BALANCED DESIGN STORM UH, RATIONAL, AND REGRESSION EQUATION METHODS

T. V. Hromadka II,[1] Member, ASCE

ABSTRACT: In practice, runoff peak flow rates are typically estimated by the rational method, a design storm unit hydrograph (UH) method, or a statistical regression equation. In this technical note, the balanced design storm UH procedure is used to derive a rational method peak flow rate equation that, in turn, is used to derive a regression equation. This new mathematical linkage across these three widely used peak flow rate estimation techniques provide a foundation as to how these approaches differ or agree, and may also provide an answer as to which method is "best"; specifically, the methods are essentially the same for many practical conditions, and where they differ, the underpinnings of their mathematical structures are illuminated.

INTRODUCTION

The rational method continues to be a widely used runoff peak flow rate estimator for designing small drainage facilities [e.g., Hromadka et al. (1987); Hromadka et al. (1994)]. The unit hydrograph (UH) balanced T-year design storm method, as described in the U.S. Army Corps of Engineers (USACE) *Training Document 15* (1982) is another widely used technique for estimating peak flow rates that involves considerably more computational effort than the rational method. Additionally, USACE (1982) has been adopted, with modifications, as the basis for a number of recently developed hydrology manuals for county flood control agencies [see Hromadka et al. (1986, 1987, 1992), among others]. Peak flow rates are also estimated by statistical regression equations (e.g., the U.S. Geological Survey equations) that are calibrated to local runoff data.

In the present technical note, the balanced design storm UH approach is used to mathematically derive a rational method equation for the two cases of catchment areas less than 2.5 km^2 (1 sq mi) [also see the derivation contained in Hromadka (1995)], and catchment areas greater than 2.5 km^2 (1 sq mi). It is shown that peak flow rates developed from the well-known TD-15 (USACE 1982) balanced design storm UH method are equal to rational method peak flow estimates, except that the underlying normalized UH (or S-graph) results in a new constant to be multiplied to the usual rational method mean rainfall intensity. The linkage developed herein between the rational method and the balanced design storm UH method also depends on the loss function used. The widely used phi-index (constant loss function) approach and the constant proportion loss functions are considered. The mathematical development results in a simple modification of the standard rational method equation structure, with the introduction of a fixed constant (multiplied to mean rainfall intensity) that corresponds to the parent normalized UH (or S-graph type) and also the rainfall depth-duration log-log exponent. For areas greater than 2.5 km^2 (1 sq mi), the effects of depth-area adjustments are included, resulting in a peak flow rate estimator that corresponds to the typical regression equation structure. Although it is often conjectured that there exists a linkage among the three considered peak flow rate estimators, it appears that a constructive mathematical derivation across these different peak flow rate estimators has not been presented in the open literature.

MATHEMATICAL DEVELOPMENT

In the following, a rational method peak flow rate estimator is derived from the balanced design storm UH method [of USACE (1982)]. This derivation is presented in detail in Hromadka (1995) and Hromadka and Whitley (1996). Only the key steps are presented for the reader's convenience, so that the subsequent extension to regression equations can be better seen.

Unit Hydrographs

UHs for a catchment may be developed from normalized S-graphs (Hromadka and Whitley 1989; USACE 1982). Generally, S-graphs can be developed that apply across large regions, e.g., several countywide hydrology manuals use S-graphs that apply to mountain, desert, foothill, or valley area catchments [see Hromadka (1986, 1987, 1992)]. The S-graph is typically expressed by $S(l)$, where l = proportion (percent) of catchment lag in which catchment lag can be related to the catchment time of concentration T_c by (Hromadka et al. 1987)

$$\text{lag} = \gamma T_c \tag{1}$$

where γ = calibration constant. Then $S(l) = S(t100/\gamma T_c)$, where the UH is expressed as a function of T_c.

