UPDATING THE RATIONAL METHOD FOR PEAK FLOW ESTIMATION

T.V. Hromadka II
Principal Engineer, Boyle Engineering Corporation,
1501 Quail Street, Newport Beach, California 92658-9020
and Professor, Department of Mathematics,
California State University, Fullerton, California 92634

Key Words: Rational Method, Unit Hydrographs, Models, Hydrology, Surface Water

Abstract

The Rational Method equation is derived from the balanced design storm unit hydrograph approach presented in the U.S. Army Corps of Engineers HEC Training Document 15. The new form of the Rational Method equation is $Q_p = (\alpha I - \phi)A$, instead of the well known $Q_p = (I - \phi)A$; or $Q_p = \alpha CIA$, instead of the well known $Q_p = CIA$; depending upon the respective loss function used in the unit hydrograph effective rainfall model. The above fixed constant α is found to depend upon the type of unit hydrograph used (i.e., S-Graph) and the log-log slope of the rainfall depth-duration curve (for durations less than about 3 hours), and is easily determined by equating to a known unit hydrograph design storm model peak flow rate result. This new development provides significant foundation for use of the well known Rational Method equation in small catchments where depth-area effects are negligible.

INTRODUCTION

The Rational Method continues to be perhaps the most widely used peak flow rate estimator in surface water hydrology studies for designing flood control facilities (Hromadka, et al, 1987, Hromadka et al, 1994, among others). In this paper, the unit hydrograph (UH) balanced T-year design storm method, as described in the U.S. Army Corps of Engineers Training Document 15 (or TD-15, 1982), is used to derive the Rational Method equation. It is shown that the well known TD-15 (1982) UH balanced design storm peak flow estimator is analogous with the Rational Method peak flow estimator, except that the underlying UH (or S-Graph) results in different constants to be applied to the Rational Method mean rainfall intensity. The linkage developed herein between the Rational Method and the T-year balanced design storm UH method is shown to also depend upon the loss function used. For analysis purposes, the phi-index (constant loss function) approach and the constant proportion loss functions are considered. The resulting mathematical development results in a modification of the standard Rational Method equation structure, with a new single fixed constant (multiplied to mean rainfall intensity) that corresponds to the parent UH or S-Graph type.

MATHEMATICAL DEVELOPMENT

It is assumed that the design storm effective rainfall pattern (rainfall less losses), e(t), and the unit hydrograph (for the catchment), v(t), are both single peaked where each curve monotonically increases to its respective maximum value and then monotonically decreases (Fig. 1). It is also assumed that the peak flow rate, Q_p, occurs by the convolution of effective rainfall with the unit hydrograph (UH) when the maximum of the effective rainfall coincides in model time with the maximum of the unit hydrograph (Fig. 1).

The peak flow rate, Q_p, is given by the integral of the product of e(t) and u(t) where u(t) is the time-reverse function representative of the UH, v(t), as shown in Fig. 2. Let time, t, be redefined by t^+ and t^- as shown in Fig. 2. Note that t^+ and t^- are both zero coincident with the maximum value of e(t) and u(t).

The time duration of the catchment UH is x_0. From Fig. 2, x_0 is split into two proportions (of constant relative size) of size rx_0 and $(1-r)x_0$ where $0<r<1$ is a constant. Then, $0 \leq t^+ \leq rx_0$, $0 \leq t^- \leq (1-r)x_0$. The value x_0 is proportional to the usual time of concentration or catchment lag (TD-15, 1982; Hromadka et al, 1987).

DESIGN STORM RAINFALL

The effective rainfall pattern, e(t), is a function of the rainfall, i(t), where i(t) is a nested storm pattern described in the HEC Training Document 15 (or TD-15 (1982)).

The i(t) is constructed, for a storm event of T-year return frequency, by nesting the peak T-year 5-minute rainfall depth within the peak T-year 10-minute depth, and so forth until the 24-hour T-year depth is obtained. With respect to Fig. 2, i(t) is resolved into components $i^+(t^+)$ and $i^-(t^-)$, respectively.

