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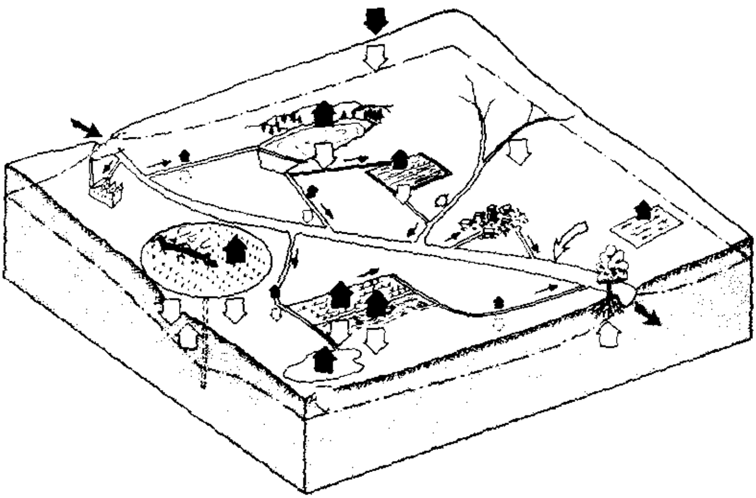

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analysis process consist of the application of alternative drilling rules and the consideration of sampling and mobilization costs when estimating expected borehole costs.

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Estimating Uncertainty in Design Storm Rainfall-Runoff Models Using a Stochastic Integral Equation

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ABSTRACT

Almost all rainfall-runoff models in use today involve the subdivision of the catchment into smaller areas, linked together by a system of channel links. These "link-node" hydrologic models represent the flow processes within the channel links by a translation (moving in time) and an attenuation (reduction of the maximum or peak flow rate) of the runoff (floodwater) hydrograph. The runoff in each subarea is based upon the available rainfall data, modified according to an assumed "loss rate" due to soil-infiltration, ponding, evaporation, and other effects. The net effect of all these approximations is a vast spectrum of possible modeling structures. Using a stochastic integral equation, we can mathematically approximate many of these rainfall-runoff modeling structures with a generalized model that is more tractable to detailed analysis of the model structure. We can then proceed to evaluate rainfall-runoff modeling uncertainty.

INTRODUCTION

Due to the nondeterministic nature of the rainfall-runoff processes occurring over the catchment, the mathematical descriptions of these processes result in stochastic equations. Additionally, the so-called deterministic rainfall-runoff models used to describe the several physical processes contain parameters or coefficients which have well-defined physically-based meanings, but whose exact values are unknown. The boundary conditions of the problem itself are unknown (e.g., the temporal and spatial distribution of rainfall over the catchment for the storm event under study and also for all prior storm events) and often exhibit considerable variations with respect to the assumed boundary conditions, the measured rainfall at a single location (e.g., Nash and Sutcliffe, 1970; Huff, 1970). Thus the physically-based parameters and coefficients, and also the problem boundary conditions, are not the assumed values used in rainfall-runoff modeling applications, but are instead random variables and stochastic processes whose variations about the assumed values are governed by certain probability distributions.

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In this paper, the uncertainty problem is addressed by providing a methodology which can be incorporated into almost all rainfall-runoff models. The methodology is based upon the standard theory of stochastic integral equations which has been successfully applied to several problems in the life sciences and chemical engineering (e.g., Tsokos and Padgett, 1974, provide a thorough development). The stochastic integral formulation is used to represent the total error between a record of measured rainfall-runoff data and the model estimates, and provides an answer to the questions: "based upon the historic rainfall-runoff data record and the model's accuracy in estimating the measured runoff, what is the distribution of probable values of the subject criterion variable given a hypothetical rainfall event?"

STOCHASTIC INTEGRAL EQUATION

Rainfall-Runoff Model Errors

Let M be a deterministic rainfall-runoff model which transforms gauged rainfall data for some storm event, i, noted by $P_g{}^i(t)$, into an estimate of runoff, $M^i(t)$, by

$$M: P_g{}^i(t) \rightarrow M^i(t) \tag{1}$$

where t is time. In our problem, rainfall data are obtained from a single rain gauge. The operator M may include loss rate and flow routing parameters, memory of prior storm event effects, and other factors.

