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A stochastic integral equation analog of rainfall-runoff processes for evaluating modeling uncertainty

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Abstract: In this paper a very general rainfall-runoff model structure (described below) is shown to reduce to a unit hydrograph model structure. For the general model, a multi-linear unit hydrograph approach is used to develop subarea runoff, and is coupled to a multi-linear channel flow routing method to develop a link-node rainfall-runoff model network. The spatial and temporal rainfall distribution over the catchment is probabilistically related to a known rainfall data source located in the catchment in order to account for the stochastic nature of rainfall with respect to the rain gauge measured data. The resulting link node model structure is a series of stochastic integral equations, one equation for each subarea. A cumulative stochastic integral equation is developed as a sum of the above series, and includes the complete spatial and temporal variabilities of the rainfall over the catchment. The resulting stochastic integral equation is seen to be an extension of the well-known single area unit hydrograph method, except that the model output of a runoff hydrograph is a distribution of outcomes (or realizations) when applied to problems involving prediction of storm runoff; that is, the model output is a set of probable runoff hydrographs, each outcome being the results of calibration to a known storm event.

Key words: Rainfall, runoff, modeling, uncertainty, stochastics, stochastic integral equations.

1 Introduction

Issues regarding rainfall-runoff modeling complexity and the apparent lack of success in achieving further improvement in modeling accuracy is well documented (for example, Jakeman and Hornberger, 1993; Loague and Freeze, 1985; Hornberger et al, 1985; Hooper et al, 1988; Beven, 1989; Hromadka and Whitley, 1989). Speaking of the use of a unit hydrograph, Jakeman and Hornberger (1993), observed a "predominant linearity in the response of watersheds over a large range of catchment scales even if only a simple adjustment is made for antecedent rainfall conditions. The linearity assumption of unit hydrograph theory therefore seems applicable in temperate catchments and works just as well for slow flow as for quick flow."

In this paper, the work of Hromadka and Whitley (1989; pgs. 169-254) is rederived in a constructive way which streamlines the accounting of rainfall variations over the catchment, resulting in a final formulation which is easier to use. The use of stochastic integral equations to model rainfall-runoff response is shown to come about from a straight-forward use of probability in modeling the uncertainty in runoff as due to the uncertainty in the distribution of rainfall (see Hromadka and Whitley (1989), pgs. 1-52, for a review of the literature regarding rainfall-runoff models and stochastic integral equations.) The unit hydrograph approach is used to generate catchment subarea runoff which is then coupled to a multi-linear channel flow routing analog to develop a link-node rainfall-runoff model network. The spatial and temporal rainfall distribution over the catchment is equated to a known rainfall data source in the catchment (i.e., the rain gauge) in order to account for the rand ture of rainfall with respect to the measured rain gauge data. (That is, the precise rainfall intensity at a specific location in a catchment is written as a function of a single rain gauge data set.) The resulting link node model structure is a series of random integral equations. A stochastic integral equation is synthesized from the above series that includes the complete spatial and temporal variabilities of the rainfall over the catchment. The resulting stochastic integral equation is seen to be an extension of the well-known single area unit hydrograph method, where the model output is a distribution of outcomes when applied to problems involving prediction of storm runoff. The distribution of outcomes can then be used to develop probability distributions for runoff criterion variables, such as peak flow rate, detention basin volume (among others), whereby confidence intervals may be developed.

2 Stochastic rainfall-runoff model development

Similar to the development in Hromadka and Whitley (1989), a stochastic integral equation will be developed under the premise that the uncertainty in the spatial and temporal distribution of rainfall, with respect to a single known rainfall data source, $p^i_g(\cdot)$, for storm event i, coupled to the uncertainty in loss rates throughout the catchment, dominates the rainfall-runoff uncertainty problem (Naef (1981), Loague (1990), Wilcox et al, (1990), Refsgaard (1994), among others). In the following analysis, it is assumed that a quasi-linear modeling structure can be used to represent the rainfall-runoff process. (From the development, the approach applies, in general, to free draining catchments in which the dominating effects of storage, such as due to dams or other similar effects, is not a significant influence).

