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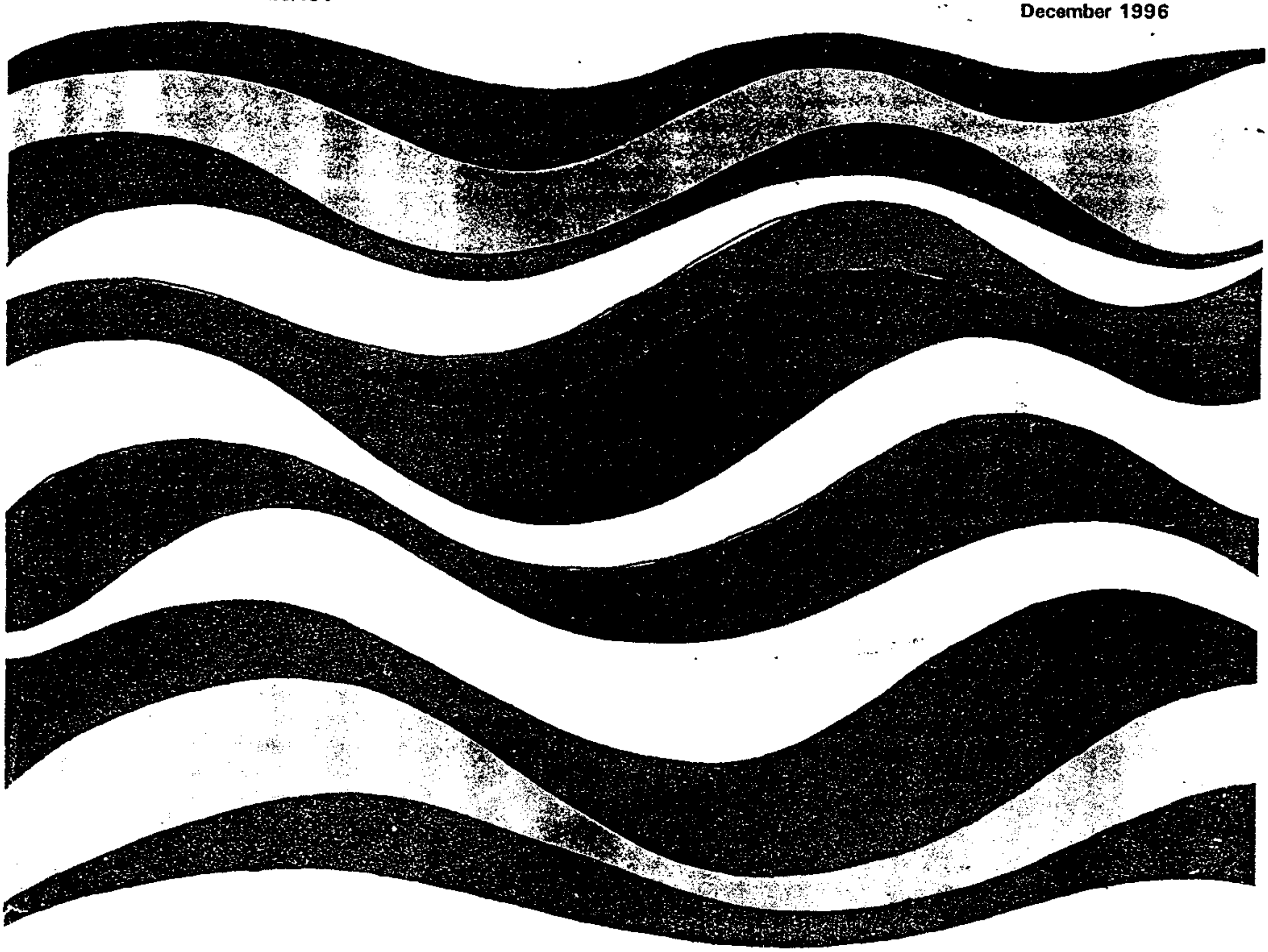

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RATIONAL-METHOD EQUATION AND HEC TD-15