Let $P_g{}^i(t)$ be the rainfall measured from storm event i, and $Q_g{}^i(t)$ be the runoff measured at the stream gauge. Various error (or uncertainty) terms are now defined such that for arbitrary storm event i,

$$Q_g{}^i(t) = M^i(t) + E_m{}^i(t) + E_d{}^i(t) + E_r{}^i(t) \tag{2}$$

where

$E_m{}^i(t)$ is the modeling error due to inaccurate approximations of the physical processes (spatially and temporally);

$E_d{}^i(t)$ is the error in data measurements of $P_g{}^i(t)$ and $Q_g{}^i(t)$ (which is assumed hereafter to be of negligible significance in the analysis);

$E_r{}^i(t)$ is the remaining "inexplainable" error, such as due to the unknown variation of effective rainfall (i.e., rainfall less losses; rainfall excess) over the catchment, among other factors.

Let $E^i(t)$ be redefined to equal the total error

$$E^i(t) = E_m{}^i(t) + E_d{}^i(t) + E_r{}^i(t) \tag{3}$$

where $E^i(t)$ is necessarily highly correlated to $E_r{}^i(t)$ due to the given assumptions. Because $E^i(t)$ depends on the model M used in Eq. (1), then Eqs. (2) and (3) are combined as

$$Q_g{}^i(t) = M^i(t) + E_M{}^i(t) \tag{4}$$

where $E_M{}^i(t)$ is a conditional notation for $E^i(t)$, given model type M.

The several terms in Eq. (4) are each a realization of a stochastic process. And for a future storm event D, the $E_M{}^D(t)$ is not known precisely, but rather is an unknown realization of a stochastic process distributed as $[E_M{}^D(t)]$ where

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)] \tag{5}$$

In Eq. (5), $[Q_M{}^D(t)]$ and $[E_M{}^D(t)]$ are the stochastic processes associated to the catchment runoff and total modeling errors, respectively, associated with model M, for hypothetical storm event D. Hence in prediction, the model output of Eq. (5) is not a single outcome, but instead is a stochastic distribution of outcomes, distributed as $[Q_M{}^D(t)]$. Should $\mathcal{A}$ be some functional operator on the possible outcome (e.g., detention basin volume; peak flow rate; median flow velocity, etc.) of storm event D, then the possible value of $\mathcal{A}$ for storm event D, noted as $A_M{}^D$, is a random variable distributed as $[A_M{}^D]$, where

$$[A_M{}^D] = \mathcal{A}[Q_M{}^D(t)] \tag{6}$$

Developing Distributions for Model Estimates

The distribution for $[E_M{}^D(t)]$ may be estimated by using the available sampling of realizations of the various stochastic processes:

$$\{E_M{}^i(t)\} = \{Q_g{}^i(t) - M^i(t)\}, \; i = 1,2,\cdots \tag{7}$$

Assuming elements in $\{E_M{}^i(t)\}$ to be dependent upon the "severity" of $Q_g{}^i(t)$, one may partition $\{E_M{}^i(t)\}$ into classes of storms such as mild, major, flood, or others, should ample rainfall-runoff data be available to develop significant distributions for the resulting subclasses. To simplify development purposes, $[E_M{}^D(t)]$ will be based on the entire set $\{E_M{}^i(t)\}$ with the underlying assumption that all storms are of "equivalent" error.

The second assumption involved is to assume each $E_M{}^i(t)$ is strongly correlated to some function of precipitation, $F^i(t) = F(P_g{}^i(t))$, where F is an operator which includes parameters, memory of prior rainfall, and other factors. Assuming that $E_M{}^i(t_0)$ depends only on the values of $F^i(t)$ for time $t \leq t_0$, then $E_M{}^i(t)$ is expressed as a causal linear filter (for only mild conditions imposed on $F^i(t)$), given by the stochastic integral equation (see Tsokos and Padgett, 1974)

$$E_M^i(t_o) = \int_{s=0}^{t_o} F^i(t_o\text{-}s)\, h_M^i(s)\, ds \qquad (8)$$

where $h_M{}^i(t)$ *is the transfer function between* $(E_M{}^i(t), F^i(t))$. Other convenient candidates to be used in Eq. (8), instead of $F^i(t)$, are the storm rainfall, $P_g{}^i(t)$, and the model estimates itself, $M^i(t)$.

