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A rainfall–runoff probabilistic simulation program: 2. Synthetic data analysis

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Abstract

The development of a best-fit unit hydrograph to rainfall–runoff data is an important problem in flood control hydrology. In this paper, a computer program is described that is useful in determining an optimum unit hydrograph to rainfall–runoff data, and the sensitivity of the unit hydrograph to various hydrologic effects. By such an analysis, priority can be placed on fine-tuning data collection where sensitivity is greatest. Published by Elsevier Science Ltd.

Keywords: Rainfall; runoff; unit hydrograph; modeling sensitivity

Software availability

Program title:	RRSTAT
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First available:	June 1996
Hardware:	IBM PC
Source language:	FORTRAN
Availability:	Public Domain

1. Introduction

The purpose of this study was to determine the sensitivity of a synthetic watershed hydrology model to various storm and watershed parameters. All rainfall and runoff data used as inputs to this study came from a computer simulation, and the terms rainfall and runoff refer to these synthetic data forms. Although use of actual rainfall–runoff data is preferable, the typical problem encountered is that sufficient data are not available for a particular catchment. Consequently, a hydrological simulation of the rainfall is used to precisely establish the rainfall 'measured' over the catchment (Hromadka, 1996). The synthetic unit hydrograph method was used in this study to relate storm rainfall to storm runoff. For a given storm, a search was made for the unit hydrograph which best predicted the runoff from the synthesized rainfall data. This search as well as the steps in the synthetic unit hydrograph method were accomplished with a computer program referred to as 'the rainfall program' or 'the program'.

Ideally, there should have been a single unit hydrograph which best predicted the actual runoff for all storms. In reality this does not occur. Instead, for a set of storms, we obtained a set containing the best hydrograph for each storm. We then examined the distribution of this set in order to find a representative unit hydrograph (Hromadka *et al.*, 1987). In this analysis, the unit hydrographs were based on the gamma probability distribution function (Hromadka and Whitley, 1989). The shape of the unit hydrographs could be varied by changing a single scalar parameter of the gamma probability distribution function, called α. Thus, examining the distribution of the unit hydrographs was equivalent to examining the distribution of their αs. The variance of the αs was then used as a measure to how ideal the unit hydrographs were for the watershed under consideration.

In this study two types of simulated storm data were used, a set of baseline storms and eight sets of modified storms. These sets were generated by the same computer simulation; however, each modified storm was generated after one modification to a storm or watershed parameter was made in the simulation. Each modified storm in a set was generated with the same modification. The set of baseline storms were used to calibrate a loss function to be applied to all storms. The baseline storm results were also used as a basis for comparison of the modified storm results. By making these comparisons, conclusions may be drawn regarding which modifications were the most significant with respect to runoff prediction. A companion paper (Hromadka, 1996) discusses the rainfall simulation and catchment response model used in this study.

2. Description of the program inputs

As mentioned earlier, a computer program was used to implement the synthetic unit hydrograph method and to conduct a search for the unit hydrograph parameter α which best predicted the actual runoff. The main inputs to this program consisted of a set of storms. Each storm was characterized by a set of data representing rainfall and a set of data representing the actual runoff as functions of time. Here, time was measured in units of 0.1 h.

Two types of storm sets were used with the program. The first type, the baseline set, was used to calibrate the loss function and to pick a range of αs. The second type consisted of modified storm sets. The modified storms were generated by modifying a single parameter associated with either the storm or the watershed. There were eight sets of modified storms and one set of baseline storms, each set containing 200 storms. The modified storm sets are as defined in Table 1 and are described in detail in the companion paper (Hromadka, 1996).

Other inputs to the program included the following parameters: area of the watershed, time unit, unit hydrograph time-to-peak, a set of trial loss functions, and a set of unit hydrograph αs. The area of the watershed and the time unit were used to compute a watershed factor in order to scale the unit hydrographs to the appropriate units. The time unit was chosen to agree with the storm data time unit, i.e. 0.1 h. The unit hydrograph time-to-peak was fixed at 1.8 h based upon analysis of the storm data. This value was obtained by taking the average of the time between peak rainfall and peak runoff. The distribution of these time differences is shown in Fig. 1. The set of trial loss functions used by the program was a set of constants. The unit hydrograph αs used were read into the program as base-10 log values.