T. V. Hromadka II * and R. J. Whitley **

ABSTRACT

The rational-method equation for estimating peak flow rates for storm-water runoff is derived from the balanced-design storm unit hydrograph approach presented in the US Army Corps of Engineers HEC Training Document 15. The new form of the rational-method equation is $Q_p = (\alpha I - \phi)A$, instead of the well known $Q_p = (I - \phi)A$; or $Q_p = \alpha CIA$, instead of the well known $Q_p = CIA$, depending on the respective loss function used in the unit hydrograph effective rainfall model. The preceding fixed constant α is found to depend on the type of unit hydrograph used (i.e., S-Graph) and the log-log slope of the rainfall depth-duration curve, and is easily determined by equating to a known unit hydrograph design storm model peak flow rate result. This new development provides a significant foundation for the use of the well-known rational-method equation in small catchments where rainfall depth-area effects are negligible.

INTRODUCTION

The rational method continues to be perhaps the most widely used peak flow rate estimator in surface-water hydrology studies for designing flood control facilities (Hromadka and others 1987, 1994, among others). In the present paper, the unit hydrograph (UH) balanced *T*-year design storm method, as described in the US Army Corps of Engineers Hydrologic Engineering Center (HEC) Training Document 15 (TD-15) ("Hydrologic" 1982), is used to derive the rational-method equation. It is shown that the well-known TD-15 UH balanced-design storm peak flow estimator is analogous with the rational-method peak-flow estimator, except that the underlying UH (or *S*-Graph) results in a new constant to be multiplied to the rational-method mean rainfall intensity. The linkage developed herein between the rational method and the *T*-year balanced-design storm UH method is shown to depend also on the loss function used. For analysis purposes, the phi-index (constant loss function) approach and the constant proportion loss functions are considered. The resulting mathematical development results in a modification of the standard rational-method equation structure, with a new single fixed constant (multiplied to mean rainfall intensity) that corresponds to the parent UH or *S*-Graph type and also the rainfall depth-duration log-log exponent.

The HEC TD-15 procedure will be briefly reviewed, and the key equations needed to resolve the HEC TD-15 into the much simpler and well-known rational-method peak flow rate equations are presented in the next section of the paper.

HEC TD-15 PROCEDURE

The HEC TD-15 manual ("Hydrologic" 1982) provides a uniform procedure for developing a runoff hydrograph, and corresponding peak flow rate, for a specified return frequency *T* (in years), at ungauged catchments. The approach presented in HEC TD-15 is a set of procedural steps that are generally described by the following: (1) The development of a balanced-design storm rainfall pattern of uniform return frequency, *T*, (2) the development of a loss rate that is used to develop rainfall excess (rainfall less losses); (3) the development of a catchment unit hydrograph (UH), such as evolved from an *S*-Graph, or by the Clark UH method; and (4) the standard convolution of rainfall excess with the developed UH, resulting in a balanced runoff hydrograph estimate for the catchment's point of concentration.

The balanced design storm is constructed directly from a specified *T*-year (e.g., 10-year, 100-year, or other) rainfall depth-duration curve. For a specified return frequency *T*, and a selected modelling unit time interval such as 5 min, successive unit interval incremental rainfall depths are

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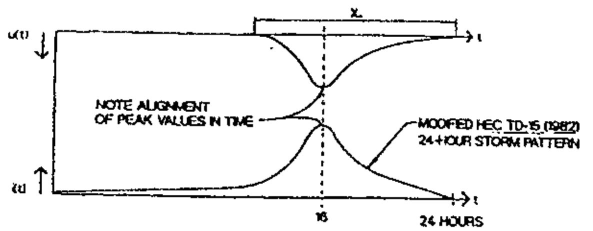

Figure 1. Time arrangement for HEC TD-15 design storm pattern and unit hydrograph, for derivation of rational-method equation.