Given a significant set of storm data, an underlying distribution $[h_M(t)]$ of the $\{h_M{}^i(t)\}$ may be identified, or the $\{h_M{}^i(t)\}$ may be used directly as in the case of having a discrete distribution of equally-likely realizations. Using $[h_M(t)]$ as notation for both cases of distributions stated above, the predicted response from M for future storm event D is estimated to be

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)] \qquad (9)$$

Combining Eqs. (8) and (9),

$$[Q_M^D(t)] = M^D(t) + \int_{s=0}^{t_o} F^D(t\text{-}s)\, [h_M(s)]\, ds \qquad (10)$$

and for the functional operation $\mathcal{A}$, Eq. (6) is rewritten as

$$[A_M{}^D] = \mathcal{A}[Q_M{}^D(t)] = \mathcal{A}\left(M^D(t) + \int_{s=0}^{t} F^D(t\text{-}s)\, [h_M(s)]\, ds\right) \qquad (11)$$

Confidence interval estimates for the chosen criterion variable can now be obtained from the frequency-distribution, $[A_M{}^D]$. It is noted that $[A_M{}^D]$ is necessarily a random variable distribution that depends on the model structure, M.

DEVELOPMENT OF TOTAL ERROR DISTRIBUTIONS

A Translation Unsteady Flow Routing Rainfall-Runoff Model

The previous concepts are now utilized to directly develop the total error distributions, $[E_M(t)]$, for a set of three idealized catchment responses. Besides providing a set of applications,

additional notation and concepts are introduced, leading to the introduction of storm classes.

Let F be a functional which operates on rainfall data, $P_g{}^i(t)$, to produce the realization, $F^i(t)$, for storm i by

$$F: P_g{}^i(t) \rightarrow F^i(t) \qquad (12)$$

The catchment R is subdivided into m homogeneous subareas, $R = \cup R_j$, such that in each R_j, the effective rainfall, $e_j{}^i(t)$, is assumed given by

$$e_j{}^i(t) = \lambda_j(1+ X_j{}^i)\, F^i(t) \qquad (13)$$

where λ_j is a constant proportion factor; and where $X_j{}^i$ is a sample of a random variable, which is constant for storm event i. The parameter λ_j is defined for subarea R_j and represents the relative runoff response of R_j in comparison to $F^i(t)$, and is a constant for all storms, whereas $X_j{}^i$ is a samples of the random variable distributed as $[X_j]$, where the set of distributions, $\{[X_j];\ j = 1,2,\cdots,m\}$ may be mutually dependent.

The subarea runoff is

$$q_j{}^i(t) = \int_{s=0}^{t} e_j{}^i(t\text{-}s)\, \phi_j^i(s)\, ds = \int_{s=0}^{t} \lambda_j(i+X_j{}^i)\, F^i(t\text{-}s)\, \phi_j^i(s)\, ds \qquad (14)$$

At this stage of development, unsteady flow routing along channel links is assumed to be pure translation. Thus, each channel link, L_k, has the constant translation time, T_k. Hence for m links,

$$Q_g^i(t) = \sum_{j=1}^{9} q_j{}^i(t\text{-}\tau_j) \qquad (15)$$

where $q_j{}^i(t - \tau_j)$ is defined to be zero for negative arguments and τ_j is the sum of link travel times.