Let the T-year peak rainfall depth be given as a function of duration, t, by

$$D(t) = at^b, \ \text{a and b both} > 0. \tag{1}$$

Equation (1) is generally found to be applicable for peak storm durations less than about 3 hours; oftentimes, (1) is applicable up to 24 hours.

As a case study, the design storm for Orange County, California, (Orange County Hydrology Manual, 1987), adjusts the 24-hour nested storm pattern of TD-15 so that 2/3 of the rainfall mass occurs prior to design storm hour 16, and the remaining 1/3 rainfall mass occurs between storm time 16 hours and 24 hours (see Fig. 2). For such a storm pattern,

$$\begin{aligned} D^+(t^+) &= a3^{b-1}(t^+)^b \\ D^-(t^-) &= a(\tfrac{3}{2})^{b-1}(t^-)^b \end{aligned} \tag{2}$$

From Fig. 2, $i^+(t^+)$ and $i^-(t^-)$ are both monotonically decreasing functions with respect to t^+ and t^-, respectively, such that from (2),

$$\begin{aligned} i^+(t^+) &= ab3^{b-1}(t^+)^{b-1} = b3^{b-1}\, I(t) \\ i^-(t^-) &= ab(\tfrac{3}{2})^{b-1}(t^-)^{b-1} = b(\tfrac{3}{2})^{b-1}\, I(t) \end{aligned} \tag{3}$$

where I(t) is the usual Rational Method mean rainfall intensity, for duration t, obtained from (1) by

$$I(t) = \frac{1}{t} D(t) = at^{b-1} \tag{4}$$

UNIT HYDROGRAPH ANALYSIS

Analogous to the rainfall mass functions, $D^{\pm}(t^{\pm})$, the u(t) function can also be represented by mass functions,

$$M^{+}(\ell^{+}) = c(\ell^{+})^{d}$$
$$M^{-}(\ell^{-}) = e(\ell^{-})^{d} \qquad (5)$$

where $\ell^{\pm}$ is time in terms of catchment lag (e.g., see TD-15, 1982), which is related to time, $t^{\pm}$, by

$$\ell^{\pm} = \frac{100t^{\pm}}{\gamma Tc} \qquad (6)$$

where

Tc = the usual time of concentration used in the Rational

γ = a calibration coefficient such that lag = γTc.

In Orange County, California, $\gamma = 0.8$ in (6) which closely follows the SCS relationship between lag and Tc (see Hromadka et al, 1987). For γ = 0.8,

$$\ell^{\pm} = \frac{c125\, t^{\pm}}{Tc} \qquad (7)$$

which will be used in the following development (other γ values can be used directly).

Combining (5) and (7),

$$M^{+}(t^{+}) = \frac{c125^{d}(t^{+})^{d}}{Tc^{d}}$$
$$M^{-}(t^{-}) = \frac{e125^{f}(t^{-})^{f}}{Tc^{f}} \qquad (8)$$

and, from Fig. 2, u(t) is the derivative of $M^{\pm}(t^{\pm})$,

$$\frac{dM^{+}(t^{+})}{dt^{+}} = \frac{cd125^{d}(t^{+})^{d-1}}{Tc^{d}}$$
$$\frac{dM^{-}(t^{-})}{dt^{-}} = \frac{ef125^{f}(t^{-})^{f-1}}{Tc^{f}} \qquad (9)$$

PEAK FLOW RATE ESTIMATE

In the following, $t^{\pm}$ will be represented by simply t when no ambiguity occurs.

The phi-index loss function is a widely used approach to estimate effective rainfall (TD-15, 1982; Hromadka and Whitley, 1989). Let $e(t) = i(t)-\phi$, where $\phi > 0$ is a constant, and $i(t) > \phi$ for $t \le x_o$. Then peak flow rate, Q_p, is given by (see Fig. 2),

$$Q_p = \int_{t=0}^{rx_o} (i^+(t) - \phi)\frac{dM^+(t)}{dt}\,dt + \int_{t=0}^{(1-r)x_o} (i^-(t) - \phi)\frac{dM^-(t)}{dt}\,dt \tag{10}$$

or

$$Q_p = \int_{t=0}^{rx_o} i^+(t)\frac{dM^+(t)}{dt}\,dt + \int_{t=0}^{(1-r)x_o} i^-(t)\frac{dM^-(t)}{dt}\,dt - \phi\left[\int_{t=0}^{rx_o}\frac{dM^+(t)}{dt}\,dt + \int_{t=0}^{(1-r)x_o}\frac{dM^-(t)}{dt}\,dt\right] \tag{11}$$

