NANIA

Comprehensive Watershed Management

Proceedings of the
Eighteenth Annual Conference
of the
Association of State Floodplain Managers

May 8-13, 1994
Tulsa, Oklahoma

Examples of Applications of These Models

Discussed below are some typical examples where more complex flow situations have been analyzed through one- and two-dimensional steady- and unsteady-flow models.

Example 1—Large Tributary Inflows to Main Stem

In this example, river flows are controlled by upstream dams and reservoirs. For this reason, tributary inflows have a significant effect on the resulting 100-year water-surface elevation in the main stem of the river. During significant flooding, flows from the tributary will cause unsteady flow in the river's main stem.

The DWOPER model was used to determine the effects of tributary inflows on the main stem of a controlled river. In this case, the tributary inflows were combined with the main stem base flow and then routed to determine the flows above and below the confluence point. The resulting flows were used in the steady-state backwater program to calculate the water-surface elevations. The main stem water-surface profile was compared to the tributary-influenced profile to determine the controlling water-surface profile for NFIP purposes.

Example 2—Effects of Levees on Peak Flows

In this example, a major levee is located on the stream. When overtopped, the levee will allow off-stream storage behind it. Flood peaks will be affected by these levee overflows and off-stream storage. Encroachments in the off-stream storage areas were evaluated to ensure that flood peaks downstream would not be increased by future development (fill) in these areas due to loss of storage.

The DWOPER model was used to simulate the progression of the 100-year flood wave through the reach of stream affected by the levee. The DWOPER model was used because it can simulate flow over and storage behind levees. These resulting peaks were used in the steady-state backwater program to calculate water-surface elevations and floodways.

Example 3—Bridge, Many Islands, and Bifurcations

In this example, a river reach that is hydraulically complex, with a bridge, many islands, and bifurcations present during 100-year flood conditions, is to be modeled. Because of the hydraulic complexity, the FESWMS-2DH model was used. For purposes of developing a floodway, the FESWMS-2DH model results were used to calibrate the 100-year water-surface elevations determined in the one-dimensional HEC-2 model. The HEC-2 model was then used to establish an equal-conveyance floodway.

A STOCHASTIC INTEGRAL EQUATION ANALOG FOR RAINFALL-RUNOFF MODELING

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Abstract

The complexity of rainfall-runoff modeling and the apparent lack of success in significantly improving the accuracy of such modeling are well documented. In this paper, a multi-linear unit hydrograph approach is used to develop subarea runoff, and is coupled with a multi-linear channel flow routing method. The spatial and temporal rainfall distribution over the catchment is equated to a known rainfall data source. The resulting model structure is a series of stochastic integral equations, one equation for each subarea. A cumulative stochastic integral equation is developed that includes the spatial and temporal variabilities of rainfall. The resulting stochastic integral equation is an extension of the well-known single-area unit hydrograph method, except that the model prediction of a runoff hydrograph is a distribution of outcomes (or realizations).

Introduction

The complexity of rainfall-runoff modeling and the apparent lack of success in improving its accuracy are well documented (for example, Jakeman and Hornberger, 1993; Loague and Freeze, 1985; Hornberger et al., 1985; Hooper et al., 1988; Beven, 1989; Hromadka and Whitley, 1989). An apparent barrier to improvement in modeling accuracy is the lack of accurate rainfall data. Raines and Valdes (1993) state that "the estimate of the rainfall parameters is the most subjective task and seems to be responsible for the major sources of error." In this paper, unit hydrographs are used to estimate subarea runoff, which is then coupled to a multi-linear channel flow routing analog to develop a link-node model network. Jakeman and Hornberger (1993) observed a "predominant linearity in the response of watershed over a large range of catchment scales even if only a simple adjustment is made for antecedent rainfall conditions. The linearity assumption of unit hydrograph theory therefore seems applicable in temperate catchments and works just as well for slow flow as for quick flow."

Stochastic Rainfall-Runoff Model Development

The catchment is divided into hydrologic subareas, R_j, such as discussed in Hromadka et al. (1987). Each R_j is homogeneous in that a single loss function transform, $F_j(\bullet)$, applies in the subarea. The effective rainfall (or rainfall less losses) is given by $e_j^i(\bullet)$, for storm event i, where

$$e_j^i(t) = \int_{R_j}\int F_j(P^i(x,y,t))\,dxdy \;/\; A_j \quad (1)$$

where A_j is the area of R_j. The point rainfall is written as a sum of proportions of the available rain gauge data by

$$P^i(x,y,t) = \sum_{k=1}^{n_p} \lambda^i_{xyk}\, P_g^i(t-\theta^i_{xyk});\; P_g^i(\cdot) = \geq 0 \quad (2)$$

where λ^i_{xyk} is a proportion factor at coordinates (x,y) for event i, and Θ^i_{xyk} is a timing offset at (x,y) for event i. Combining (1) and (2),

$$A_j\, e_j^i(t) = \int_{R_j} F_j\Big[\sum_{k=1}^{n_p} \lambda^i_{xyk}\, P_g^i(t+\theta^i_{xyk})\Big]\, dR_j \quad (3)$$

Let F_j satisfy the conservative property

$$F_j\Big[\sum_{k=1}^{n_p} \lambda^i_{xyk}\, P_g^i(t+\theta^i_{xyk})\Big] = \sum_{k=1}^{n_p} \lambda^i_{xyk}\, F_j\,(P_g^i(t+\theta^i_{xyk})) \quad (4)$$

(An example of such a loss transform is $F_j(\bullet) = C_j(\bullet)$, where C_j is a constant for R_j.)