The stochastic integral equation rainfall-runoff model is developed with respect to a distributed parameter link-node model setting, including nonhomogeneous loss functions, multi-linear subarea runoff response, multi-linear channel flow routing, and the random processes involved with the spatial and temporal variation of rainfall over the entire catchment. In this way, the randomness of the problem's initial and boundary conditions (i.e., the prior and current rainfall over the catchment) is combined with the integration of the various mutually dependent random components of the runoff process, resulting in a stochastic integral equation.

To begin, let the catchment be divided into hydrologic subareas, R_j, such as discussed in Hromadka et al (1987). Each R_j is homogeneous in that a single loss function transform, $F_j(\cdot)$, operates on the subarea point rainfall uniformly. The effective rainfall (or rainfall less losses) is given by $e^i_j(\cdot)$, for storm event i, where

$$e^i_j(t) = \int_{R_j}\int F_j(P^i(x,y,t))\,dxdy/A_j \tag{1}$$

and A_j is the area of subarea, R_j. It is assumed that the point rainfall can be written as a sum of fractions of translates of the available rain gauge data:

$$P^i(x,y,t) = \sum_k \lambda^i_{xyk}\, P^i_g(t-\theta^i_{xyk}) \tag{2}$$

where λ^i_{xyk} is a proportion factor at coordinates (x,y) for event i, each θ^i_{xyk} is a positive timing offset at (x,y) for event i, and $P^i_g(u) = 0$ for u<0. Combining (1) and (2),

$$A_j\, e^i_j(t) = \int_{R_j} F_j\left[\sum_k \lambda^i_{xyk}\, P^i_g(t-\theta^i_{xyk})\right] dR_j \tag{3}$$

Let F_j satisfy the property that

$$F_j\left[\sum_k \lambda^i_{xyk}\, P^i_g(t-\theta^i_{xyk})\right] = \sum_k \lambda^i_{xyk}\, F_j(P^i_g(t-\theta^i_{xyk})) \tag{4}$$

(An example of such a loss function is $F_j(\cdot) = C_j(\cdot)$, where C_j is a constant for R_j). This loss function is analogous to the Rational Method C-coefficient.

3 Subarea runoff contribution for event i

The unit hydrograph model applied to the runoff contribution for subarea j gives

$$q^i_j = \int_{s=0}^{t} e^i_j(t-s)\phi_j(s)ds = \int_{s=0}^{t}\int_{R_j}\frac{1}{A_j}\sum_k \lambda^i_{xyk}\, F_j(P^i_g(t-\theta^i_{xyk}-s))\,\phi_j(s)\,dR_j ds \tag{5}$$

$$= \int_{s=0}^{t} F_j(P^i_g(t-s))\int_{R_j}\frac{1}{A_j}\sum_k \lambda^i_{xyk}\,\phi_j(s-\theta^i_{xyk})\;dR_j ds \tag{6}$$

$$= \int_{s=0}^{t} F_j(P^i_g(t-s))\,\psi^i_j(s)\,ds \tag{7}$$

where

$$\psi^i_j(s) = \int_{R_j}\frac{1}{A_j}\sum_k \lambda^i_{xyk}\,\phi_j(s-\theta^i_{xyk})dR_j \tag{8}$$

and where $\phi_j(u) = 0$ for u<0.

A non-linear model for $q^i_j(\cdot)$, can come about by the choice of transfer functions $\phi_j(\cdot)$ which depend upon the magnitude of $e^i_j(\cdot)$; one method is to define a set of subarea transfer functions according to the severity of storm; i.e., by storm class (e.g., mild, moderate, severe, flooding, etc.). (It is noted that the $\phi_j(\cdot)$ may, in turn, be statistically regressed against catchment parameters such as lag, area, etc.)

From (7) and (8), randomness is inherent in the λ^i_{xykj} and θ^i_{xyk} values, for each storm event i. That is, for prediction of runoff, the λ^i_{xykj} and θ^i_{xyk} values are samples of random variables distributed as $[\lambda_{xyk}]$ and $[\theta_{xyk}]$, respectively, where the notation $[\cdot]$ refers to both the random process and its distribution.