The second half of (11) is solved by integration, giving

$$\phi\left[M^+(t)\Big|_{t=0}^{rx_o} + M^-(t)\Big|_{t=0}^{(1-r)x_o}\right] = \phi A \tag{12}$$

where A is catchment unit area (units throughout this development are flowrate per unit area), and where it is understood that a necessary condition is $i^{\pm}(t) \ge \phi$. The remaining components of (11) are evaluated by

$$\int_0^{rx_o} i^+(t)\frac{dM^+(t)}{dt}\,dt + \int_0^{(1-r)x_o} i^-(t)\frac{dM^-(t)}{dt}\,dt$$

$$= ab3^{b-1}\frac{cd125^d}{Tc^d}\int_0^{rx_o} t^{b+d-2}\,dt + ab\left(\frac{3}{2}\right)^{b-1}\frac{ef125^f}{Tc^f}\int_0^{(1-r)x_o} t^{b+f-2}\,dt$$

$$= \frac{ab3^{b-1}cd125^d}{(b+d-1)Tc^d}(rx_o)^{b+d-1} + \frac{ab\left(\frac{3}{2}\right)^{b-1}ef125^f}{(b+f-1)Tc^f}((1-r)x_o)^{b+f-1} \qquad (13)$$

Letting $x_0 = \eta Tc$, η a fixed constant of proportion for a given S-Graph,

$$\text{Eq. (13)} = \frac{ab3^{b-1}cd125^d(r\eta)^{b+d-1}Tc^{b+d-1}}{(b+d-1)Tc^d} + \frac{ab\left(\frac{3}{2}\right)^{b-1}ef125^f((1-r)\eta)^{b+f-1}Tc^{b+f-1}}{(b+f-1)Tc^f}$$

$$= \left[\frac{b3^{b-1}cd125^d(r\eta)^{b+d-1}}{b+d-1} + \frac{b\left(\frac{3}{2}\right)^{b-1}ef125^f((1-r)\eta)^{b+f-1}}{b+f-1}\right] I(Tc = \alpha I(Tc) \qquad (14)$$

THE RATIONAL METHOD EQUATION

From (11), (12) and (14), the peak flow rate, Q_p, is given by

$$Q_p = (\alpha I(Tc) - \phi)A \qquad (15)$$

where A is as defined above (see (12)), and α is a fixed constant. It is noted that in (14), the terms b, c, d, e, f are constants used in the mass functions for rainfall and the UH, and r and η are fixed constants of proportion that describe the duration of the catchment UH, for a given S-Graph, about either side of the UH peak value, with respect to Tc. Thus, only I(Tc) is variable in (14) and hence α is a constant that can be evaluated by equating (11) to a known value of Q_p computed by the usual UH convolution process rather than computing directly by use of (14). It is noted that from

(14) and (15) α also depends upon the log-log slope (b) of the rainfall depth-duration curve (see (1)); however, this slope is generally a constant for large regions (e.g., b = 0.427 throughout Orange County, California Valley areas) and hence is essentially a constant for our analysis purposes.

The results of (14) are based upon the loss function $e(t) = i(t) - \phi$ to be used in the balanced T-year design storm UH method. Another popular loss function is the constant proportion loss function. If effective rainfall, e(t), is given by

$$e(t) = Ci(t) \tag{16}$$

where C is a constant, then it is readily determined, analogous to (11) through (14), that,

$$Q_p = C(\alpha I(Tc))A \tag{17}$$

where α is the same constant used in (15). It is noted that there is no limitation imposed on (17) such as imposed following (12), (i.e., $i(x_o) \geq \phi$).

