STORM DURATION AND THE RUNOFF CRITERION VARIABLE

by

T. V. Hromadka\textsuperscript{1}, J. J. DeVries\textsuperscript{2}, and K. Loague\textsuperscript{3}

ABSTRACT

There are many runoff criterion variables of interest in stormwater management, including peak flow rate, peak flow depth in a channel, mean flow velocity for flow rates exceeding a threshold flow rate, flood control basin maximum runoff volume, among many others. The relationship between a runoff criterion variable and the duration of storms is complex but is suitable for analysis using traditional statistical techniques. The results of such an analysis lead to the design storm approach commonly used in flood control design and planning. The complete design storm approach includes three components: (1) a rainfall depth, given a return frequency, for each storm duration; (2) the mean rainfall pattern shape, given the return frequency, for each storm duration; and (3) the distribution of storm pattern shape variations about the mean storm pattern shape, for each storm duration and return frequency. By testing the stormwater system with each element of the complete set of “design storm” patterns, a distribution of possible outcomes of the selected runoff criterion variable can be estimated, the maximum value of which is the target design value for a selected return frequency.

In this paper, the focus of the discussion is upon the runoff criterion variable: flood control basin volume. It will be shown that the well known “balanced” design storm of HEC Training Document 15 is an efficient estimator of the complete design storm distribution and that a prescribed fixed design storm duration, such as 2 hours, may not necessarily develop the selected return frequency value of the flood control basin maximum runoff volume.

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AND THE RUNOFF CRITERION VARIABLE

T. V. Hromadka

J. J. DeVries

K. Loague
Relationship Between a Runoff Criterion Variable and Duration of Storms is Complex

Traditional Statistical Techniques Most Suitable for Analysis

Results of Such an Analysis Lead to the Design Storm Approach

Very Commonly Used In Flood Control Design And Planning.
Runoff Criterion Variables Of Interest In Stormwater Management

- Peak Flow Rate
- Peak Flow Depth In A Channel
- Mean Velocity For Flows Exceeding A Threshold Flow Rate
- Flood Control Basin Maximum Runoff Volume
- and Many Others
Fig. 1.7. FREQUENCY ANALYSIS OF FLOOD DATA (REF: HEC HYDROLOGIC ENGINEERING METHODS FOR WATER RESOURCES: VOLUME 5—HYPOTHETICAL FLOODS; MARCH, 1975)
Fig. 1.8. THE BALANCED FLOOD HYDROGRAPH (REF: HEC HYDROLOGIC ENGINEERING METHODS FOR WATER RESOURCES: VOLUME 5—HYPOTHETICAL FLOODS; MARCH, 1975)
The complete design storm approach includes three components:

1. A rainfall depth, given a return frequency, for each storm duration
2. The mean rainfall pattern shape, given the return frequency, for each storm duration
3. The distribution of storm pattern shape variations about the mean storm pattern shape, for each storm duration and return frequency.
The steps in developing a hypothetical storm rainfall include:

- Determine total storm duration
- Determine the time interval for subdividing the storm
- Extract data from the appropriate NWS or NOAA publications
- Adjust for area
- Adjust for annual series, if necessary
- Develop relation for accumulated depth versus time
- Increment depths for each period
- Arrange storm
A.6. Rainfall Value for the Study Area  
(Northeast Corner of Texas Panhandle)

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<th>Duration</th>
<th>15-min</th>
<th>60-min</th>
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<th>3-hr</th>
<th>6-hr</th>
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<td>100-year</td>
<td>1.82 in.</td>
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<td>4.35 in.</td>
<td>4.65 in.</td>
<td>5.30 in.</td>
<td>6.00 in.</td>
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- 100-year return period
- 12-hour duration
- NE corner of Texas Panhandle
- Area = 100 sq. miles
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<tr>
<th>Period (hours)</th>
<th>Accum. 1/ Point Rainfall (in.)</th>
<th>Point Rainfall Factor (100 mi²)</th>
<th>Accum. Depth (in.)</th>
<th>Incre. Depth (in.)</th>
<th>Arranged Incre. Depth (in.)</th>
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<td>0.810 3/</td>
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1/ From TP-40, HYDRO-35.
2/ Figure A.3
3/ Interpolated from plot of 0.5-, 1-, 3-, 6-, 24-hour adjustments.
4/ Interpolated from Figure A.7 for intermediate values.
A.5. Example of Arrangement of Hourly Rainfall Over a 24-Hour Period
Fig. 2.15b. SCS Storm Type Designations for Geographic Regions
Theodore V. Hromadka II
Robert J. Whitley

Stochastic Integral Equations
and Rainfall-Runoff Models

With 78 Figures

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Fig. 5.1. LOCATING THE PEAK AREA OF $F_1(t)$, FOR DURATION, $\delta$. 
Fig. 5.2. RESOLVING $F_i^\delta(\cdot)$ INTO COMPONENTS FOR STATISTICAL ANALYSIS.