The runoff contribution for subarea j is given by

$$q_j^i(t) = \int_{s=0}^{t} e_j^i(t-s)\,\phi_j(s)\,ds = \int_{s=0}^{t}\int_{R_j}\sum_{k=1}^{n_p}\lambda^i_{xyk}\, F_j\,(P_g^i(t-\theta^i_{xyk}-s))\,\phi_j(s)\,dR_j\,ds \quad (5)$$

$$= \int_{s=0}^{t} F_j(P_g^i(t-s))\,\psi^i_j(s)\,ds \quad (6)$$

We can introduce nonlinearity with the $\phi_j(\bullet)$ based upon the magnitude of $e_j^i(\bullet)$, such as $\phi'_j(\bullet) = (\phi_j(\bullet) \mid e_j^i(\bullet))$. One method is to define subarea transfer functions according to the severity of storm, i.e., by storm class (e.g., mild, moderate, severe, flooding, etc.). From (6), randomness is inherent in the λ^i_{xyk} and θ^i_{xyk} values, for each storm event i.

Channel Flow Routing

Using a multilinear flow routing analog, without channel losses, (e.g., see Doyle et al., 1983; Becher and Kundzewicz, 1987),

$$Q_{j+1}^i(t) = q_{j+1}^i(t) + \sum_{k=1}^{n_r} \alpha_k\; Q_j^i(t-\beta_k) \quad (7)$$

where the link is known given nodes j, j+1; node j+1 is downstream of node j, n_r is the number of flow routing translates used in the analog; and the α_k and β_k are constants. The Convex, Muskingum, and many other flow routing techniques are given by (7).

Runoff at node j is given by upstream contributions of runoff

$$Q_j^i(t) = \sum_{\iota=1}^{n_j}\Big(\sum_{<k>_{j\iota}} \alpha'_{<k>_{j\iota}}\, q_\iota^i(t-\beta'_{<k>_{j\iota}})\Big) \quad (8)$$

where n_j is the number of subareas tributary to node j; the $<k>_{j\iota}$ is index notation for runoff contributions as summed over index ι, for index k.

Rewriting,

$$Q_j^i(t) = \sum_{\iota=1}^{n_j}\int_{s=0}^{t} F_\iota(P_g^i(t-s)) \sum_{<k>_{j\iota}} \alpha'_{<k>_{j\iota}}\, \psi_j^{i\iota}(s-\beta'_{<k>_{j\iota}})\, ds \quad (9)$$

$$= \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^{i}(t-s))\, \Psi_j^{i'}(s)\, ds; \; \Psi_j^{i'}(s) = \sum_{<k>_\gamma} \alpha'_{j<k>_\gamma} \psi_j^{i'}(s-\beta'_{<k>_\gamma}) \tag{10}$$

Runoff Prediction on a Storm Class Basis

In prediction, the distribution of $P^i(x,y,t)$ is unknown. The possible outcome for runoff, at node j, is a distribution of realizations given by $[Q_j^{*o}(\bullet)]$ where

$$[Q_j^{*o}(t)] = \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^{*}(t-s))\, [\Psi_j^{o}(s)]\, ds \tag{11}$$

where $[\Psi_j^o(s)]$ is the stochastic process of realizations from storm class o, where for node j,

$$[\Psi_j^{o}(s)] = \sum_{<k>_\gamma} \alpha^o_{<k>_\gamma} [\Psi_j^{o}(s-\beta^o_{<k>_\gamma})] \tag{12}$$

The expectation is given for (11) by

$$E\,[Q_j^{*o}(t)] = \sum_{\ell=1}^{n_j} \int_{s=0}^{t} F_\ell(P_g^{*}(t-s))\, E[\Psi^{o}(s)]\, ds \tag{13}$$

Equation (13) forms a basis of the unit hydrograph procedure commonly used for flood control design and planning.

The Unit Hydrograph Method (Single Area)

The well-known single-area unit hydrograph (UH) method may be developed by the expectation, for the case of prediction of runoff for rainfall event $P_g^*(\bullet)$,

$$E[Q_g^{*}(t)] = \int_{s=0}^{t} F(P_g^{*}(t-s))\, E[\Phi(s)]\, ds \tag{14}$$

where $E[Q_g^*(\bullet)]$ is a single runoff hydrograph (usually filtered); and $E[\Phi(\bullet)]$ is the calibrated transfer function. In order for $E[\Phi(\bullet)]$ to be a UH, normalization is needed by letting

$$\eta = \int_{s=0}^{\infty} E[\Phi(\cdot)]\, ds \tag{15}$$

and the UH is simply $\frac{1}{\eta} E[\phi(\cdot)]$

Conclusions and Discussion

Methods have been in use for decades for transferring UH relationships to locations where stream gauge data are not available (for example, see Hromadka et al., 1987). In order to transfer the stochastic relationships of variability in the $[\Phi(\bullet)]$, the same UH transferability techniques may be used. That is, by scaling the distribution of $[\Phi(\bullet)]$ outcomes with respect to $E[\Phi(\bullet)]$, then as $E[\Phi(\bullet)]$ is transferred in UH form, so is the distribution $[\Phi(\bullet)]$. This approach has been implemented in the recent hydrology manuals for the counties of Kern (1992) and the largest county in the mainland United States, San Bernardino (1993). The approach is currently being developed for the hydrology manual of the county of San Joaquin (1993).

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