From (15) and (17), the Rational Method equation peak flowrate can be derived from the unit hydrograph method for small areas which have no depth-area adjustment, and where a phi-index (constant loss) or constant fraction loss function is used to develop effective rainfall. It is also noted that the Rational Method results depend upon the underlying unit hydrograph. For example, in Orange County, California, the "Valley Developed" (or "Urban") unit hydrograph results in α = 0.98, whereas the "Valley Undeveloped" UH results in α = 0.86; that is,

$$Q_p = \begin{cases} (I-\phi)A, \text{ for Urban areas, for } i(x_o) > \phi; & (18a) \\ (0.86I-\phi)A, \text{ for developing areas, for } i(x_o) > \phi. & (18b) \end{cases}$$

$$Q_p = \begin{cases} CIA, \text{ for Urban areas;} & (19a) \\ 0.86CIA, \text{ for developing areas} & (19b) \end{cases}$$

where (18) and (19) apply in Orange County, California, and $I = I(Tc)$, the usual Rational Method mean rainfall intensity corresponding to peak duration Tc.

The accuracy of Q_p estimates from (18) and (19) were evaluated by comparisons to actual HEC-1 Q_p estimates (see Example 2). It is recalled that in (18), Tc estimates are limited in application by the requirement that $i(x_o) = i(\eta Tc) \geq \phi$.

In (18) and (19), it is obvious that other α values would result had other unit hydrograph types (or S-Graphs) been used in the UH method. However, once an α is determined for each UH type, a Rational Method equation results. For example, the Los Angeles District Office of the U.S. Army Corps of Engineers has developed a family of S-Graphs (see ref. 6), for UH development, dependent upon the region and conditions a catchment is in. From the above, each such S-Graph would have a corresponding value of α for use in both (18) and (19).

Example 1: Loss Rate Constraints

Let $x_o = \eta Tc$ where $\eta = 1.267$. For T = 100-years, $i(t) = 6.636t^{-0.573}$ inch/hour in Orange County, California. For $i(\beta) = \phi$, we obtain $\beta = 451$ minutes for $\phi = 0.2$; $\beta = 136$ minutes for $\phi = 0.4$; $\beta = 40.1$ minutes for $\phi = 0.8$. Thus, the Rational Method (for a phi-index loss function) is limited in application according to $i(t) \geq \phi$. For T = 25-years, $i(t) = 5.14t^{-0.573}$ giving, $\beta = 289$ minutes for $\phi = 0.2$; $\beta = 86$ for $\phi = 0.4$; $\beta = 25.7$ for $\phi = 0.8$. Thus, the Rational Method is significantly constrained for frequent storms with high ϕ loss rates.

Example 2: Peak Flow Rate Comparisons

The Orange County, California, Hydrology Manual (1987) uses the previously discussed TD-15 (1982) design storm methods with the discussed "Valley Developed" (or Urban) and "Valley Undeveloped" S-Graphs. To simplify calculations, the Hydrology Manual includes peak flow rate curves for a variety of Tc and loss rate conditions. Equations 18 can be used to compare with these peak flow rate curves for T = 100-years and T = 25-years, and use of the "Valley Developed" S-Graph. In the Hydrology Manual (1987), a two-component loss function, f(t), is used defined by

$$f(t) = \begin{cases} \phi, \text{ for } \bar{Y}i(t) \geq \phi; \\ \bar{Y}i(t), \text{ otherwise;} \end{cases} \tag{20}$$

where $\bar{Y} = 1\text{-}Y$, where Y is the catchment yield computed using a 24-hour storm rainfall by the standard SCS curve number relationship (see Hromadka et al, 1987). Consequently, the comparison between (18a) and the Hydrology Manual would be valid for Tc values that satisfy both conditions of $i(x_o) \geq \phi$ and $\bar{Y}i(x_o) \geq \phi$. Figures 3 and 4 show the comparisons between (18a) and the Hydrology Manual, for $\bar{Y} = 1$ and $\phi = 0$, $\bar{Y} = 1$ and $\phi = 0.2$, $\bar{Y} = 1$ and $\phi = 0.4$; for T = 100-years and 25-years, respectively (use of $\bar{Y} = 1$ in (20) results in a standard phi-index loss function). In Fig. 3, it is noted that $i(t) = \phi = 0.4$ at 2.3 hours; and departure between Q_p estimates will occur for Tc values larger than $2.3/\eta = 1.82$ hours. In Fig. 4, $i(t) = \phi = 0.4$ at 1.43 hours; and departure between Q_p estimates occur for $Tc = 1.43/\eta = 1.13$ hours, as discussed previously.