4 Channel flow routing

In the link-node network model, there is accumulating runoff contributions at nodes, with flow routing along each link.

Using a multilinear flow routing analog, without channel losses, (e.g., see Doyle et al (1983), Becker and Kundzewicz (1987)),

$$Q^i_{j+1}(t) = q^i_{j+1}(t) + \sum_k \alpha^i_k\, Q^i_j(t+\beta^j_k) \tag{9}$$

where the link is known given nodes j, j+1; node j+1 is downstream of node j, and the sum is over the number of flow routing translates used in the analog; and the α^j_k and β^j_k are constants. The Convex, Muskingum, and many other flow routing techniques are given by (9). The parameters α^j_k, β^j_k are link dependent on the discharge $Q^i_j(\cdot)$ at the j^{th} node, due to the i^{th} storm class. For a known discharge $Q^i_j(\cdot)$, the α^j_k and β^j_k are constant, but their values may differ for different discharges. They may be defined, in prediction, according to the storm class system used for the $\phi^i_j(\cdot)$ realizations.

Thus we can correlate the α^j_k and β^j_k value, for each storm class to the k measured rainfall, $P^i_g(\cdot)$, so that α^j_k and β^j_k can be taken to be (different) constants for each $P^i_g(\cdot)$. Of course, inherent in these α^j_k and β^j_k are channel geometric parameters, among others; however, on a link basis, the α^j_k and β^j_k represent the cumulative effect of all influences.

5 Link-node model

For subarea 1, contributing runoff at node 1,

$$Q_1^i(t) = q_1^i(t) = \int_{s=0}^{t} F_1(P_g^i(t-s))\psi_1^i(s)ds \tag{10}$$

Routing $Q_1^i(\cdot)$ to node 2, using (9), and adding $q_2^i(\cdot)$:

$$Q_2^i(t) = q_2^i(t) + \sum_{k_1} \alpha_{k_1}^1 q_1^i(t+\beta_{k_1}^1) \tag{11}$$

Routing $Q_2^i(\cdot)$ to node 3, and adding $q_3^i(\cdot)$:

$$Q_3^i(t) = q_3^i(t) + \sum_{k_2} \alpha_{k_2}^2 Q_2^i(t+\beta_{k_2}^2) \tag{12}$$

$$= q_3^i(t) + \sum_{k_2} \alpha_{k_2}^2 q_2^i(t+\beta_{k_2}^2) + \sum_{k_2}\sum_{k_1} \alpha_{k_2}^2\alpha_{k_1}^1 q_1^i(t+\beta_{k_1}^1+\beta_{k_2}^2) \tag{13}$$

In general the runoff at node j is given by the upstream runoff contribution to this node:

$$\begin{aligned} Q_j^i(t) = {} & q_j^i(t) + \sum_{k_{j-1}} \alpha_{k_{j-1}}^{j-1}\, q_{j-1}^i(t+\beta_{k_{j-1}}^{j-1}) \\ & + \sum_{k_{j-1}}\sum_{k_{j-2}} \alpha_{k_{j-1}}^{j-1}\, \alpha_{k_{j-2}}^{j-2}\, q_{j-2}^i(t+\beta_{k_{j-1}}^{j-1}+\beta_{k_{j-2}}^{j-2}) \\ & + \ldots\ldots \\ & + \sum_{k_{j-1}}\sum_{k_{j-2}} \cdots \sum_{k_1} (\alpha_{k_{j-1}}^{j-1}\, \alpha_{k_{j-2}}^{j-2}\, \alpha_{k+j-1}^{j-1}\, \alpha_{k+j-2}^{j-2} \ldots \alpha_{k_1}^1)\, q_1^i(t+\beta_{k_{j-1}}^{j-1}+\ldots+\beta_{kk_1}^1) \end{aligned} \tag{14}$$