Table 1
Test case descriptions

Test case number	Description
0	Baseline model
1	Initial abstraction (IA) set uniformly to 0.5 inches
2	Antecedent moisture condition (AMC) assumed negligible (set to zero)
3	Storm velocity set uniformly at 7 miles per hour
4	Phi index loss function used, with phi = 0.3 inch h^{-1}
5	Peak storm rainfall set to occur at 0.5 h
6	Storm durations set at 4 h
7	All subarea T_c values set at 0.83 h
8	Only the rain gauge at centroid of catchment used

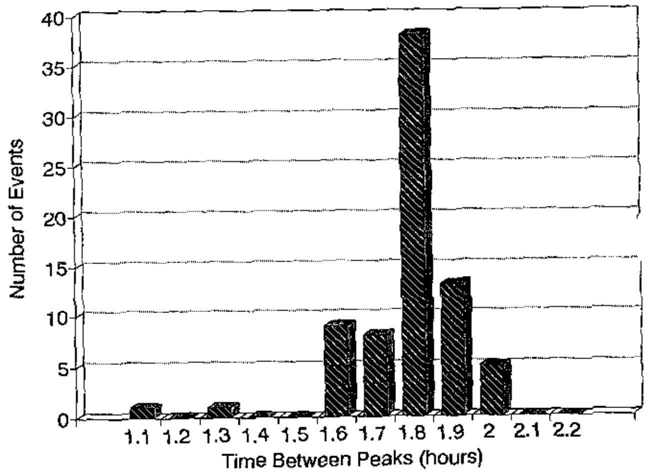

Fig. 1. Time between rainfall and runoff peaks.

Because the α parameter is essential to the program, a few words regarding its relationship to the unit hydrograph are in order. The unit hydrograph was approximated by a gamma distribution which was scaled for our particular watershed by the watershed factor. The shape of the gamma distribution, and hence the unit hydrograph, was completely determined by the value of α chosen. (Sample unit hydrographs are shown in Fig. 2(a)–(c) for various αs.) The important point to be noted in these figures is that the unit hydrograph increases in height and decreases in width as α increases, yet the area under the unit hydrograph remains constant.

3. Description of the program steps

The rainfall program consisted of two major steps. In the first step, calibration, 200 baseline storms were

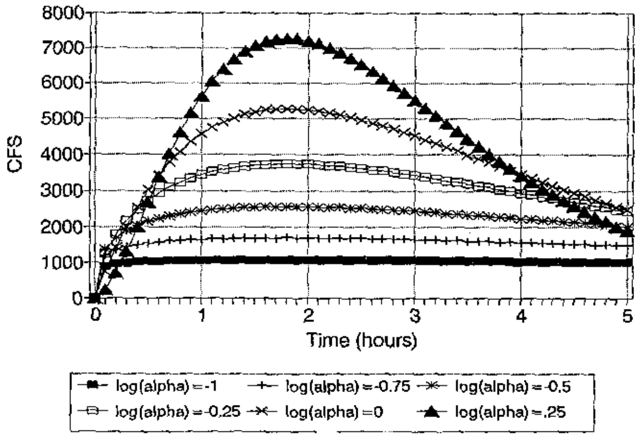

Fig. 2. (a) Unit hydrographs (1 of 3).

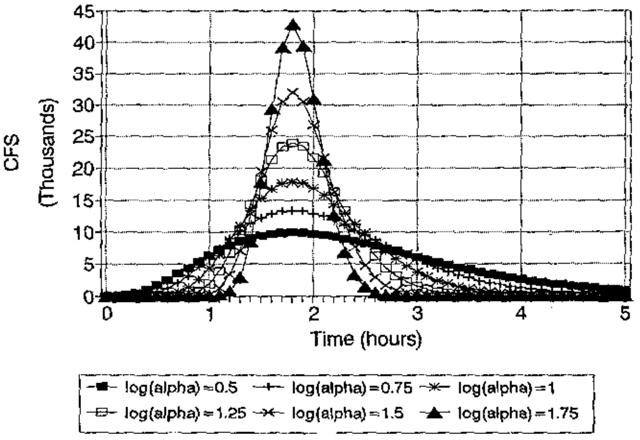

Fig. 2. (b) Unit hydrographs (2 of 3).

