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THE APPROXIMATE BOUNDARY METHOD FOR ERROR REDUCTION IN THE COMPLEX VARIABLE BOUNDARY ELEMENT METHOD: OVERVIEW

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Abstract

The Complex Variable Boundary Element Method or CVBEM has been shown to be an effective computational tool for approximately solving two-dimensional problems involving the Laplace or Poisson equations. In this paper, the Approximate Boundary Method (ABM) for error reduction in the CVBEM boundary condition approximation is further examined. A norm for evaluating accuracy and approximation improvement is used to assess the utility of the ABM.

INTRODUCTION

The objective of this paper is to present the results of a study into the error reduction afforded by utilization of the approximative boundary method to refining approximations of the Complex Variable Boundary Element Method or CVBEM (Hromadka and Lai, 1987). The CVBEM is a mathematical modeling technique that approximates solutions to boundary value problems, such as two-dimensional ideal fluid flow and steady state heat transfer, which are governed by the two-dimensional Laplace equation. Given known and usually mixed conditions at specified points, or nodes, on the boundary of a singly or multiply connected domain, the CVBEM utilizes the Cauchy integral formula to produce an approximation function which is analytic on the interior of the problem domain, continuous on the problem boundary, and thus satisfies the two-dimensional Laplace equation. The approximation function produced by the CVBEM approaches the analytic solution as the number of specified nodal values increases and the boundary element lengths decrease.

THE COMPLEX VARIABLE BOUNDARY ELEMENT METHOD

Let $\omega(z) = \phi(x,y) + i\psi(x,y)$ be a complex variable function which is analytic on $\Gamma \cup \Omega$, where Ω is a simply connected domain enclosed by the simple closed boundary Γ (Fig. 1.). We define $\phi(x,y)$ to be the state variable and $\psi(x,y)$ the stream function, where ϕ and ψ are two-dimensional real valued functions. Since ω is analytic, ϕ and ψ are related by the Cauchy-Reimann equations

$$\frac{\delta\phi}{\delta x} = \frac{\delta\psi}{\delta y} \text{ and } \frac{\delta\phi}{\delta y} = -\frac{\delta\psi}{\delta x} \tag{1}$$

and thus satisfy the two-dimensional Laplace equations

$$\frac{\delta^2\phi}{\delta x^2} = \frac{\delta^2\phi}{\delta y^2} \text{ and } \frac{\delta^2\psi}{\delta x^2} = -\frac{\delta^2\psi}{\delta y^2} = \tag{2}$$

The Cauchy integral theorem states that if we know the value of the complex function ω on the boundary Γ, and if ω is analytic on $\Omega \cup \Gamma$, then ω on Ω is given by

$$\omega(z) = \frac{1}{2\pi i}\int_{\Gamma} \frac{\omega(\zeta)d\zeta}{\zeta - z}, \; z \in \Omega, z \notin \Gamma \tag{3}$$

The CVBEM forms $\hat{\omega}$, an approximation of ω, using known values of either ϕ or ψ on the boundary Γ, and uses the Cauchy integral (3) to determine approximate values for ω on $\Omega \cup \Gamma$. The approximator, $\hat{\omega}$, is a two-dimensional analytic function in Ω that can be differentiated, integrated, or otherwise manipulated to obtain higher order operator relationships (Hromadka and Lai, 1987).

Example 1: $\omega(z) = z + 1/z$ (Ideal fluid flow over a cylinder).

Figure 3 shows the problem boundary, the 17 nodes initially defined on Γ, and the known values of ϕ and ψ. Both ϕ and ψ are known at the points (100,0), (100,100), (0,100), and (0,1). The 25-node and 40-node approximate boundaries are shown in figures 4 and 5, respectively. The maximum errors for each approximation are E_{25} = .3502 and E_{40} = .2534, indicating a 27.6 percent reduction in approximation error. Figure 6 depicts the boundary condition error values on Γ.

REFERENCES

1. Hromadka II, T.V., and Lai, C., *The Complex Variable Boundary Element Method in Engineering Analysis*, Springer-Verlag, New York, 1987.
2. Harryman III, R.R., Hromadka II, T.V., Vaughn, J.L., and Watson, D.P., *The CVBEM for Multiply Connected Domains Using a Linear Trial Function*, Applied Mathematical Modeling, Vol. 14, February 1990.
3. Churchill, R.V. and Brown, J.W., *Complex Variables and Applications*, McGraw-Hill, New York, 1984.