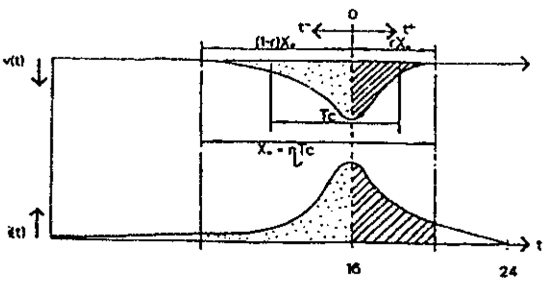

Figure 2. Definition of terms used in mathematical development (Note time reversal plot of unit hydrograph with respect to figure 1).

obtained by reading values directly from the rainfall depth-duration curve. A 24-h-duration design storm is then assembled by placing the peak 5-min (as an example unit interval size) unit rainfall to begin at a storm time of 12 h. The next largest 5-min rainfall is placed to occur prior to the maximum unit rainfall. The third largest unit rainfall is placed to occur immediately after the maximum unit rainfall. Continuing in this fashion, a 24-h balanced-design storm is assembled that is essentially symmetric at about hour 12, with the peak rainfall occurring at hour 12. (In this paper, an extension of the HEC TD-15 balanced-design storm is used to allow a redefinition as to when in model storm time the peak rainfall occurs, and then to assemble the design storm rainfall pattern in T-year rainfall mass proportional to the time ratio of when the peak rainfall occurs divided by the total 24-h design storm duration.) Figure 1 depicts the case, for example, of specifying the peak rainfall to occur at hour 16, and distributing the rainfall mass such that two-thirds occurs prior to hour 16, and the remaining one-third occurs after hour 16.

The next step in the HEC TD-15 procedure is to develop a rainfall excess distribution by subtracting losses from the rainfall design storm pattern. The constant loss function or phi-index technique is a popular choice for a loss function. The phi-index technique is considered in the present paper, along with a constant rainfall fraction loss rate.

The development of the UH from catchment characteristics is fully described in the HEC TD-15 and standard texts such as Hromadka and others (1987, 1989, 1994). Similarly, the convolution of the catchment UH with the balanced-design storm rainfall excess results in the T-year balanced runoff hydrograph.

It is readily apparent that the effort in the development of a peak flow rate using the procedures of HEC TD-15 is considerably more than that involved in using the rational method. Furthermore, the design storm UH procedures are typically considered to be more accurate than the use of a rational equation. [For example, see the literature review in Hromadka and Whitley (1989)]. In the present paper, however, we will show that both of these methods are mathematically equivalent for many situations, and that in many cases an equality can be achieved by simply calibrating the rational method with respect to a new calibration constant as derived herein.

MATHEMATICAL DEVELOPMENT

Unit hydrographs

Unit hydrographs (UHs) for a catchment may be developed from normalized S-Graphs (Hromadka and Whitley 1989, "Hydrologic" 1982). The S-Graph, which is developed from regional rainfall-runoff data, is typically expressed by $S(l)$ where l is a proportion (per cent) of catchment lag. Catchment lag is related to the usual time of concentration, T_c, by (Hromadka and others 1987)

$$\text{lag} = \gamma T_c \tag{1}$$

In several flood-control districts in California, $\gamma = 0.80$ (Hromadka 1987, 1986, 1992). Then $S(l) = S(t100/\gamma T_c)$, in which now the UH is a function of T_c and is the derivative of $S(t)$ with respect to time t.

For $T_c = 1$ and catchment area $A = 1$, a normalized UH results, $U(t)$. For $T_c \neq 1$ or $A \neq 1$, the catchment UH, $u(t, T_c, A)$, is related to $U(t)$ by

$$u(t, T_c, A) = U(t / T_c)A / T_c \tag{2}$$

where

$$\int_0^\infty u(t, T_c, A)\, dt = A \int_0^\infty U\left(\frac{t}{T_c}\right) \frac{dt}{T_c} = AU_0 \tag{3}$$

in which U_0 is a constant dependent upon the units used. Hereafter, the catchment UH, $u(t, T_c, A)$, will simply be written as $u(t)$ where no confusion occurs.