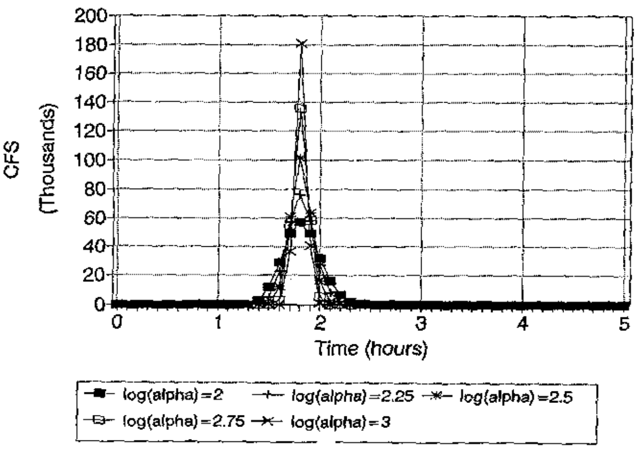

Fig. 2. (c) Unit hydrographs (3 of 3).

used as inputs to the program. In the second step, post-calibration, the eight sets of modified storms were used as program inputs. The purpose of the calibration step was to identify the set of unit hydrographs and the best loss function to be used in the post-calibration step. The purpose of the post-calibration step was to determine if the modification associated with each set of modified storms had a significant effect on runoff. (A flowchart of the program is given in Fig. 3.)

Each storm in a set consisted of rainfall and actual runoff measurements as functions of time. These measurements were taken at intervals of 0.1 h for the duration of the storm. The rainfall and runoff data were read into the program as n-dimensional column vectors where n was the number of time intervals corresponding to the duration of the storm. The effective rainfall vector was computed by subtracting the loss function from the rainfall vector with the provision that no element in the effective rainfall vector should be negative.

In the calibration step, the program processed each baseline storm with a set of unit hydrographs. However, because the gamma distribution derived unit hydrograph is characterized by the distribution parameter α, the sets of unit hydrographs used in the calibration step were actually sets of gamma distribution αs. Thus, there was a one-to-one correspondence between the unit hydrographs and the αs. By using the parameter α, a scalar quantity, the program was able to compile statistics on α (mean and variance) which were later used in determining the best loss function for the watershed. The α was used to select a particular distribution from a family of gamma distributions (where the independent variable of the distributions was time). Points along the selected distributions were then computed for the same time intervals used in the effective rainfall vector. The computed gamma distribution points were then assembled into a column vector which was then scaled by the watershed factor. The resulting vector was the unit hydrograph associated with the parameter α.

The unit hydrograph and the effective rainfall were used by the program to produce the predicted runoff vector. This was done by transforming the effective rainfall vector into a convolution matrix and multiplying the matrix with the unit hydrograph vector to yield the predicted runoff. The predicted and the actual runoffs were then used to calculate an error vector by subtracting the non-zero components in the actual runoff from the corresponding non-zero components in the predicted runoff. (This restriction regarding non-zero components was adopted to ensure that only those instances when both predicted and actual runoffs had data which would contribute to the error.) The inner product of the error vector was then computed. This, in turn, was divided by the number of non-zero components in the runoff vectors to obtain the final runoff error expressed as a scalar. Because the unit hydrograph was used to derive the predicted runoff, which was then used to obtain the runoff error, the magnitude of the runoff error was taken as an indication of the suitability of the unit hydrograph to the storm under consideration. Because the unit hydrograph was charac-