For $T_c = 1$ and catchment area $A = 1$, a normalized UH results, $U(t)$. For $T_c \neq 1$ or $A \neq 1$, the catchment UH, $u(t, T_c, A)$, is given by

$$u(t) = u(t, T_c, A) = \frac{A}{T_c} U\left(\frac{t}{T_c}\right) \tag{2}$$

where, by definition

$$\int_0^\infty u(t, T_c, A)\, dt = A \int_0^\infty U\left(\frac{t}{T_c}\right)\frac{dt}{T_c} = AU_o \tag{3}$$

where U_o = constant; and $u(t, T_c, A)$ may be written as $u(t)$.

Rainfall Depth-Duration Relationships

Precipitation depth-duration relationships, for a given return frequency, are generally given by the power law (Hromadka and Whitley 1996)

$$D(\tau) = a\tau^b \tag{4}$$

where $a > 0$ = function of return frequency and is assumed constant for a selected design storm return frequency; "b" is typically a constant for large regions (e.g., entire counties); $D(\tau)$ = rainfall depth corresponding to peak duration τ; and τ = selected duration of time of peak rainfall depth.

[1]Principal, Failure Analysis Assoc., Newport Beach, CA; and Prof. of Mathematics and Envir. Studies, Dept. of Mathematics, California State Univ., Fullerton, CA 92634.

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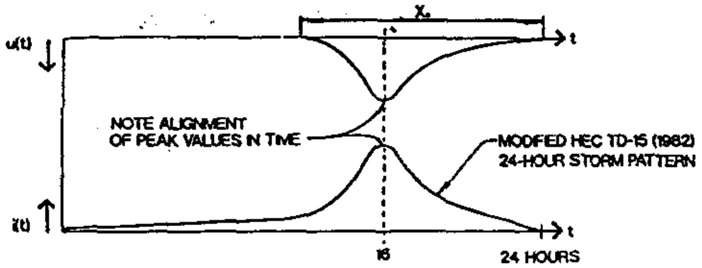

FIG. 1. Alignment of Unit Hydrograph and Balanced Design Storm Used in Eqs. (10)–(17)

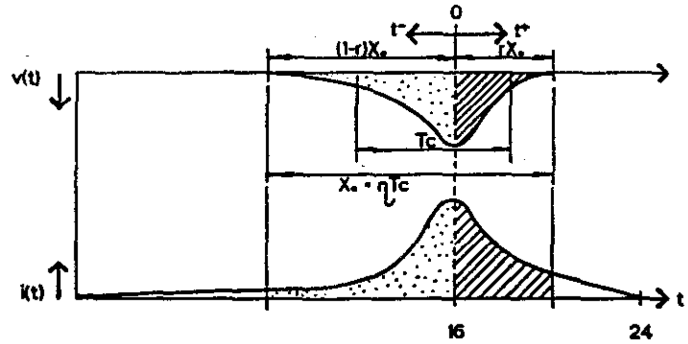

FIG. 2. Definition of Time Scale with Respect to Time-of-Concentration Parameter, T_c

Mean rainfall intensity $I(t)$ is

$$I(\tau) = \frac{1}{\tau} D(\tau) = a\tau^{b-1} \quad (5)$$

and instantaneous rainfall intensity $i(t)$ is

$$i(\tau) = \frac{d}{d\tau} D(\tau) = ab\tau^{b-1} = bI(\tau) \quad (6)$$

It is noted that $I(\tau)$ is the usual mean rainfall intensity used in the rational method for a T_c value of τ.

The balanced design storm effective rainfall pattern (i.e., rainfall less losses, or rainfall excess) $e(t)$ is a function of the instantaneous rainfall, which is formulated into a nested storm pattern as described in USACE (1982). Fig. 1 illustrates an extension of the TD-15 balanced design storm pattern that is defined to have a peak at storm hour 16 (rather than at hour 12) and where rainfall is uniformly distributed with 2/3 of its mass preceding the peak (rather than being symmetrical about the peak).

With respect to Fig. 2, the nested design storm rainfall intensity can be resolved into components $i^+(t^+)$ and $i^-(t^-)$, respectively.