Rainfall depth-duration description

Precipitation depth-duration relationships, for a given return frequency, are generally given by the power law analog (Hromadka and others 1994)

$$D(\tau) = a\tau^b \tag{4}$$

where $a > 0$ = function of return frequency, held constant for a selected design storm return frequency; b is typically a constant for large regions (e.g., entire counties); $D(\tau)$ = rainfall depth; and τ = selected duration of time of peak rainfall depth.

Mean rainfall intensity, $I(\tau)$, is

$$I(\tau) = (1/\tau)\, D(\tau) = a\tau^{b-1} \tag{5}$$

and instantaneous rainfall intensity, $i(\tau)$, is

$$i(\tau) = (d/d\tau)\, D(\tau) = ab\tau^{b-1} = bI(\tau) \tag{6}$$

It is noted that time τ in (4)-(6) differs from the t in (2) and (3). In (4)-(6), τ refers to a duration of time in which the maximum precipitation depth occurs. The $i(\tau)$ relationship is used directly to define mathematically the rainfall intensity of the balanced-design storm pattern.

The balanced-design storm effective rainfall pattern (i.e., rainfall less losses, or rainfall excess) $e(t)$, is a function of the rainfall, $i(t)$, in which $i(t)$ is a nested storm pattern described in the HEC Training Document 15 ("Hydrologic" 1982).

The $i(t)$ is constructed for a storm event of T-year return frequency, by nesting the peak T-year 5-min rainfall depth within the peak T-year 10-min depth, and so forth until the 24-h T-year depth is obtained from the US Army Corps of Engineers Los Angeles District Office computer program. With respect to figure 2, $i(t)$ is resolved into components $i^+(t^+)$ and $i^-(t^-)$, respectively. The superscript ($^+$) refers to model time beyond the time of peak rainfall; the superscript ($^-$) analogously refers to prior model storm time as depicted in figure 2.

With respect to HEC TD-15, a balanced design storm pattern (of nested uniform return frequency rainfall depths) can be described by the time coordinates $t^\pm$ shown in figure 2. For a proportioning of rainfall quantities by allocation of a θ-proportion prior to time $t^\pm = 0$ (see figure 2 for the case of $\theta = 2/3$), instantaneous rainfall intensifies are given by

$$i^-(t^-) = i^-(\theta t) = i(t) \tag{7}$$

or

$$i^-(t^-) = i(t^-/\theta) = (1/\theta)^{b-1}\, i(t^-) \tag{8}$$

Similarly

$$i^+(t^+) = (1/1 - \theta)^{b-1}\, i(t^+) \tag{9}$$

In HEC TD-15, a symmetrical balanced-design storm rainfall pattern is given by (7)-(9) by setting parameter θ = 0.5; similarly, for the case of figures 1 and 2, parameter θ = 0.667.

Peak flow rate estimates from balanced design storm unit hydrograph procedure

Let $v(t) = v(\eta T_c - t)$ where $v(t)$ = a time-reversed plot (see figure 2) of UH, $u(t)$; and $X_0 = \eta T_c$ is the total duration of UH where η is a constant for a given S-Graph type. From figure 2, and aligning the UH peak to occur at time $t^\pm = 0$ (see figure 1)

$$v^+(t^+) = u(T_p - t^+) \tag{10a}$$

$$v^-(t^-) = u(T_p + t^-) \tag{10b}$$

Then the balanced design storm UH procedure estimates the peak flow rate, Q_p, by integration of the product of rainfall excess and the time reversed UH

$$Q_p = \int_{t^+=0}^{T_p} e^+(t^+)\,v^+(t^+)\,dt^+ + \int_{t^-=0}^{\eta T_c - T_p} e^-(t^-)\,v^-(t^-)\,dt^- \tag{11a}$$

$$= \int_0^{T_p} i^+(t^+)\,v^+(t^+)\,dt^+ + \int_0^{\eta T_c - T_p} i^-(t^-)\,v^-(t^-)\,dt^- - \phi\left[\int_0^{T_p} v^+(t^+)\,dt^+ + \int_0^{\eta T_c - T_p} v^-(t^-)\,dt^-\right] \tag{11b}$$

where T_p = usual time-to-peak of UH, given by $T_p = rX_0$ as shown in figure 2. In (11b) a " phi-index" (or constant) loss function is used to compute rainfall excess; also, a necessary constraint imposed is that $i(\eta T_c) \geq \phi$ (that is, rainfall excess must be nonnegative).