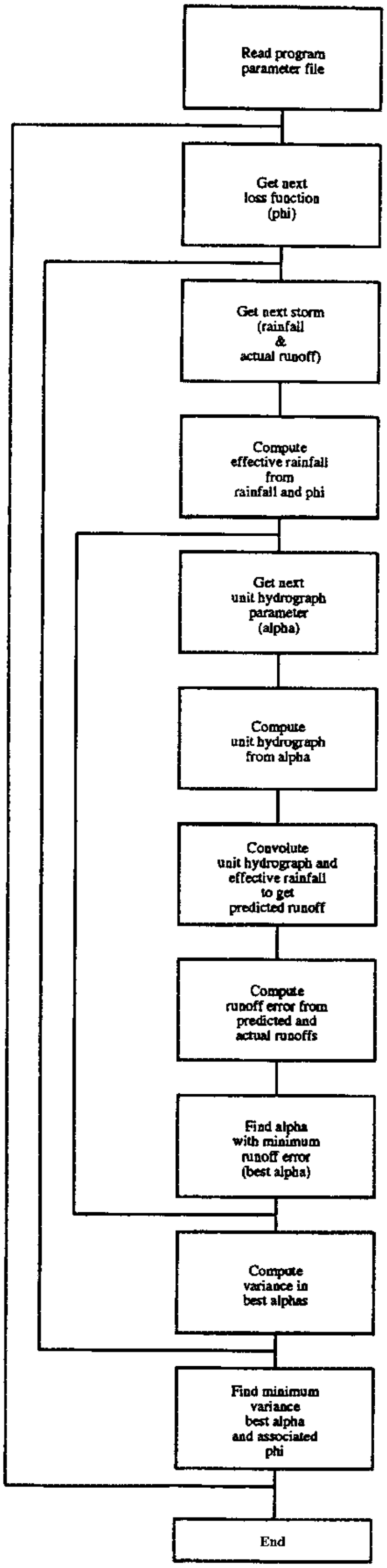

Fig. 3. Program flowchart.

terized by its α parameter, the α which resulted in the smallest runoff error was deemed to be the most suitable for the given storm.

For each storm processed by the program, a set of αs were also processed. The base-10 log of the αs in this set ranged in value from −1.0 to 3.0 in increments of 0.25. For each α, the program calculated a runoff error and thus an indication of how well the α predicted the runoff for the storm. After the entire set of αs was processed in this way, the program identified the α with the smallest runoff error. This α was then considered to be the 'best-α' for the given storm, i.e. the storm was paired with the best-α.

The program repeated this process for each storm in the baseline set provided the storm had measurable rainfall. Eighty of the 200 baseline storms fell into this category. For each of these storms a best-α was identified. The program then computed the mean and the variance of these best-αs. The mean was interpreted to be the unit hydrograph which was the best predictor of runoff for all the storms considered, while the variance gave an indication of how representative the mean was.

This process of computing the mean of the variance of the best-αs over all the baseline storms was repeated for each loss function in the set of trial loss functions. These loss functions ranged in value from 0.0 in h^{-1} to 1.0 in h^{-1} in increments of 0.01 in h^{-1}. For each loss function, the program processed the entire storm set and the entire set of αs in order to produce two numbers: the mean and the variance of the best-αs. In this way each loss function was associated with a mean and variance pair. After all of the loss functions were similarly processed, the program identified the one with the minimum variance as the 'best loss function'. In other words, the criterion for selecting the best loss function out of the set of trial loss functions was to find the one which resulted in the smallest variation in unit hydrograph αs over all the storms considered. The calibration step was completed once this best loss function was found.

With one difference, the program computations in the post-calibration step were identical to those in the calibration step. The difference was that post-calibration assumed the existence of a single loss function for all of the storms. This loss function was the best loss function found during calibration. Post-calibration made use of the same set of unit hydrograph αs that were used in calibration. As with calibration, the outputs of post-calibration consisted of the mean and the variance of the distribution of best-αs. By comparing the variance in best-αs for a particular modified storm set with the result from calibration, conclusions could be made as to the significance of the modification. If a modified storm set resulted in a change in variance, then the modification was considered significant in proportion to the amount of change observed. If little or no change resulted, then the modification was considered to be of relatively minor significance.