For a proportioning of rainfall quantities by allocation of a θ-proportion (for all durations) prior to time $t^{\pm} = 0$ (see Fig. 2 for the case of $\theta = 2/3$), instantaneous rainfall intensities are given by

$$i^-(t^-) = i^-(\theta t) = i(t) \quad (7)$$

or

$$i^-(t^-) = i\left(\frac{t^-}{\theta}\right) = \left(\frac{1}{\theta}\right)^{b-1} i(t^-) \quad (8)$$

Similarly

$$i^+(t^+) = \left(\frac{1}{1-\theta}\right)^{b-1} i(t^+) \quad (9)$$

In the preceding, the USACE (1982) balanced design storm instantaneous rainfall intensities, given a power law relationship of (6), is obtained by $\theta = 1/2$.

Peak Flow Rate Estimates from Balanced Design Storm Unit Hydrograph Procedure and Rational Method

Let $v(t) = v(\eta T_c - t)$, where $v(t)$ = time-reversed plot of the UH $u(t)$; and $X_o = \eta T_c$ = total duration of the UH, where η is a constant for a given S-graph. From Fig. 2 and to obtain a peak flow estimate, aligning the UH peak to occur at time $t^{\pm} = 0$ (see Fig. 1)

$$v^+(t^+) = u(T_p - t^+), \quad 0 \le t^+ \le T_p \quad (10)$$

$$v^-(t^-) = u(T_p + t^-), \quad 0 \le t^- \le X_o - T_p = \eta T_c - T_p \quad (11)$$

where T_p = time-to-peak of the UH. Then, the peak flow rate from the balanced design storm UH procedure (in this case, for a constant loss rate "phi-index" model) is given by

$$Q_p = \int_{t^+=0}^{T_p} e^+(t^+)v^+(t^+)\,dt^+ + \int_{t^-=0}^{\eta T_c - T_p} e^-(t^-)v^-(t^-)\,dt^- \quad (12)$$

or

$$Q_p = \int_0^{T_p} i^+(t^+)v^+(t^+)\,dt^+ + \int_0^{\eta T_c - T_p} i^-(t^-)v^-(t^-)\,dt^- - \phi\left[\int_0^{T_p} v^+(t^+)\,dt^+ + \int_0^{\eta T_c - T_p} v^-(t^-)\,dt^-\right] \quad (13)$$

where in (13) a phi-index (or constant) loss function is used to compute rainfall excess; also, a necessary constraint imposed is that $i(\eta T_c) \ge \phi$.

Introducing a local time coordinate s defined by

$$s = \frac{t}{T_c} \quad (14)$$

then $t = sT_c$, $dt = T_c ds$.

The balanced design storm instantaneous rainfall intensities $i^{\pm}(t^{\pm})$ can be rewritten in terms of $s^{\pm}$ (analogous to $t^{\pm}$) where $s^{\pm} = t^{\pm}/T_c$ by

$$i^+(t^+) = \left(\frac{1}{1-\theta}\right)^{b-1} ab(s^+T_c)^{b-1} = \left(\frac{T_c}{1-\theta}\right)^{b-1} i(s^+) \quad (15)$$

and

$$i^-(t^-) = \left(\frac{T_c}{\theta}\right)^{b-1} i(s^-) \quad (16)$$

For a given S-graph t_p and η are constants. For a given precipitation region, log-log exponent b is a constant. Following the derivation presented in Hromadka (1995), (13) can be simplified by including (5) as

$$Q_p = [\alpha I(T_c) - \theta U_o]A \quad (17)$$

where α = derived constant for the given S-graph and precipitation region.

In English units, $U_o = 1$ and $Q_p[\alpha I(T_c) - \phi]A$, which is the usual form of this type of rational method peak flow rate estimator.

Another popular loss function is a constant proportion loss rate given by

$$e(t) = ki(t) \quad (18)$$

where k = constant dependent on catchment land use and soil cover.