For the above particular assumptions,

$$Q_g^i(t) = \sum_{j=1}^{9} \int_{s=0}^{t} \lambda_j(1+X_j{}^i)\, F^i(t\text{-}s)\, \phi_j^i(s\text{-}\tau_j)\, ds$$
$$= \int_{s=0}^{t} F^i(t\text{-}s) \left(\sum_{j=1}^{9} \lambda_j\, (1+X_j{}^i)\ \phi_j^i(s\text{-}\tau_j)\right) ds \qquad (16)$$

In a final form, the runoff response for the given simplification is

$$Q_g^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{9} \lambda_j \; \phi_j^i(s-\tau_j)\, ds$$

$$+ \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{9} \lambda_j X_j^i \, \phi_j^i(s-\tau_j)\, ds \tag{17}$$

In the above equations, the samples $\{X_j^i\}$ are unknown to the modeler for any storm event i. From Eq. (17), the model structure M, used in design practice is

$$M^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{9} \lambda_j \, \phi_j^i(s-\tau_j)\, ds \tag{18}$$

Then, $Q_g^i(t) = M^i(t) + E_M^i(t)$ where

$$E_M^i(t) = \int_{s=0}^{t} F^i(t-s)\, h_M^i(s)\, ds \tag{19}$$

where $h_M^i(s)$ follows directly from Eqs. (17) and (18).

Should the subarea UH all be assumed fixed, (i.e., $\phi_j^i(t) = \phi_j(t)$, for all i), as is assumed in practice, then the above equations can be further simplified as

$$M^i(t) = \int_{s=0}^{t} F^i(t-s)\, \Phi(s)\, ds \tag{20}$$

where $\Phi(s) = \sum_{j=1}^{9} \lambda_j \, \phi_j(s - \tau_j)$. Additionally, the distribution of the stochastic process $[h_M(t)]$ is readily determined for this simple example,

$$[h_M(t)] = \sum_{j=1}^{9} [X_j] \, \lambda_j \, \phi_j(t-\tau_j) \tag{21}$$

where $[h_M(t)]$ is directly equated to the 9 random variables, $\{X_j, j=,1,2,\cdots,9\}$. It is again noted that the random variables, X_j, may be all mutually dependent.

In prediction, the estimated runoff hydrograph is the distribution $[Q_M{}^D(t)]$ where $[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)]$, and M refers to the model structure of Eqs. (18) or (20).

For this example problem, the stochastic integral formlulation is

$$[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s)\, \Phi(s)\, ds + \int_{s=0}^{t} F^D(t-s)\, [h_M(s)]\, ds \tag{22}$$

where the error distribution, $[E_M{}^D(t)]$, is assumed to be correlated to the model input, $F^D(t)$, as provided in Eqs. (19) and (21).

Multilinear unsteady flow routing and storm classes

The above equation is now extended to include the additional assumption that the channel link travel times are strongly correlated to some set of characteristic descriptions of the runoff hydrograph being routed, such as some weighted mean flow rate of the associated hydrograph. For example, the widely used Convex Routing technique (Mockus, 1972) often utilized the 85-percentile of all flows in excess of one-half of the peak flow rate as a statistic used to estimate the routing parameters. But by the previous development (i.e., definition of $e_j^i(t)$), all runoff hydrographs in the link-node channel system would be highly correlated to an equivalent weighting of the model input, $F^i(t)$. Hence, storm classes, $[\xi_z]$, of "equivalent" $F^i(t)$ realizations could be defined where all elements of $[\xi_z]$ have the same characteristic parameter set, $\mathcal{C}(F^i(t))$, by

$$[\xi_z] = \{F^i(t) \mid \mathcal{C}(F^i(t)) = z\} \tag{23}$$

And for all $F^i(t) \in [\xi_z]$, each respective channel link travel time is identical, that is $T^k = T_{k_z}$ for all for all $F^i(t) \in [\xi_z]$. In the above definition of storm class, z is a characteristics parameter set in vector form.