Figure 3 also contains a comparison of (18b) for $C = Y = 0.9$ (i.e., $\bar{Y} = 0.1$) and $\phi = 0.4$. Figure 4 compares (18) for $C = Y = 0.9$ ($\bar{Y} = 0.1$) and $\phi = 0.2$, $C = Y = 0.9$ and $\phi = 0.4$. Note that in these last three cases $\bar{Y}i(t) < \phi$ is essentially satisfied, and hence (18b) applies. Obviously, other choices for constant "C" values can be used in the derived formulae. For the Orange County Hydrology Manual loss function, low magnitude design storm conditions (e.g., T = 2-, 5- or even 10-year storms) oftentimes only involve the loss function of $f(t) = \bar{Y}i(t)$, which has the corresponding Rational Method equation of (18b).

CONCLUSIONS

The Rational Method is derived from the balanced design storm unit hydrograph approach presented in the U.S. Army Corps of Engineers HEC Training Document 15. The new form of the Rational Method equation is $Q_p = (\alpha I - \phi)A$, instead of the well known $Q_p = (I - \phi)A$; or $Q_p = \alpha CIA$, instead of the well known $Q_p = CIA$; depending upon the respective loss function used in the unit hydrograph effective rainfall model. The above fixed constant α is found to depend upon the type of unit hydrograph used, and is easily determined by equating to a known unit hydrograph design storm model peak flow rate result. This new development provides significant foundation for use of the well known Rational Method in small catchments where depth-area effects are negligible. Examples provided herein demonstrate the linkage between the balanced design storm unit hydrograph procedure of TD-15 and the new Rational Method equations.

REFERENCES

1. Hromadka II, T.V., R.H. McCuen, Durbin, T.J., DeVries, J.J., 1994, *Computer Methods in Water Resources and Environmental Engineering*, Lighthouse Publications, 608 pages.
2. Hromadka II, T.V., and Whitley, R.J., 1989, *Stochastic Integral Equations in Rainfall-Runoff Modeling*, Springer-Verlag.
3. Hromadka II. T.V., McCuen, R.H., and Yen, C.C., 1987, *Computational Hydrology in Flood Control Design and Planning*, Lighthouse Publications.
4. Hromadka II, T.V., 1987, Orange County Hydrology Manual, County of Orange, California.
5. Hydrologic Analysis of Ungauged Watersheds Using HEC-1, U.S. Army Corps of Engineers, Hydrologic Engineering Center (HEC) Training Document TD-15, April 1982.
6. U.S. Army Corps of Engineers, Los Angeles, California, District Office, computer program "LAPRE1": Los Angeles Pre-Processor to HEC-1.

Figure 1. Illustration of Time Arrangement for the HEC TD-15 (1982) Design Storm Pattern and the Unit Hydrograph, for Derivation of Rational Method Equation.

Figure 2. Definition of Terms Used in the Mathematical Development. (Note time reversal plot of the unit hydrograph with respect to Fig. 1.)

Figure 3. Comparison of Equation (18a) to Design Storm UH Method.

Figure 4. Comparison of Equation (18a) to Design Storm UH Method.

