplification of (19) is to assume that the rainfall-runoff model will be used only within a single storm class. Thus, the model is linear over the entire storm class. Using a superscript "o" notation for this case,

$$Q_j^{io}(t) = \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^i(t-s))\, \Psi_j^{io}(s)\, ds \qquad (21)$$

where $\psi_j^{io}(s)$ follows from (20).

Equation (21) is the case typically considered for flood control purposes, where only severe storm data are used for analysis purposes.

3. Runoff Prediction on a Storm Class Basis

In prediction, the distribution of $P^i(x,y,t)$ is unknown with respect to a future measured data $P_g^*(\cdot)$. In examining (8), (18) and (21), the possible outcome for runoff, at node j, given the simplifications leading to (21), is a distribution of realizations given by $[Q_j^{*o}(\cdot)]$ where

$$[Q_j^{*o}(t)] = \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^*(t-s))\, [\Psi_j^o(s)]\, ds \tag{22}$$

where $[\psi_j^o(s)]$ is the stochastic process of realizations from storm class o, where for node j,

$$[\Psi_j^o(s)] = \sum_{<k>j\ell} \alpha^o_{<k>j\ell} [\Psi_j^o(s+\beta^o_{<k>j\ell})] \tag{23}$$

where from (8)

$$[\Psi_j^o(s)] = \int_{R_j} \sum_{n=1}^{n_p} [\lambda_{xyk}]\, \phi_j^o([\theta_{xyk}] + s)\, dR_j \tag{24}$$

(Again $[\cdot]$ refers to both the random process and its distribution, respectively. Using the notation $[\cdot]$ aids in identifying stochastic variables in the mathematical development and subsequent equations.)

The expectation is given for (22) by

$$E\,[Q_j^{*o}(t)] = \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^*(t-s))\, E[\Psi^o(s)]\, ds \tag{25}$$

Equation (25) forms a basis of the unit hydrograph procedure commonly used for flood control design and planning.

4. Distributions of Runoff Criterion Variables

Assume a free flowing catchment such that the modeling assumptions leading to (17) applies. Further assume that each loss function transfer, $F_i(\cdot)$, can be written as a linear combination of a single loss function, $F(\cdot)$. Given catchment runoff at a stream gauge location, with runoff $Q_g^i(\cdot)$ for storm i, and given associated rainfall, $P_g^i(\cdot)$, (17) can to used to relate runoff to rainfall by

$$Q_g^i(t) = \int_{s=0}^{t} F\,(P_g^i(t\text{-}s))\,\Phi^i(s)\,ds \tag{26}$$

where $\Phi^i(\cdot)$ is a transfer function for storm event i, and it is required that runoff at $Q_g^i(\cdot)$ does not occur in time prior to initiation of rainfall, $P_g^i(\cdot)$. From (17), $\Phi^i(\cdot)$ includes all the proper samplings of the various mutually dependent random processes and variables, for storm i, used in the previous stochastic integral equation development leading to (17).

Since the only available rainfall data are from the single rain gauge, storm classes are defined on that rain gauge data. For this application, only "severe" storms are being considered for flood protection purposes. Define the storm class S by

$$S = \{P_g^i(\cdot)\,|\,P_g^i(\cdot) \text{ is considered severe; } j = i=1,2,...m\} \tag{27}$$

For each event $P_g^i(\cdot) \in S$, resolve the unique transfer function $\Phi^i(\cdot)$ by solving (26), resulting in m equally likely realizations of the transfer function. Define the discrete distribution $[\Phi^i(\cdot)]$ by

$$[\Phi^i(\cdot)] = \{\Phi^i(\cdot);\ i=1,2,...,m\} \tag{28}$$

where each $\Phi^i(\cdot)$ in (28) is a solution of (26) for a $P_g^i(\cdot) \in S$. Note that in (28), the distribution $[\Phi(\cdot)]$ is dependent upon the loss function, F, chosen in (26); (that is $[\Phi(\cdot)] = [\Phi(\cdot)\,|\,F]$; however where understood, the added condition notation will be omitted in the following.

Each sample $\Phi^i(\cdot)$ from $[\Phi(\cdot)]$ provides a proper sampling of all the mutually dependent random variables and processes incorporated in (17). Additionally, as with any stochastic process, the distribution $[\Phi(\cdot)]$, in (28), depends on the sample size, m, available from the rainfall-runoff data.