Use (7) and (8), together with (14), to write

$$\begin{aligned} Q_j^i(t) = {} & \int_{s=0}^{t} F_j\,(P_g^i(t-s))\psi_j^i(s)\,ds \\ & + \sum_{k_{j-1}} \alpha_{k_{j-1}}^{j-1} \int_{s=0}^{t} F_{j-1}(P_g^i(t-s))\psi_{j-1}^i(s+\beta_{k_{j-1}}^{j-1})ds \\ & + \sum_{k_{j-1}}\sum_{k_{j-2}} \alpha_{k_{j-1}}^{j-1}\alpha_{k_{j-2}}^{j-2} \int_{s=0}^{t} F_{j-2}(P_g^i(t-s))\psi_{j-2}^i(s+\beta_{k_{j-1}}^{j-1}+\beta_{k_{j-2}}^{j-2})ds \\ & + \ldots\ldots \\ & + \sum_{k_{j-1}}\sum_{k_{j-2}} \cdots \sum_{k_1} (\alpha_{k_{j-1}}^{j-1}\, \alpha_{k_{j-2}}^{j-2} \ldots \alpha_{k_1}^1) \times \int_{s=0}^{t} F_j\,(P_g^i(t-s))\psi_1^i(s+\beta_{k_{j-1}}^{j-1}+\ldots+\beta_1^1)ds \\ = {} & \sum_m \int_{s=0}^{t} F_m\,(P_g^i(t-s))\psi_m^i(s)\,ds \end{aligned} \tag{15}$$

where

$$\psi_m^i(s) = \sum_{k_{j-1}}\sum_{k_{j-2}} \cdots \sum_{k_m}^{m} (\alpha_{k_{j-1}}^{j-1}\, \alpha_{k_{j-2}}^{j-2} \ldots \alpha_{k_m}^m)\, \psi_m^i(s+\beta_{k_{j-1}}^{j-1}+\ldots+\beta_{k_m}^m)) \tag{16}$$

Since the numbers, $\alpha_{k_j}^j$ $\beta_{k_j}^j$ and the function ψ_m^i are all dependent on the storm classes in which the several upstream subarea realizations $e_j^i(\cdot)$ belong, $\psi_m^i(s)$ is itself so dependent. Also, the random spatial and temporal variation of point rainfall for storm event i, namely $P^i(x,y,t)$, is given by the probability distributions of the λ_{xyk} and θ_{xyk} of equation (2).

Note that from Eqs. (2) and (4), due to the variation in point rainfall, $P^i(x,y,t)$, the various subarea runoff contributions do not directly correlate to the rainfall data, $P_g^i(t)$ measured at the gauge. Consequently, the various flow routing parameters and subarea transfer functions all depend upon the cumulative effects of the upstream λ_{xyk}^i and θ_{xyk}^i values. Indeed, some subareas may have zero runoff due to occurrence of negligible rainfall, and the flow routing and subarea transfer functions will be inaccurate because the only known value is the possibly severe rainfall data $P_g^i(\cdot)$.

6 Simplifications

1. Storm Class Determined by $P_g^i(\cdot)$

A simplification of Eq. (15) is to neglect the temporal and spatial variation of point rainfall, $P^i(x,y,t)$, in the choice of storm classes for determination of the α_k^j, β_k^j, and ψ_k^j. This is reasonable because the variation of $P^i(x,y,t)$ is obviously unknown. A suitable choice for determining parameter storm classes is to simply use the rainfall data itself, $P_g^i(\cdot)$. For example, if $P_g^i(\cdot)$ is severe, all parameters are based on a severe storm class, and if $P_g^i(\cdot)$ is mild, all parameters are based on mild storm class values. Using $P_g^i(\cdot)$ for determining the storm class simplifies ψ_m^j, as given by (16), in that the α_k^j, β_k^j, and ψ_k^j now only depend on the storm class of $P_g^i(\cdot)$.

This model is still multi-linear, due to the use of the storm classes, but differs from the model of (15) and (16), in that the effects of sampling the various distributions of $[\lambda_{xyk}]$ and $[\theta_{xyk}]$ are essentially ignored.

2. Single Storm Class

A further simplification is to assume that the rainfall-runoff model will be used only within a single storm class. This is the case typically considered for flood control purposes, where only severe storm data are used for analysis purposes; here the coefficients α_k^j and β_k^j are the same for all the storms considered.