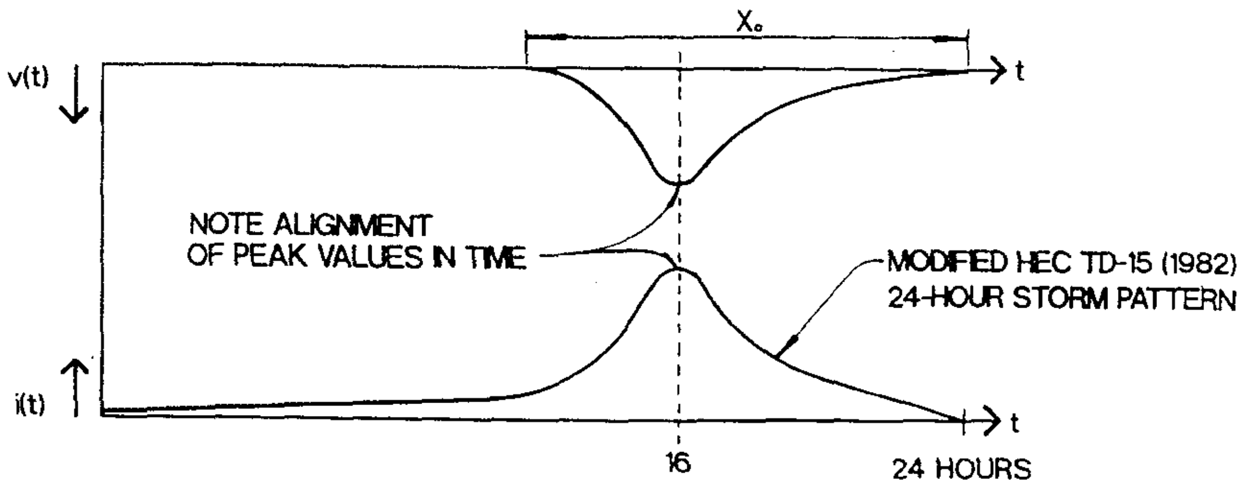

FIGURE 1

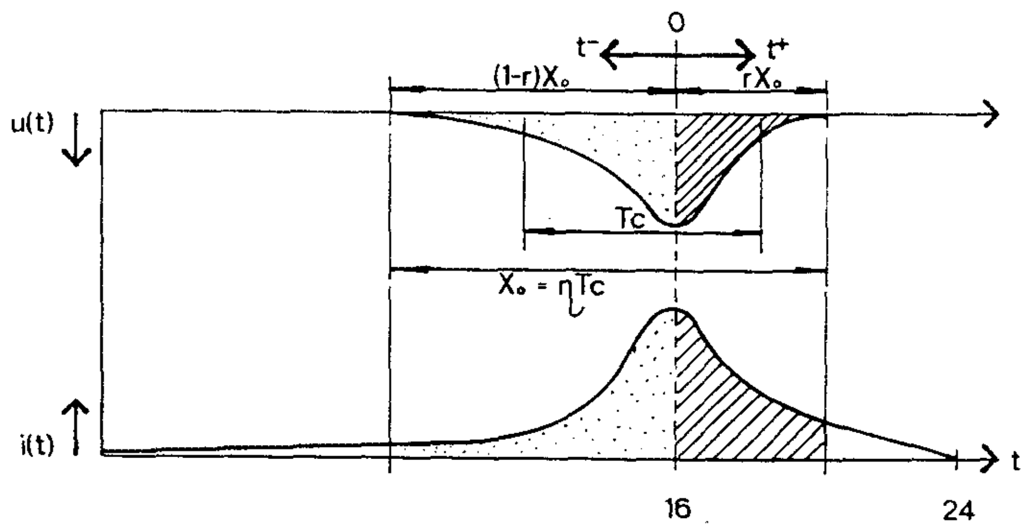

FIGURE 2

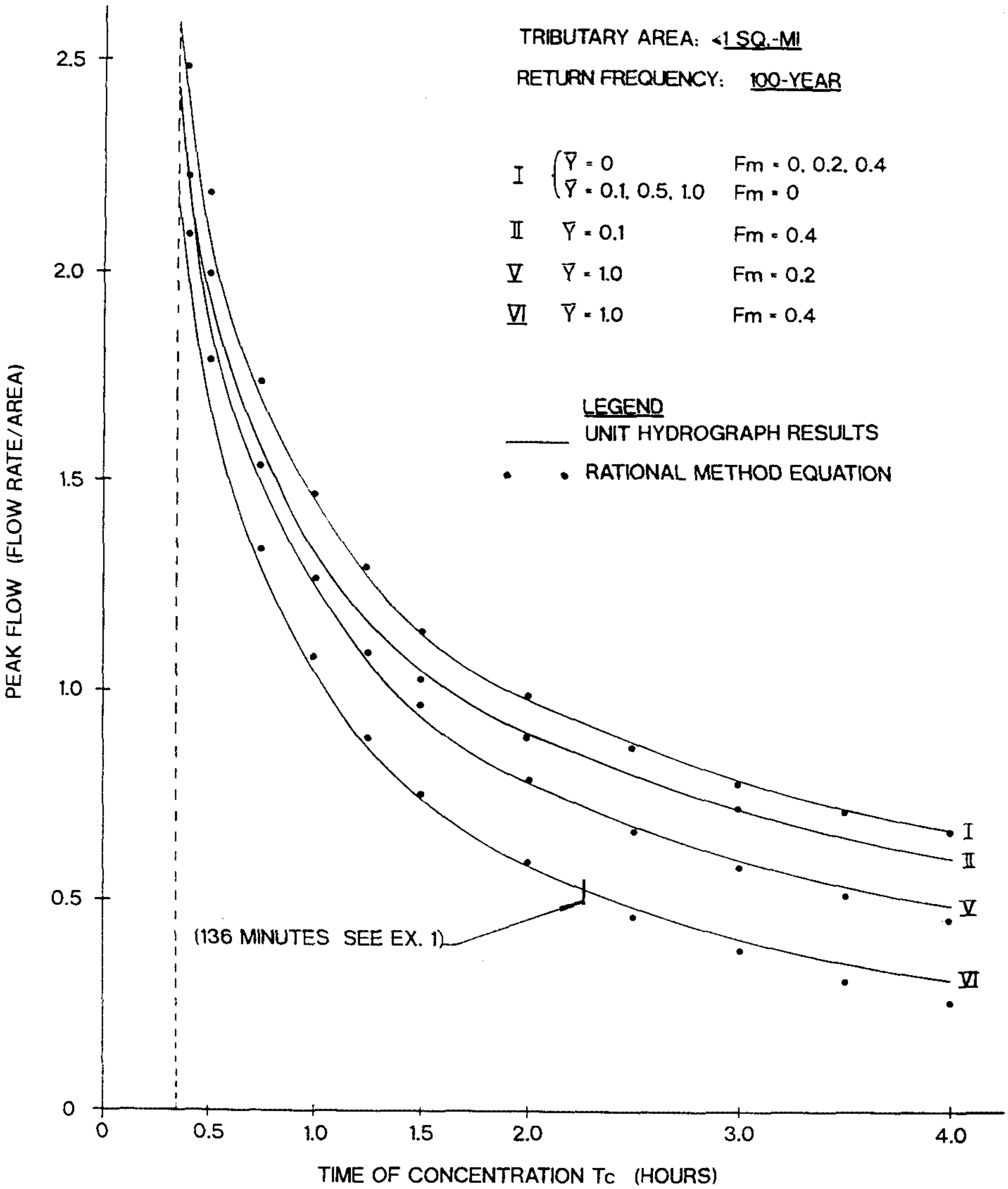
TRIBUTARY AREA: <1 SQ.-MI
RETURN FREQUENCY: 100-YEAR
I (Ȳ = 0 Fm = 0, 0.2, 0.4
Ȳ = 0.1, 0.5, 1.0 Fm = 0)
II Ȳ = 0.1 Fm = 0.4
V Ȳ = 1.0 Fm = 0.2
VI Ȳ = 1.0 Fm = 0.4
LEGEND
UNIT HYDROGRAPH RESULTS
RATIONAL METHOD EQUATION
(136 MINUTES SEE EX. 1)
PEAK FLOW (FLOW RATE/AREA)
0
0.5
1.0
1.5
2.0
2.5
0
0.5
1.0
1.5
2.0
3.0
4.0
I
II
V
VI
TIME OF CONCENTRATION Tc (HOURS)