4. Discussion of results

After running the program to calibrate the loss function, a loss function value of 0.49 was found to minimize the variances of the best-αs over the range of loss function from 0.3 to 0.6. This was considered a reasonable range for the given watershed. Figure 4 shows a plot of the mean and variances of the base-10 logs of the best-αs with respect to loss function. In examining this figure it can be seen that for extreme values of the loss function, exceedingly small variances result. These extreme values were rejected when determining the best loss function for two reasons. For extremely large values it was observed that only four of the 200 baseline storms were used in deriving the best-α statistics. This was considered an unrepresentative sample and therefore rejected. For sufficiently small loss function values, it was observed that a number of the αs tended towards values below the minimum allowed value. In such cases the program assigned the minimum allowed value to these αs. This resulted in a skewing of the α statistics towards zero.

Once the best loss function was determined it was used in the program for all storm sets (including the aseline). Figure 5(a)–(i) shows the results of post-calibration with respect to the distribution of the best-αs. Each figure is a histogram of the best-αs for one of the storm sets, baseline or modified.

5. Conclusions

After examining the outputs of the eight sensitivity tests, three of the sensitivity tests were deemed significant, four of the tests were judged to be of little or no significance, and one was considered a borderline case. The significant sensitivities included storm duration, storm velocity, and storm time-to-peak intensity. The sensitivities of little significance included initial abstraction, antecedent moisture content (AMC), rain gauge location, and time of concentration. The border-

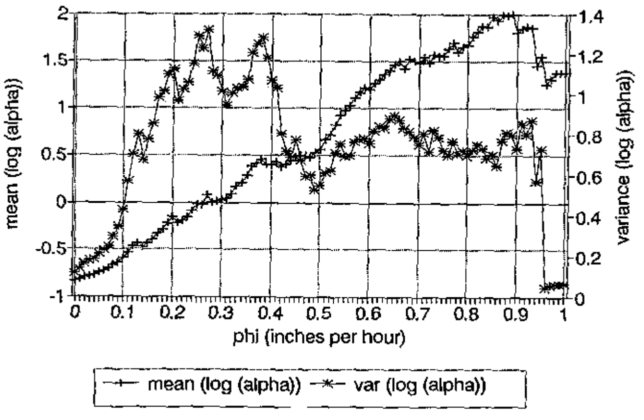

Fig. 4. Mean and variance of log (alpha) versus phi.

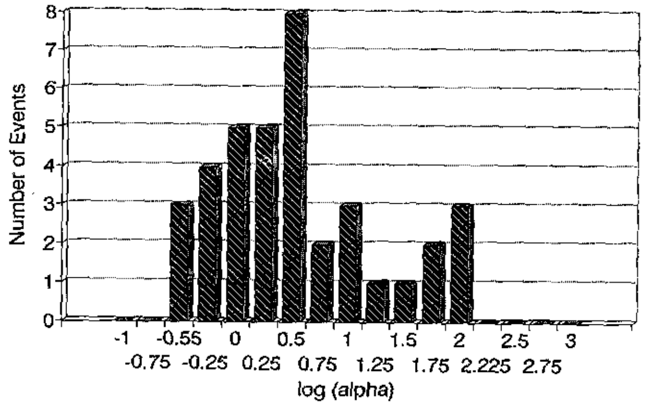

Fig. 5. (a) Distribution of log (alpha) BASE.DAT (mean = 0.527, variance = 0.530).

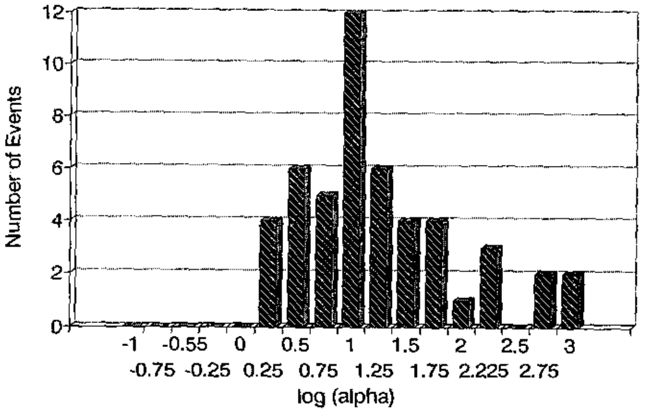

Fig. 5. (b) Distribution of log (alpha) GAUGE.DAT (mean = 1.23, variance = 0.517).