3. Runoff Prediction on a Storm Class Basis

In prediction, the distribution of $P^i(x,y,t)$ is unknown even if the future measured data $P_g^*(\cdot)$ is assumed known. In examining (8), (15) and (16), the possible outcome for runoff, at node j, given the simplifications of 2 above, is a distribution of realizations given by $[Q_j^{*o}(\cdot)]$ where

$$[Q_j^{*o}(t)] = \sum_{\ell} \int_{s=0}^{t} F_\ell\,(P_g^*(t-s))\; [\Psi_j^o(s)]\; ds \tag{17}$$

where $[\psi_j^o(s)]$ *is the stochastic process of realizations from storm class o*, where for node j, the coefficients α_k^j and β_k^j in (14) are for that storm class, and from (8) the distribution $[\psi_j^o(\cdot)]$ is given by

$$[Q_1^o(s)] = \frac{1}{A_j} \int_{R_j} \sum_{n} [\lambda_{xyk}]\; \phi_j^o(s-[\theta_{xyk}])dR_j \tag{18}$$

(Again $[\cdot]$ refers to both the random process and its distribution, respectively. Using the notation $[\cdot]$ aids in identifying *stochastic variables in the mathematical development and subsequent* equations.)

The expectation of (17) is:

$$E\,[Q_j^{*o}(t)] = \sum_{\ell} \int_{s=0}^{t} F_\ell\,(P_g^*(t-s))\; E\,[\psi_j^o(s)]\; ds \tag{19}$$

Equation (19) forms a basis of the unit hydrograph procedure commonly used for flood control design and planning.

4. Distributions of Runoff Criterion Variables

Assume a free flowing catchment such that the modeling assumptions leading to (15) and (16) applies. Further assume that each loss function $F_k(\cdot) = a_k\, F(\cdot) + b_k$ for a single loss function, $F(\cdot)$. That is, each subarea k loss function, $F_k(\cdot)$, is a linear function of a reference loss function, $F(\cdot)$. Given catchment runoff at a stream gauge location, with runoff $Q_g^i(\cdot)$ for storm i, and given associated rainfall, $P_g^i(\cdot)$, (15) can to used to relate runoff to rainfall by

$$Q_g^i(t) = \int_{s=0}^{t} F\,(P_g^i(t-s))\ \Phi^i(s)\, ds \tag{20}$$

where $\Phi^i(\cdot)$ is a transfer function for storm event i, and it is required that runoff at $Q_g^i(\cdot)$ does not occur in time prior to initiation of rainfall, $P_g^i(\cdot)$. From (15), $\Phi^i(\cdot)$ includes all the proper samplings of the various mutually dependent random processes and variables, for storm i, used in the previous stochastic integral equation development leading to (15).

Since the only available rainfall data are from the single rain gauge, storm classes are defined on that rain gauge data. For this application, only "severe" storms are being considered for flood protection purposes. Define the storm class S by

$$S = \{P_g^i(\cdot) | P_g^i(\cdot) \text{ is considered severe; } i = 1,\ 2,\ \ldots\ m\} \tag{21}$$

For each event $P_g^i(\cdot) \in S$, resolve the unique transfer function $\Phi^i(\cdot)$ by solving (20), resulting in what are assumed to be m equally likely realizations of the transfer function. Define the discrete distribution $[\Phi^i(\cdot)]$ by indicating its equally likely values:

$$[\Phi(\cdot)] = \{\Phi^i(\cdot);\ 1,\ 2,\ \ldots\ m\} \tag{22}$$

where each $\Phi^i(\cdot)$ in (22) is a solution of (20) for a $P_g^i(\cdot) \in S$. Note that in (22), the distribution $[\Phi(\cdot)]$ is dependent upon the loss function, F, chosen.

Each sample $\Phi^i(\cdot)$ from $[\Phi(\cdot)]$ is from all the mutually dependent random variables and processes incorporated in (15). Additionally, as with any stochastic process, the discrete distribution $[\Phi(\cdot)]$, in (22), depends on the sample size, m, available from the rainfall-runoff data.