It is noted that the assumption that each $F_j(\cdot)$ be a linear combination of a reference $F(\cdot)$ may be accommodated by directly relating effective rainfall, as a point function, in (2), by an analogous

$$e_j^i = \sum_{k=1}^{n_e} \int_{s=0}^{t} \lambda^i_{xyk} \, F\,(P_g^{\,i}(t+\theta^i_{xyk})).$$

In prediction, where a future storm $P_g^*(\cdot) \in S$ is contemplated at the rain gauge, the estimated distribution of runoff realizations at the stream gauge is given by the stochastic integral equation,

$$[Q_g^*(t)] = \int_{s=0}^{t} F(P_g^*(t-s))\,[\Phi(\cdot)]\,ds \qquad (29)$$

where $[Q_g^*(\cdot)]$ is the distribution of outcomes, based on the available rainfall-runoff data. For m discrete events in $[\Phi(\cdot)]$, there will be m discrete outcomes in $[Q_g^*(\cdot)]$. Oftentimes a filter is used with $[Q_g^*(\cdot)]$ such that for each $Q_g^*(\cdot) \in [Q_g^*(\cdot)]$,

$$Q_g^{*\prime}(t) = \begin{cases} Q_g^*(t), \text{ if positive} \\ 0, \text{ otherwise} \end{cases} \qquad (30)$$

where $Q_g^{*\prime}(\cdot)$ refers to a filtered realization of $Q_g^*(\cdot)$.

For the criterion variable of peak flow rate, Q_p, the distribution $[Q_p]$ is determined by the operator $\mathcal{A}_1$ on $[Q_g^{*\prime}(\cdot)]$, where

$$[Q_p] = \mathcal{A}_1[Q_g^{*\prime}(\cdot)] \qquad (31)$$

where $[Q_p]$ is the probability distribution of peak flow rates, $\mathcal{A}_1$ is the operation of finding the peak flow rate from any realization distributed as $[Q_g^{*'}(\cdot)]$. Confidence intervals can then be computed for $[Q_p]$ by the usual methods.

As another criterion variable, let $\mathcal{A}_2$ now be the operation of finding the maximum ponded depth of floodwater in a detention/retention basin. Then the distribution of basin peak flood depth, [depth], is similarly given by

$$[\text{depth}] = \mathcal{A}_2[Q_g^{*'}(\cdot)] \tag{32}$$

Note that since $\mathcal{A}_2$ is nonlinear, the expectations are related by

$$E[\text{depth}] = E(\mathcal{A}_2[Q_g^{*'}(\cdot)]) \tag{33a}$$

where

$$E(\mathcal{A}_2[Q_g^{*'}(\cdot)]) \neq \mathcal{A}_2(E[Q_g^{*'}(\cdot)]) \tag{33b}$$

5. <u>The Unit Hydrograph Method (Single Area)</u>

From (29), the well-known single area unit hydrograph (UH) method may be developed by the expectation, for the case of prediction of runoff for rainfall event $P_g^*(\cdot)$,

$$E[Q_g^*(t)] = \int_{s=0}^{t} F(P_g^*(t-s))\, E[\Phi(s)]\, ds \tag{34}$$

where $E[Q_g^*(\cdot)]$ is a single runoff hydrograph (usually filtered); and $E[\Phi(\cdot)]$ is the calibrated transfer function. In order for $E[\Phi(\cdot)]$ to be a UH, normalization is needed by letting

$$\eta = \int_{s=0}^{\infty} E[\Phi(\cdot)]\, ds \tag{35}$$

and the UH is simply $\frac{1}{\eta} E[\Phi(\cdot)]$, where the loss function is modified by multiplying by the constant, η.

6. Transferability of the Stochastic Integral Equation Method

The applications 4 and 5 develop a stochastic integral equation, and a UH method, at a stream gauge location given available rain gauge data.

Methods have been in use for decades for transferring UH relationships to locations where stream gauge data are not available [for example, see Hromadka et al, 1987], and need not be discussed here. In order to transfer the stochastic relationships of variability in the $[\Phi(\cdot)]$, as developed in (29), the same UH transferability techniques may be used. That is, by scaling the distribution of $[\Phi(\cdot)]$ outcomes with respect to $E[\Phi(\cdot)]$, then as $E[\Phi(\cdot)]$ is transferred in UH form, so is the distribution $[\Phi(\cdot)]$. This approach has been implemented in the recent hydrology manuals for the counties of Kern (1992) and the largest county in the mainland United States, San Bernardino (1993). The approach is currently being developed for the Hydrology Manual of the County of San Joaquin (1993).

Conclusions

The stochastic integral equation rainfall-runoff model is developed with respect to a distributed parameter link-node model setting, including nonhomogeneous loss functions, multilinear subarea runoff response, multi-linear channel flow routing, and the random processes involved with spatial and temporal variation of rainfall over the entire catchment. In this way, the randomness of the problem's initial and boundary conditions (i.e., the prior and current rainfall over the catchment) is properly accounted until the integration of the various mutually dependent random components result in the stochastic integral equation. The applications considered in this paper derive the classic unit hydrograph method as the expectation of the stochastic integral equation, and also discuss transferability methods to apply the uncertainty distributions at locations where runoff gauge data are not available. Use of the stochastic integral equation method should be of no significant increase in effort over that employed in the usual unit hydrograph method for free flowing catchments.

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