Fig. 5.3. T-YEAR DISTRIBUTIONS OF ANNUAL $\overline{F_i^\delta(\cdot)}$ VALUES.
5.6. Estimation of T-Year Values of the Criterion Variable

For each peak duration, $I_{\delta}$, the samples of $F_{1}(\cdot)$ (see Eqs. (5.14) and (5.15)) are now analyzed to determine the underlying distribution of the annual outcomes of the values, $I(F_{1}(\cdot))$. From these distributions of mean intensity of $I_{\delta}$ model inputs, T-year values, $I_{T}^{\delta}$ of the $I(F_{1}(\cdot))$ can be derived (Fig. 5.4) and the unique T-year $F_{T}^{\delta}(\cdot)$ defined by:

$$I(F_{T}^{\delta}(\cdot)) = I_{T}^{\delta}$$  (5.20)

Given $I_{T}^{\delta}$, $F_{T}^{\delta}(\cdot)$ is defined and also both the corresponding $E_{T}(\cdot)$ and the distribution $[\varepsilon_{T}(\cdot)]$. The "T-year $I_{\delta}$ model input", $S_{T}^{\delta}(\cdot)$, is defined as

$$S_{T}^{\delta}(\cdot) = F_{T}^{\delta}(\cdot) + E_{T}(\cdot)$$  (5.21)

Figure 5.5 shows a set of $S_{T}^{\delta}(\cdot)$ for $T = 100$ years, and various $\delta$, using the data of application 3, and the model structure of Eqs. (4.54) and (4.58). The T-year $I_{\delta}$ model input, $S_{T}^{\delta}(\cdot)$, varies in both shape and mass as either $T$ or $\delta$ varies. The distribution $[Q_{T}(\cdot)]$ of realizations of $Q_{1}(\cdot)$ is now written from Eqs. (5.9), (5.19), and (5.21) as

$$[Q_{T}(t)] = \int_{s=0}^{t} \left[ F_{T}^{\delta}(s) + E_{T}(s) + [\varepsilon_{T}(s)] \right] [\eta_{T}(t-s)] ds$$  (5.22)

where $F_{T}^{\delta}(\cdot)$ is the mean intensity of the model input, $F_{1}(\cdot)$, over the time interval $0 \leq t \leq \delta$ (where $F_{1}(\cdot)$ has been translated to begin at time $t = 0$); $E_{T}(\cdot)$ is the expected shape of all possible $\delta$-interval peak durations of model inputs with the same total mass of $F_{T}^{\delta}(\cdot)$; $\varepsilon_{T}(\cdot)$ is the variation of $\Delta F_{1}(\cdot)$ about the expected shape, $E_{T}(\cdot)$; and $\eta_{T}(\cdot)$ is the necessary multilinear model transfer function realization for the parent annual event $F_{1}(\cdot)$, in some storm class $[\xi_{T}]$. 
Fig. 5.4. PLOTS OF $\Delta F^\delta_1(t)$ TRANSLATED IN TIME ($\delta = 1$ HOUR).

Fig. 5.5. PLOTS OF $S_1^\delta(t)$ FOR $T = 100$ YEARS, AND VARIOUS VALUES OF $\delta$. 
5.7. T-Year Estimate Model Simplifications

The earlier sections dealt with uncertainty in predictions of the operator $A$, which necessitated the inclusion of the stochastic process $[\eta_Z(\cdot)]$ in the final model formulation.

Equation (5.23) can be considerably simplified if it is assumed that

$$A_T^\delta = A\left( E(Q_T^\delta(\cdot)) \right) \quad (5.25)$$

in which case the joint effect of $[\varepsilon_T(\cdot)]$ and $[\eta_Z(\cdot)]$ cancel, and Eqs. (5.23) and (5.25) can be combined as

$$A_T^\delta = A \left( \int_{s=0}^{t} S_T^\delta(t-s) \eta_Z(s) \, ds \right) \quad (5.26)$$

If furthermore it is assumed that the storm classes of model input, $[\xi_Z]$, are highly correlated to T-year values of model input mean intensity, then storm classes of T-year model input can be defined, $[\xi_T]$, (perhaps on a duration basis such as 1-hour, 3-hour, etc.; see Scully and Bender, 1969, for the case of Eq. (4.58)), and Eq. (5.26) becomes

$$A_T^\delta = A \left( \int_{s=0}^{t} S_T^\delta(t-s) \eta_T(s) \, ds \right) \quad (5.27)$$

Finally, if it is assumed that the T-year value of $[A^\delta]$ monotonically increases as T increases in Eq. (5.27), then the $T_o$ return frequency value of $A$ is

$$A_{T_o} = \max_{\delta} A \left( \int_{s=0}^{t} S_T^\delta(t-s) \eta_{T_o}(s) \, ds \right) \quad (5.28)$$

where $\eta_{T_o}(\cdot)$ is the expected realization of a multilinear rainfall-runoff model response corresponding to storm class $[\xi_{T_o}]$. Equation (5.28) is a multilinear form of the well-known design storm single area unit hydrograph procedures (e.g., Hromadka et al., 1987; also, see chapter 1).
We can test the stormwater system with each element of the complete set of “design storm” patterns.

This permits a distribution of possible outcomes of the selected runoff criterion variable to be estimated.

The maximum value of the distribution of possible outcomes of the selected runoff criterion is the target design value for a selected return frequency.
The focus of this discussion is on one runoff criterion variable:

Flood Control Basin Volume

The well known “balanced” design storm of HEC Training Document 15 is an efficient estimator of the complete design storm distribution.

This means that a prescribed fixed design storm duration (say 2 hours) may not necessarily develop the selected return frequency value of the flood control basin maximum runoff volume.