It is noted that the assumption that each $F_k(\cdot)$ be a linear combination of a reference $F(\cdot)$ may be accommodated by directly relating effective rainfall, as a point function, in (1), by the analogous formula

$$e_j^i(t) = \sum_k \int_{s=0}^{t} \lambda_{xyk}^i\ F(P_g^i(t-\theta_{xyk}^i)).$$

In prediction, where a future storm $P_g^*(\cdot) \in S$ is contemplated at the rain gauge, the estimated distribution of runoff realizations at the stream gauge is given by the stochastic integral equation,

$$[Q_g^*(t)] = \int_{s=0}^{t} F(P_g^*(t-s))[\Phi(\cdot)]\, ds \tag{23}$$

where $[Q_g^*(\cdot)]$ is the distribution of outcomes, based on the available rainfall-runoff data. For m discrete events in $[\Phi(\cdot)]$, there will be m discrete outcomes in $[Q_g^*(\cdot)]$. Oftentimes a filter is used with $[Q_g^*(\cdot)]$ such that for each $Q_g^*(\cdot) \in [Q_g^*(\cdot)]$,

$$Q_g^{*'}(t) = \begin{cases} Q_g^*(t), & \text{if positive} \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

where $Q_g^{*'}(\cdot)$ refers to a filtered realization of $Q_g^*(\cdot)$.

For the criterion variable of peak flow rate, Q_p, the distribution $[Q_p]$ is determined by the operator A_1 on $[Q_g^{*'}(\cdot)]$, where

$$[Q_p] = A_1[Q_g^{*'}(\cdot)] \tag{25}$$

where $[Q_p]$ is the probability distribution of peak flow rates, A_1 is the operation of finding the peak flow rate from any realization distributed as $[Q_g^{*'}(\cdot)]$. Confidence intervals can then be computed for $[Q_p]$ by the usual methods.

As another example, let A_2 be the operation of finding the maximum ponded depth, γ, of floodwater in a detention/retention basin. Then the distribution of basin peak flood depth, $[\gamma]$, is similarly given by

$$[\gamma] = A_2[Q_g^{*'}(\cdot)] \tag{26}$$

Note that since A_2 is nonlinear, the expectations are related by

$$E[\gamma] = E(A_2[Q_g^{*'}(\cdot)]) \tag{27}$$

where

$$E(A_2[Q_g^{*'}(\cdot)]) \neq A_2(E[Q_g^{*'}(\cdot)]) \tag{28}$$

5. The Unit Hydrograph Method (Single Area)

From (23), the well-known single area unit hydrograph (UH) method may be developed by using the expectation, for the case of prediction of runoff for rainfall event $P_g^*(\cdot)$,

$$E[Q_g^*(t)] = \int_{s=0}^{t} F(P_g^*(t-s))\ E[\Phi(s)]\, ds \tag{29}$$

where $E[Q_g^*(\cdot)]$ is a single runoff hydrograph (usually filtered); and $E[\Phi(\cdot)]$ is the calibrated transfer function. In order for $E[\Phi(\cdot)]$ to be a UH, it needs to be normalized, which is done by letting

$$\eta = \int_{s=0}^{\infty} E[\Phi(\cdot)]\, ds \tag{30}$$

and taking the UH to be $\frac{1}{\eta}\, E[\Phi(\cdot)]$, and the loss function is also modified by multiplying by the constant η.

6. Transferability of the Stochastic Integral Equation Method

The applications 4 and 5 develop a stochastic integral equation, and a UH method, at a stream gauge location given available rain gauge data.

Methods have been in use for decades for transferring UH relationships to locations where stream gauge data are not available [for example, see Hromadka et al, 1987], and need not be discussed here. In order to transfer the stochastic relationships of variability in the $[\Phi(\cdot)]$, appearing in (29), the same UH transferability techniques may be used. That is, by scaling the distribution of $[\Phi(\cdot)]$ outcomes with respect to $E[\Phi(\cdot)]$, then as $E[\Phi(\cdot)]$ is transferred in UH form, so is the distribution $[\Phi(\cdot)]$. This approach has been implemented in the recent hydrology manuals for the counties of Kern (1992) and the largest county in the mainland United States, San Bernardino (1994). The approach is currently being developed for the Hydrology Manual of the County of San Joaquin (1994).