Using (18) and (13), and repeating the previous mathematical derivation results in the balanced design storm UH procedure, peak flow rate estimator Q_p given by

$$Q_p = k\alpha I(T_c)A \quad (19)$$

where in (19), α is the same constant (and same values) used in (17). The corresponding rational method peak flow rate es-nator Q_R is $Q_R = kI(T_c)A$. Note that in (17) and (19) the ape factor θ, used to define the balanced design storm shape in (8) and (9), is absorbed into the single constant α.

It is noted that the derived constant α is a function of only the S-graph type (e.g., mountain, valley, desert, etc.) and the regional rainfall log-log equation exponent (which typically is constant for large regions). The reader is referred to Hromadka (1995) regarding application of (17) and (19) and the calibration of the constant α to the balanced design storm UH method.

Including Rainfall Depth-Area Effects

The balanced design storm UH procedure includes rainfall depth-area effects for catchment areas greater than 1 sq mi [see USACE (1982)]. Depth-area adjustment reduces area-averaged T-year point rainfall values according to catchment area. Several California flood control agencies (Hromadka 1986, 1987) use depth-area curves derived from a major regional storm called the Sierra-Madre storm event (California) of 1943). The one- and three-hour depth-area curves are plotted in Fig. 3 and demonstrate a strong logarithmic relationship

$$\Delta(A) = eA^f \quad (20)$$

where e and f = constants; A = catchment area; and $\Delta(A)$ = depth-area adjustment factor for a given peak storm duration. Such a logarithmic relationship is typically found in most depth-area curve sets. The influence of either curve (shown in Fig. 3) on the balanced design storm UH method peak flow 'e strongly depends on the catchment area and the time of ncentration T_c. For T_c values less than about two hours, the one-hour depth-area curve provides the dominant influence. For T_c values greater than two hours (and less than five hours), the three-hour depth-area curve provides the dominant influence. For simplicity we will focus on T_c values less than two hours (and where the one-hour depth-area curve is dominant); this case applies for the majority of runoff studies in California that use the Sierra-Madre depth-area curves (obviously, the three-hour depth-area curve, or other duration, can be used accordingly in the following development). Depth-area adjustment is accomplished by multiplying the depth-area factor with the rainfall and then by using the modified rainfall values for loss rate calculations.

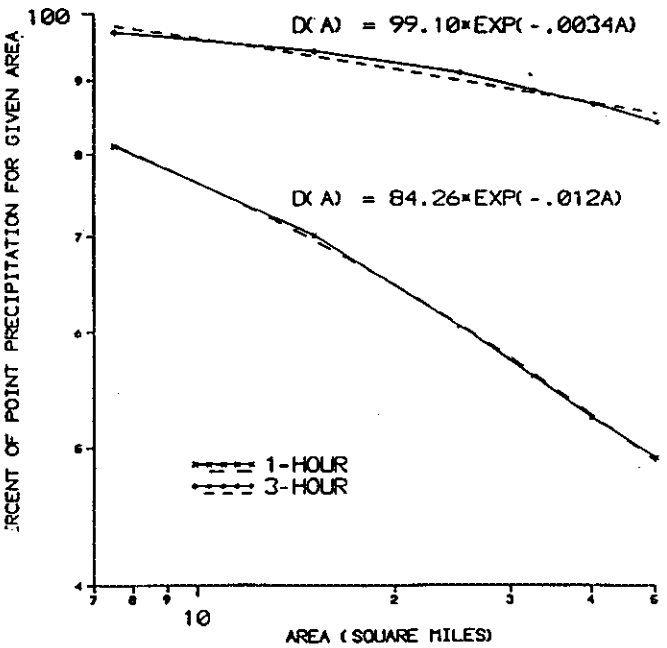

FIG. 3. U.S. Army Corps of Engineers Sierra Madre Depth-Area Curves

By combining (17) and (20), a peak flow rate estimator is (for catchments greater than 1 sq mi, and T_c less than two hours)

$$Q_p = [\alpha e A^f I(T_c) - \phi]A \quad (21)$$

Similarly, combining (19) and (20) gives

$$Q_p = ek\alpha I(T_c)A^{1+f} \quad (22)$$

Eqs. (21) and (22) provide an extension of the rational method to larger catchment sizes and is mathematically derived from the extended USACE (1982) balanced design storm UH method peak flow rate estimator.