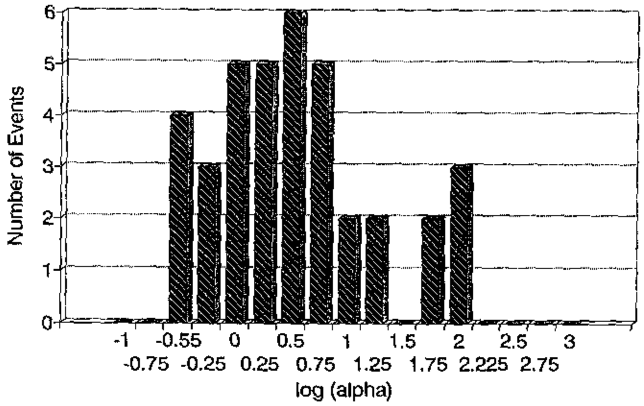

Fig. 5. (c) Distribution of log (alpha) IA.DAT (mean = 0.520, variance = 0.529).

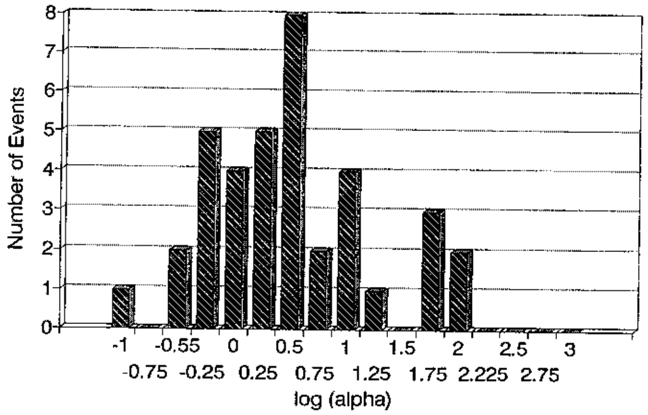

Fig. 5. (d) Distribution of log (alpha) AMC.DAT (mean = 0.486, variance = 0.535).

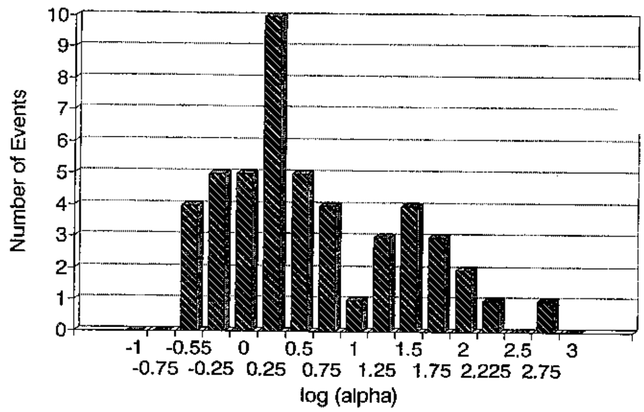

Fig. 5. (g) Distribution of log (alpha) DURATION.DAT (mean = 0.620, variance = 0.670).

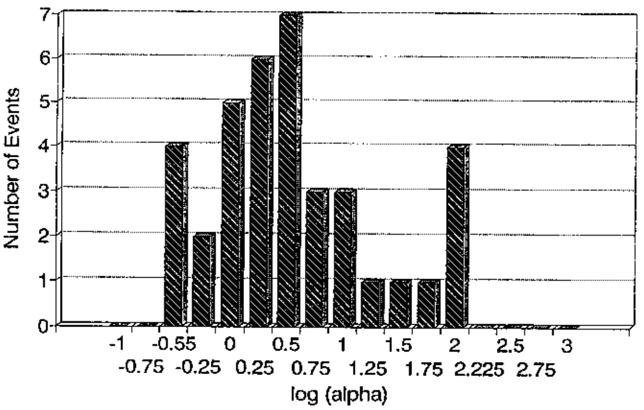

Fig. 5. (e) Distribution of log (alpha) TC.DAT (mean = 0.547, variance = 0.548).