The last term of (11b) is solved by

$$\phi\left[\int_0^{T_p} v^+(t^+)\,dt^+ + \int_0^{\eta T_c - T_p} v^-(t^-)\,dt^-\right]$$

$$= \phi\left[\int_0^{\infty} u(t)\,dt\right] = \phi A U_0 \tag{12}$$

The first two integrals of (11b) are rewritten by including (8) and (9)

$$\int_0^{T_p} i^+(t^+)\,v^+(t^+)\,dt^+ = (1/1 - \theta)^{b-1}\int_0^{T_p} i(t^+)\,v^+(t^+)\,dt^+ \tag{13a}$$

$$\int_{t^-=0}^{\eta T_c - T_p} i^-(t^-)\,v^-(t^-)\,dt^- = (1/\theta)^{b-1}\int_{t^-=0}^{\eta T_c - T_p} i(t^-)\,v^-(t^-)\,dt^- \tag{13b}$$

The next step in the mathematical development is to introduce a dimensionless variable defined by

$$s = t \,/\, T_c \tag{14}$$

The balanced-design storm instantaneous rainfall intensifies, $i^\pm(t^\pm)$, can now be rewritten in terms of $s^\pm$ (analogous to $t^\pm$) where $s^\pm = t^\pm / T_c$ by

$$i^+(t^+) = (1/1 - \theta)^{b-1}\, ab(s^+T_c)^{b-1} = (T_c/1 - \theta)^{b-1} = i(s^+) \tag{15a}$$

and

$$i^{\cdot}(t^{\cdot}) = (T_c/\theta)^{b-1}\, i(s^{\cdot}) \tag{15b}$$

Similarly, the $v^{\pm}(t^{\pm})$ functions can be rewritten in terms of coordinates $s^{\pm}$ by

$$v^{+}(t^{+}) = u(T_p - t^{+}) = \frac{A}{T_c}\,U\left(\frac{T_p - t^{+}}{T_c}\right) = \frac{A}{T_c}\,U(t_p - s^{+}) \tag{16a}$$

$$v^{-}(t^{-}) = u(T_p + t^{-}) = \frac{A}{T_c}\,U\left(\frac{T_p + t^{-}}{T_c}\right) = \frac{A}{T_c}\,U(t_p + s^{-}) \tag{16b}$$

where $t_p = T_p\,/\,T_c$ is a constant for a given S-graph type. (Note that the S-graph selection applies to large regions such as mountainous areas, foothills, deserts, or valley areas, among others.)

Combining (11)-(16) with respect to local coordinates $s^{\pm}$ gives

$$Q_p = (T_c/1-\theta)^{b-1}A\int_0^{t_p} ab(s^{+})^{b-1}(1/T_c)U(t_p - s^{+})T_c\,ds^{+} + (T_c/\theta)^{b-1}A\int_0^{\eta - t_p} ab(s^{-})^{b-1}(1/T_c)U(t_p + s^{-})T_c\,ds^{-} - \phi A U_0 \tag{17}$$

where it is recalled that it is assumed $i(\eta T_c) \geq \phi$ (for selected constant loss function).