Linkage to Peak Flow Rate Regression Equations

By substituting (5) into (21) and (22), respectively

$$Q_p = [\alpha e a A^f (T_c)^{b-1} - \phi]A \quad (23)$$

or

$$Q_p = aek\alpha(T_c)^{b-1}A^{1+f} \quad (24)$$

USACE use an estimator for catchment lag of the form (Hromadka et al. 1987, 1994; Hromadka and Whitley 1989, 1996; Hromadka 1986, 1987, 1992, 1995; USACE 1982)

$$\text{lag} = 24\bar{n}\left(\frac{L \cdot L_c}{\sqrt{S}}\right)^{\beta} \quad (25)$$

where $\bar{n}$ = basin factor, representative of system's hydraulic response (selected from a calibrated set of values); L = length of longest watercourse; L_c = length along longest watercourse to catchment centroid; S = slope of longest watercourse; and β = calibration exponent (constant).

Use of (25) is usually appropriate for larger catchments where depth-area effects are also important. From (1) and (25) an estimator for T_c is

$$T_c = \frac{24\bar{n}}{\gamma}\left(\frac{L \cdot L_c}{\sqrt{S}}\right)^{\beta} \quad (26)$$

where $S = H/L$; and H = drop in elevation along the longest watercourse. Then

$$T_c = \frac{24\bar{n}}{\gamma} L^{3\beta/2} L_c^{\beta} H^{-\beta/2} \quad (27)$$

Eqs. (24) and (27) can be combined as

$$Q_p = aek\alpha\left(\frac{24\bar{n}}{\gamma}\right)^{(b-1)} L^{3\beta(b-1)/2} L_c^{\beta(b-1)} H^{\beta(1-b)/2} A^{1+f} \quad (28)$$

A similar extension for (23) follows directly.

In (28) the several parameters are included for rainfall (a, b), depth-area effects (e, f), loss rate (k), normalized UH type (α), balanced design storm shape (θ), catchment timing via a lag estimation ($\bar{n}$, L, L_c, H, β, γ), and catchment area (A).

A power law regression equation corresponding to (28) is

$$Q_{reg} = C_o L^{P_1} L_c^{P_2} H^{P_3} A^{P_4} \quad (29)$$

Assuming that the ratio L_c/L is approximately constant (true for watersheds having similar shapes) and recalling that catchment slope $S = H/L$, (29) may be rewritten as

$$Q_{reg} = C_o L^p S^q A^r \quad (30)$$

which is of the form of many peak flow rate regression equations in use today.

Eq. (30) completes the constructive mathematical linkage among the rational method, the balanced design storm UH method as presented in USACE (1982), and peak flow rate regression equations for both small and large catchments. Although many regression equations use a daily or annual precipitation value, such a variable can be included directly in (30).

CONCLUSIONS

Runoff peak flow rates are typically estimated by the rational method, a design storm (UH) method, or a regression equation. In this technical note, the balanced design storm UH procedure is used to derive a rational method peak flow rate equation that, in turn, is used to derive a regression equation. This new linkage across these three widely used peak flow rate estimation techniques provides a foundation as to how these approaches differ or agree, and may also provide an answer as to which method is best; specifically, the methods are identical for most practical conditions, and where they differ, the underpinnings of their mathematical structures are illuminated. (From the practitioner's viewpoint the best method may be based on the availability of hydrologic data; scope and level of detail called for by a study; or time and funds available.) The fact that all of the three previously cited techniques continue to be widely used for peak flow rate estimation by flood control public agencies demonstrates the utility of the three methods in practice. It is anticipated that the derived mathematical linkage will initiate research into improving all three modeling approaches by inverse methods in parameter estimation (i.e., having calibrated one of the three techniques, the other two techniques can be calibrated), among other topics.

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