FIGURE 3

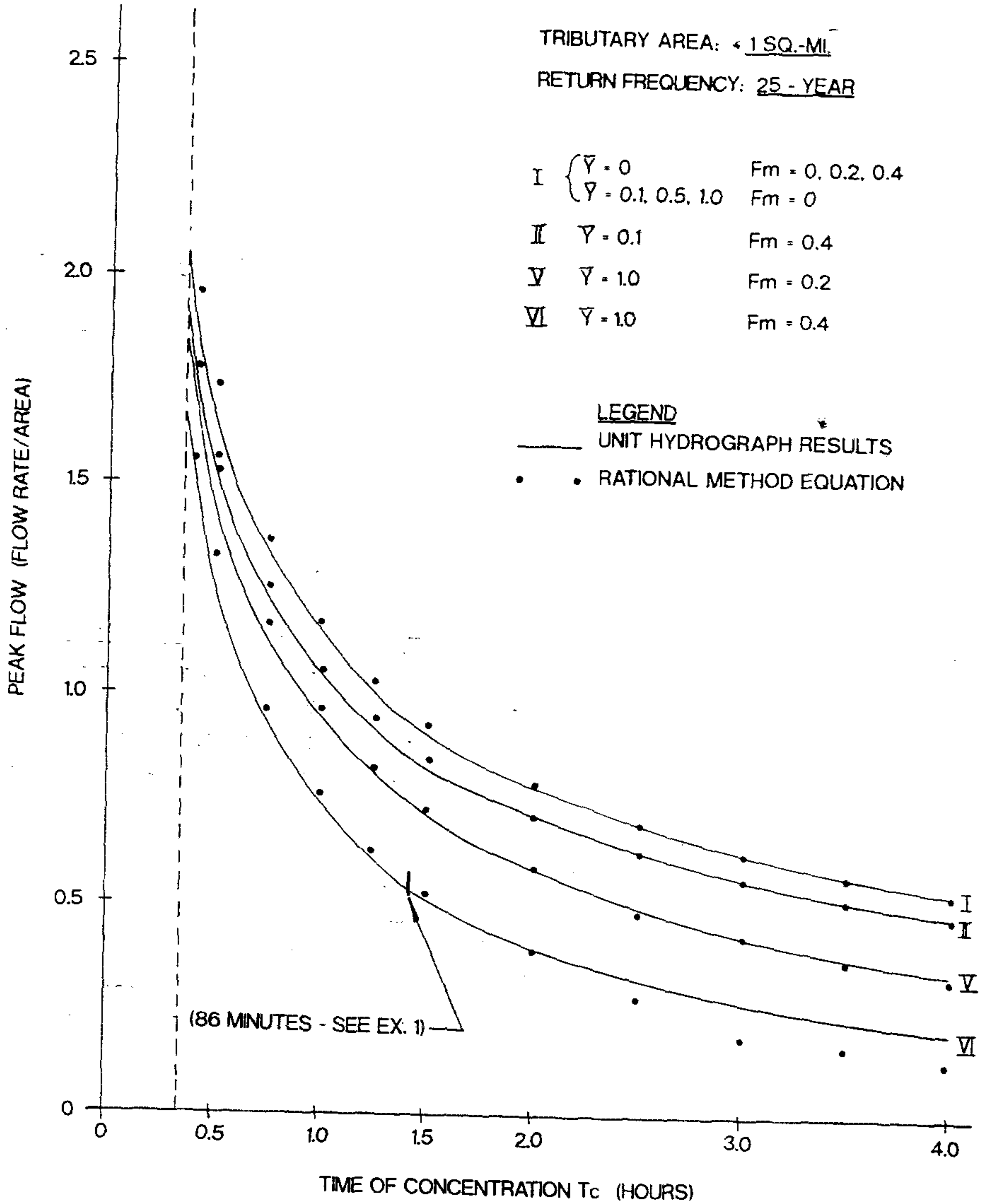
TRIBUTARY AREA: < 1 SQ.-MI.
RETURN FREQUENCY: 25 - YEAR
I { Ȳ = 0 Fm = 0, 0.2, 0.4
Ȳ = 0.1, 0.5, 1.0 Fm = 0 }
II Ȳ = 0.1 Fm = 0.4
V Ȳ = 1.0 Fm = 0.2
VI Ȳ = 1.0 Fm = 0.4
LEGEND
UNIT HYDROGRAPH RESULTS
RATIONAL METHOD EQUATION
PEAK FLOW (FLOW RATE/AREA)
2.5
2.0
1.5
1.0
0.5
0
(86 MINUTES - SEE EX. 1)
I
II
V
VI
0
0.5
1.0
1.5
2.0
3.0
4.0
TIME OF CONCENTRATION Tc (HOURS)

FIGURE 4