Equation (17) is rearranged to give

$$Q_p = A\,a(T_c)^{b-1}\left[(1/1-\theta)^{b-1}\int_0^{t_p} b(s^{+})^{b-1}U(t_p - s^{+})\,ds^{+} + (1/\theta)^{b-1}\int_0^{\eta - t_p} b(s^{-})^{b-1}U(t_p + s^{-})\,ds^{-}\right] - \phi A U_0 \tag{18a}$$

or, $$Q_p = A[a(T_c)^{b-1}\,\alpha] - \phi A U_0 \tag{18b}$$

where α = calibration constant. For a given S-graph, and a given precipitation region in which exponent b is a constant, then t_p and η are constants, and (18) can be further simplified by including (5) as

$$Q_p = [\alpha I(T_c) - \phi U_0]A \tag{19}$$

where α = a calibration constant for the given S-graph and precipitation region. The derivation of (19) implies that the peak flow rate estimate, obtained by use of the balanced-design storm unit hydrograph technique as developed in HEC TD-15, is directly given by the simple rational-method peak flow rate equation, which bypasses several computational steps used in HEC TD-15.

For English units, U_0 is simplified in practice to be simply $U_0 = 1$. Then

$$Q_p = [\alpha I(T_c) - \phi]A \tag{20}$$

where again α is the constant determined in (18a, b).

In comparison, a rational-method peak flow rate estimator, for an equivalent mathematical structure for estimating rainfall excess by a phi-index (constant loss function), is

$$Q_R = [I(Tc) - \phi]A \tag{21}$$

Application

In (20), the value of the constant a can be determined by equating (20) to (11a) for a single peak flow rate estimate (again, observing $i(\eta T_c) - \phi$). Several California Hydrology Manuals (Hromadka 1987, 1986, 1992) use two S-Graphs, one for "urbanized" and another for "undeveloped" regions. By equating (20) to (11a), α = 0.99 for the "urbanized" S-graph, and α = 0.86 for the "undeveloped" S-graph. In these determinations, the rainfall exponent (b) of (4)-(6) was $b = 0.55$. Additionally, the constraint of $\eta T_c \geq \phi$ resulted in T_c limitations of 45 min to 180 min for 10-year to 100-year storm events (and typical loss rates of 0.4 in./h.), respectively. It is noteworthy that in the urbanized setting, α = 0.99 is essentially α = 1.0, which results directly in the standard rational equation of (21). This close match between the two different methods perhaps explains the continued widespread use of the simple rational method in flood-control design practice; namely, that the rational method provides reliable results in many situations.

Constant fraction loss rate

Another popular loss function is to use a constant proportion loss rate to estimate rainfall excess, given by

$$e(t) = ki(t) \tag{22}$$

where k = constant dependent upon catchment land development and soil cover.

Using (22) and repeating the above development results in the balanced design storm UH procedure peak flow rate estimator, Q_p, given by

$$Q_p = k\alpha I(T_c)A \tag{23}$$

where in (23), α is the same constant (and same values) used in (20), and, of course, the constraint of $i(\eta T_c) \geq \phi$ is eliminated due to the different loss function used. The corresponding well-known rational-method peak flow rateestimator, Q_R, is

$$Q_R = kI(T_c)A \tag{24}$$

From the preceding application problem, (23) results in

$Q_p = kI(T_c)$, for urbanized areas, and

$Q_p = 0.86\ kI(T_c)$, for undeveloped areas (25)

where again in (25), the rainfall exponent is $b = 0.55$.

CONCLUSIONS

The rational-method peak flow rate equation is derived from the balanced-design storm unit hydrograph approach presented in the US Army Corps of Engineers HEC Training Document 15 ("Hydrologic" 1982). The new form of the rational-method equation is $Q_p = (\alpha I - \phi)A$, instead of the well-known $Q_p = (I - \phi)A$; or $Q_p = \alpha CIA$, instead of the well-known $Q_p = CIA$; depending on the respective loss function used in the unit hydrograph effective rainfall model. The preceding fixed constant α is found to depend on the type of unit hydrograph used, and is easily determined by equating to a known unit hydrograph design storm model peak flow rate result. This new development provides a significant foundation for use of the well-known rational method in small catchments where depth-area effects are negligible.

The mathematical derivation provided herein provides a direct link between the rational method and the balanced design storm unit hydrograph procedure of HEC TD-15 in estimating runoff peak flow rates.

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