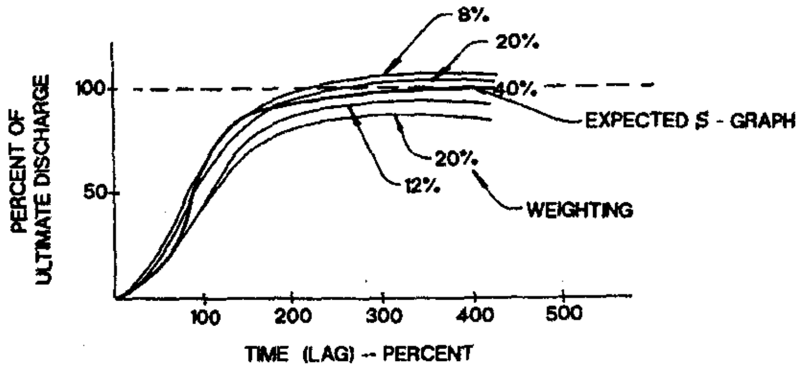

Figure 1. S-graph weighting for Southern California (note masses of S-graph do not necessarily equal 1.0).

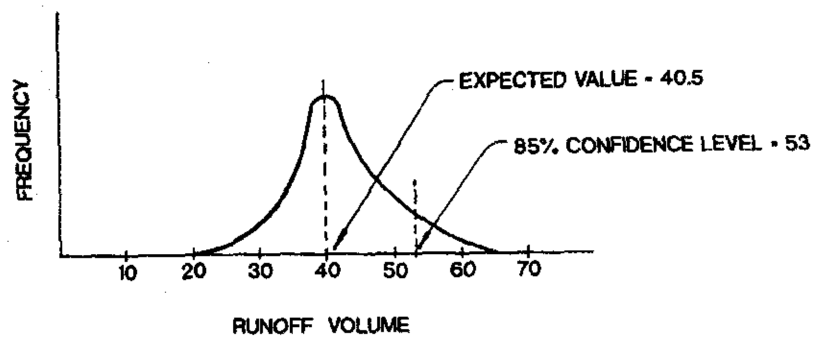

Figure 2. Distribution of flood control basin runoff volume using Figure 1 S-graphs.

7 Application

A distribution of S-graphs has been developed from catchment rainfall-runoff data in the preparation of the San Bernardino County Hydrology Manual (1994) (see Figure 1). Each S-graph is considered to have the probability weighting shown in Fig. 1. In the estimation of runoff, each S-graph is used, in turn, to develop a respective runoff hydrograph with a probability weighting equal to the parent S-graph weighting. If peak flow rate is of interest, each runoff hydrograph peak flow rate value is considered a sample with the weighting equal to the S-graph weighting. A probability distribution function is then contemplated for the sampled data (and weightings) and inferences drawn.

For example, for a detention basin problem, the maximum storm event runoff volume is of interest and its value depends upon the sampled S-graph of Fig. 1. Figure 2 shows a typical distribution of detention basin maximum volumes, using Fig. 1 S-graphs, for a single hypothetical design storm event. The expected value, and 85-percent upper confidence limit, of maximum detention basin volume (for the subject design storm event) is included in Fig. 2.

8 Conclusions

The stochastic integral equation rainfall-runoff model is developed with respect to a distributed parameter link-node model setting, including nonhomogeneous loss functions, multilinear subarea runoff response, multi-linear channel flow routing, and the random processes involved with spatial and temporal variation of rainfall over the entire catchment. In this way, the randomness of the problem's initial and boundary conditions (i.e., the prior and current rainfall over the catchment) is properly accounted for and the integration of the various mutually dependent random components result in the stochastic integral equation. The applications considered in this paper derive the classic unit hydrograph method as the expectation of the stochastic integral equation, and also discuss transferability methods to apply the uncertainty distributions at locations where runoff gauge data are not available. Use of the stochastic integral equation method is only marginally more difficult then the usual unit hydrograph method for free flowing catchments, and provides an estimate of the uncertainty in the usual estimates for criteria variables.

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