This extension of the translation unsteady flow routing algorithm to a multilinear formulation (involving a set of link translation times) modifies the previous runoff equations (20) and (21) to be,

$$Q_g^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{9} \lambda_j \; \phi_j^i(s-\tau_j^z)\, ds = \int_{s=0}^{t} F^i(t-s)\, \Phi_z(s)\, ds; \; F^i(t) \in [\xi_z] \tag{24}$$

where $\Phi_z(s) = \sum_j \lambda_j \phi_j(s-\tau_j^z)$, and

$$E_M^i(t) = \int_{s=0}^{t} F^i(t-s)\, h_{M,z}{}^i(s)\, ds; \; F^i(t) \in [\xi_z] \tag{25}$$

The structure of the new set of equations motivates an obvious extension of the definition of the subarea UH, the subarea λ_j proportion factor, and the subarea random variable distribution $[X_j]$, to all be also defined on the storm class basis of $[\xi_z]$. Thus, Eq. (24) is extended as

$$M^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_j \lambda_j^z \phi_j^z(s-\tau_j^z)\, ds$$

$$= \int_{s=0}^{t} F^i(t-s)\, \Phi_z(s)\, ds;\ F^i(t) \in [\xi_z] \quad (26)$$

The stochastic process $[h_{M_z}(t)]$ is distributed as

$$[h_{M_z}(t)] = \sum_j [X_j^z]\, \lambda_j^z\ \phi_j^z\,(s - \tau_j^z);\ Fi(t) \in [\xi_z] \quad (27)$$

And in prediction,

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)];\ F^D(t) \in [\xi_D] \quad (28)$$

where

$$E_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s)\, [h_M(s)]\, ds;\ F^D(t) \in [\xi_D] \quad (29)$$

A. Multilinear Rainfall-Runoff Model

Each subarea's effective rainfall, $e_j^i(t)$, is now defined to be the sum of proportions of $F^i(t)$ translates by

$$e_j^i(t) = \sum_k \lambda_{jk}\,(1 + X_{jk}{}^i)\, F^i(t - \theta_{jk}{}^i);\ F^i(t) \in [\xi_z] \quad (30)$$

where $X_{jk}{}^i$ and $\theta_{jk}{}^i$ are samples of the random variables distributed as $[X_{jk}]$ and $[\theta_{jk}]$, respectively. In the above equation and all equations that follow, it is assumed that a storm class system is defined, $[\xi_z]$, such that for $F^i(t) \in [\xi_z]$, all parameters and probabilistic distributions are uniquely defined, and there is no loss in understanding by omitting the additional notation needed to indicate the storm class.

The subarea runoff is

$$q_j^i(t) = \int_{s=0}^{t} \sum_k \lambda_{jk}\,(1 + X_{jk}{}^i)\, F^i(t - \theta_{jk}{}^i - s)\, \phi_j(s)\, ds \quad (31)$$

or in a simpler form,

$$q_j^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_k \lambda_{jk}\,(1 + X_{jk}{}^i)\, \phi_j(s-\theta_{jk}{}^i)\, ds \quad (32)$$

It is assumed that the unsteady flow channel routing effects are highly correlated to the magnitude of runoff in each channel link, which is additionally correlated to the magnitude of the model input realization, $F^i(t)$. On a storm class basis, each channel link is assumed to respond linearly in that (e.g., Doyle et al, 1983)

$$O_1{}^i(t) = \sum_{\ell} a_\ell I_1{}^i(t - \alpha_\ell) \quad (33)$$

where $O_1{}^i(t)$ and $I_1{}^i(t)$ are the outflow and inflow hydrographs for link 1, and storm event i; and $\{a_\ell\}$ and $\{\alpha_\ell\}$ are constants which are defined on a storm class basis which is also used for the model input, $F^i(t)$. Thus, the channel link flow routing algorithm is multilinear with routing parameters defined according to the storm class, $[\xi_z]$ (see Becker and Kundezewicz, 1987, for an analogy based on multilinear approximation of nonlinear routing).