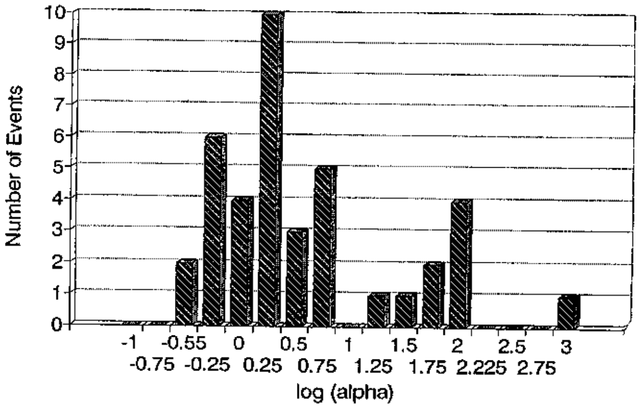

Fig. 5. (h) Distribution of log (alpha) VELOCITY.DAT (mean = 0.577, variance = 0.711).

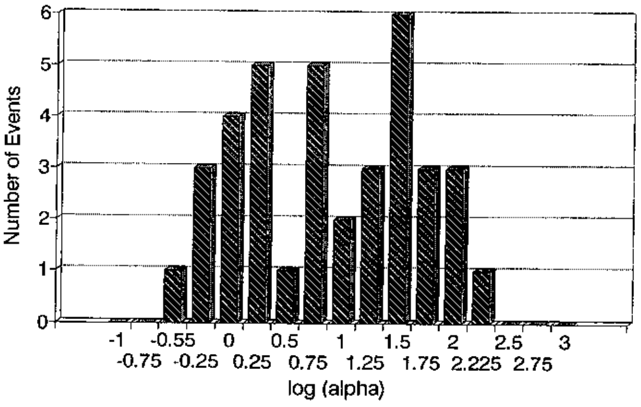

Fig. 5. (f) Distribution of log (alpha) PHI.DAT (mean = 0.878, variance = 0.603).

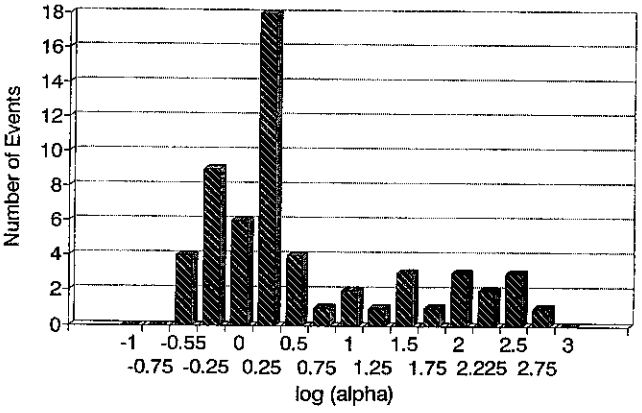

Fig. 5. (i) Distribution of log (alpha) PEAKTIME.DAT (mean = 0.573, variance = 0.825).

line case pertained to the use of a constant loss function for the modified storms. A ranking of the best-α variances in relation to the baseline is shown in Fig. 6. A final observation: the three sensitivities exhibiting the most significance were characteristics associated with the storm, whereas the modifications exhibiting little significance were characteristics associated with the watershed. This is interesting because the criterion for determining significance was based on examining the unit hydrograph statistics. Also, the unit hydrograph

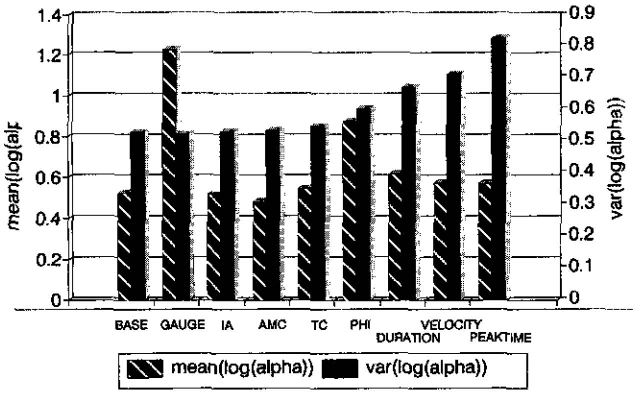

Fig. 6. Mean and variance of log (alpha) for the various storm sets.

is supposed to be a function of the watershed, rather than the storms.

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