Should the above outflow hydrograph, $O_1(t)$, now be routed through another link (number 2), then $I_2(t) = O_1(t)$ and from the above

$$O_2(t) = \sum_{\ell_2=1}^{n_2} a_{\ell_2} I_2(t - \alpha_{\ell_2})$$

$$= \sum_{\ell_2=1}^{n_2} a_{\ell_2} \sum_{\ell_1=1}^{n_1} a_{\ell_1} I_1(t - \alpha_{\ell_1} - \alpha_{\ell_2}) \quad (34)$$

for L links, each with their respective stream gauge routing data, the above linear routing techniques result in the outflow hydrograph for link number L, $O_L(t)$, being given by

$$O_L(t) = \sum_{\mathcal{A}_L=1}^{n_L} a_{\mathcal{A}_L} \sum_{\mathcal{A}_{L-1}=1}^{n_{L-1}} a_{\mathcal{A}_{L-1}} \ldots$$

$$\ldots \sum_{\mathcal{A}_2=1}^{n_2} a_{\mathcal{A}_2} \sum_{\mathcal{A}_1=1}^{n_1} a_{\mathcal{A}_1} I_1(t - \alpha_{\mathcal{A}_1} - \alpha_{\mathcal{A}_2} - \ldots - \alpha_{\mathcal{A}_{L-1}} - \alpha_{\mathcal{A}_L}) \tag{35}$$

Using an index notation, the above $O_L(t)$ is written as

$$O_L(t) = \sum_{<\mathcal{A}>} a_{<\mathcal{A}>} I_1(t - \alpha_{<\mathcal{A}>}) \tag{36}$$

For subarea R_j, the runoff hydrograph for storm i, $q_j^i(t)$, flows through L_j links before arriving at the stream gauge and contributing to the total modeled runoff hydrograph, $M^i(t)$. All of the parameters $a^i_{<\mathcal{A}>}$ and $\alpha^i_{<\mathcal{A}>}$ are constants on a storm class basis. Consequently from the linearity of the routing technique, the m-subarea link node model is given by the sum of the m, $q_j^i(t)$ contributions,

$$M^i(t) \sum_{j=1}^{m} \sum_{<\mathcal{A}>_j} a^i_{<\mathcal{A}>_j} q_j^i(t - \alpha^i_{<\mathcal{A}>_j}) \tag{37}$$

Finally, the predicted runoff response for storm event D is the stochastic integral formulation

$$[Q_M^D(t)] = \int_{s=0}^{t} F^D(t-s)\Big(\sum_{j=1}^{m} \sum_{<\mathcal{A}>_j} a^i_{<\mathcal{A}>_j} \sum_{k} \lambda_{jk}(1+[X_{jk}])\, \phi_j(s-[\theta_{jk}] - \alpha^i_{<\mathcal{A}>_j})\Big)\, ds;\ F^D(t) \in [\xi_D] \tag{38}$$

Given $F^i(t) \in [\xi_z]$, all subarea runoff parameters $\{\lambda_{jk}, \phi_j(t)\}$ and distributions $\{[X_{jk}], [\theta_{jk}]\}$ are uniquely defined $j = 1,2,\cdots,m$; and all link routing parameters $\{a_\ell, \alpha_\ell\}$ are uniquely defined. Then the entire link-node model is linear on a storm class basis and once more Eqs. (26)-(29) apply without modification.

Our final model structure can be used to study the effect on the runoff prediction (at the stream gauge) from arbitrary model M, due to the randomness exhibited by the mutually dependent set of random variables, $\{X_{jk}, \theta_{jk}\}$. Hence for any operator, $\mathcal{A}$, on the predicted runoff response of Eq. (38), the outcome of $\mathcal{A}$ for design storm event $P_g^D(t)$ is the distribution $[A_M^D]$, where for all model parameters defined,

$$[A_M^D] = \mathcal{A}[M_D(t)] = \mathcal{A}(\{[X_{jk}], [\theta_{jk}]\}) \tag{39}$$

STOCHASTIC INTEGRAL EQUATIONS AND UNCERTAINTY ESTIMATES

A stochastic integral equation that is equivalent to Eq. (38) is

$$[Q_M^D(t)] = \int_{s=0}^{t} F^D(t-s)\,[\eta(s)]\, ds;\ F^D(t) \in [\xi_D] \tag{40}$$

where now $[\eta(s)]$ is the distribution of the stochastic process representing the random variations from the set of mutually dependent random variables, $\{X_{jk}, \theta_{jk}\}$, defined on a storm class basis. (It is recalled that on a storm class basis, the hydraulic parameters of a $a^i_{<\mathcal{A}>j}$ and $\alpha^i_{<\mathcal{A}>j}$, and the $\phi_j(s)$, do not vary.) In prediction, the expected runoff estimate for storm events that are elements of $[\xi_D]$ is

$$E[Q_M^D(t)] = \int_{s=0}^{t} F^D(t-s)\, E[\eta(s)]\, ds;\ F^D(t) \in [\xi_D] \tag{41}$$

which is a multilinear version of the well-known unit hydrograph method, which is perhaps the most widely used rainfall-runoff modeling approach in use today.

Then the model M structure of Eq. (38), when unbiased, is given from Eq. (39), by

$$M^D(t) = E[Q_M^D(t)] \tag{42}$$

The total error distribution (for the subject model M) can be developed by

$$[E_M^D(t)] = [Q_M^D(t)] - E[Q_M^D(t)] \tag{43}$$

where all equations are defined on the storm class basis used in the previous equations. Given sufficient rainfall-runoff data, the total error distributions can be approximately developed by use of Eq. (43). Should another rainfall-runoff model structure be used, then $E[Q_M^D(t)]$ is replaced by the alternative model, and another set of realizations of $[E_M^D(t)]$ is obtained from (43). Equation (43) is important in that given a specified model, the total error in model estimation is approximately given by a stochastic process. And similar to any sampling process, the modeling total error distribution becomes better defined as the sampling population increases. Through the equivalence between Eqs. (38) and (40), the uncertainty of the rainfall-runoff model of Eq. (38) can be evaluated by use of Eq. (40). That is, due to the limited data available, one cannot evaluate the total model error, as developable from Eq. (43).

The various stochastic distributions utilized are estimated from regional rainfall-runoff data and the chosen model structure. Because runoff data are available for the precise catchment point under study (i.e., we have a stream gauge), the various distributions involved can be rescaled to correspond to the selected study point. However, in order to utilize these distributions at ungauged points in the catchment, or at other catchments where there are no runoff data, a method of transferring these distributions is needed. That is, a method is needed for estimating the expected values for discharge (or other description variables used) for the point under study. Given these estimates, the various distributions can be rescaled, and a distribution [$\eta(s)$] can be estimated from the rainfall-runoff data pool.

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FLOOD FREQUENCY ANALYSIS WITH DISTRIBUTIONS OF FRACTIONAL ORDER STATISTICS

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ABSTRACT

Distributions of fractional order statistics (FOS) have been recently introduced as plausible models for flood frequency analysis. These distributions should be of interest to hydrologists for a number of reasons, but can be difficult to fit to observed data sequences. This paper presents an overview of the class of FOS distributions and describes some of the difficulties encountered when dealing with the problem of parameter estimation.

INTRODUCTION

Within the class of parametric statistical methods for estimation of extreme flood magnitudes, it is possible to differentiate between two schools of thought. The first school holds to the class of exponential distributions, of which the Lognormal and Log Pearson Type 3 are members. This school is exemplified by the recommendations of the U.S. Water Resources Council (WRC, 1981), now the Interagency Advisory Committee on Water Data (IACWD). The second school, which is exemplified by the recommendations of the U.K. Natural Environment Research Council (NERC, 1975), maintains that the class of extremal distributions should be applied. Because the flood analysis problem deals with extremes that have occurred within each calendar or water year, followers of this second school seem to be gaining a significant degree of momentum. The fact that extreme value distributions are often fairly easy to work with has, without doubt, also contributed to this effect.

Of course, there are many other distributions that have been suggested as well for flood frequency analysis. Among these are the Wakeby (Houghton, 1978) and Two-Component Extreme Value (TCEV) (Rossi et al., 1984) distributions, though the latter may be considered to fall in the extreme value class. However, despite any arguments that one may advance to theoretically justify the use of a particular distribution, in practice one is forced to make a choice between them. It is generally believed that no distribution is universally applicable, and hence that selection of

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