Complex Variable Boundary Element Method Error Analysis Using Taylor Series

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Abstract

The CVBEM is a numerical approach to solving boundary value problems of two-dimensional Laplace and Poisson equations. The CVBEM estimator exactly solves the governing partial differential equations in the problem domain, but only approximately satisfies the problem boundary conditions. In this paper, a new CVBEM error measure is used in aiding in the development of improved CVBEM approximators. The new approach utilizes Taylor series theory, and can be readily programmed into computer software form. Based on numerous test applications, it appears that use of this new CVBEM error measure leads to the development of significantly improved CVBEM approximation functions.

1. INTRODUCTION

1.1. Definition of Working Space, W_Ω

Let Ω be a simply connected convex domain with a simple closed piecewise linear boundary Γ with centroid located at $0+0i$. Then in this paper, $\omega \in W_\Omega$ has the property that $\omega(z)$ is analytic over $\Omega \cup \Gamma$.

1.2. Definition of the Function $||\omega||$

For $\omega \in W_\Omega$, the symbol $||\omega||$ is notation for

$$||\omega|| = \left[\int_{\Gamma_\phi} (\mathrm{Re}\,\omega)^2 \, d\mu + \int_{\Gamma_\psi} (\mathrm{Im}\,\omega)^2 \, d\mu \right]^{1/2}$$

where Γ_ϕ and Γ_ψ are both a finite number of subsets of Γ that intersect only at a finite number of points in Γ.

The symbol $||\omega||_p$ for $\omega \in W_\Omega$ is notation for

$$||\omega||_p = \left[\int_\Gamma |\omega(\zeta)|^p \, d\mu \right]^{1/p}, \quad p \geq 1$$

Of importance is the case of p = 2:

$$\|\omega\|_2 = \left[\int_\Gamma \left\{ [\mathrm{Re}\,\omega]^2 + [\mathrm{Im}\,\omega]^2 \right\} d\mu \right]^{1/2}$$

1.3. Almost Everywhere (ae) Equality

A property which applies everywhere on a set E except for a subset E' in E such that the Lebesque measure $m(E') = 0$ is said to apply almost-everywhere (ae). Because sets of measure zero have no effect on integration, almost-everywhere (ae) equality on Γ indicates the same class of element. Thus for $\omega \in W_\Omega$, $[\omega] = \{\omega \in W_\Omega : \omega(\zeta)$ are equal ae for $\zeta \in \Gamma\}$. For example, $[0] = \{\omega \in W_\Omega : \omega(\zeta) = 0$ ae, $\zeta \in \Gamma\}$. When understood, the notation '[]' will be dropped.

2. MATHEMATICAL DEVELOPMENT

2.1. Introduction

The H^p spaces (or Hardy spaces) are well documented in the literature (e.g. Duren, 1970). Of special interest are the $E^p(\Omega)$ spaces of complex valued functions. If $\omega \in E^2(\Omega)$, then ω satisfies the conditions of Definition 1.3 on W_Ω, where $\|\omega(\delta\zeta)\|_2$ is bounded as $\delta \to 1$. Finally, if $\omega \in E^2(\Omega)$ then the Cauchy integral representation of $\omega(z)$ for $z \in \Omega$ applies. It is seen that $W_\Omega \subset E^2(\Omega)$.

2.2. Theorem (boundary integral representation)

Let $\omega \in W_\Omega$ and $z \in \Omega$. Then

$$\omega(z) = \frac{1}{2\pi i} \int_\Gamma \frac{\omega(\zeta)\, d\zeta}{\zeta - z} \tag{1}$$

Proof

For $\omega \in W_\Omega$, then $\omega \in E^2(\Omega)$ and the result follows immediately.

2.3. Almost everywhere (ae) equivalence

For $\omega \in W_\Omega$, functions $x \in W_\Omega$ which are equal to ω ae on Γ represent an equivalence class of functions which may be noted as $[\omega]$. Therefore, functions x and y in W_Ω are in the same equivalence class when

$$\int_\Gamma |x - y| \, d\mu = 0$$

For simplicity, $\omega \in W_\Omega$ is understood to indicate $[\omega]$. This follows directly from the fact that integrals over sets of measure zero have no effect on the integral value.

2.4. Theorem (uniqueness of zero element in W_Ω)

Let $\omega \in W_\Omega$ and $\phi = 0$ ae on Γ_ϕ and $\psi = 0$ ae on Γ_ψ. Then $(\omega,\omega) = 0 \Rightarrow \omega = [0]$.

Let $\omega \in W_\Omega$.

Green's theorem states, let F and G be continuous and have continuous first and second partial derivatives in a simply-connected region R bounded by a simple closed curve C. Then

$$\oint_C F\left(\frac{\partial G}{\partial y}\,dx - \frac{\partial G}{\partial x}\,dy\right) = -\iint_R \left[F\left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}\right) + \left(\frac{\partial F}{\partial x}\frac{\partial G}{\partial x} + \frac{\partial F}{\partial y}\frac{\partial G}{\partial y}\right)\right] dx\,dy$$

Let $F = \phi$, $G = \phi$. Then

$$\int_\Gamma \phi \frac{\partial \phi}{\partial n}\,d\Gamma = \int_\Omega \phi \nabla^2 \phi \,d\Omega + \int_\Omega \left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right] d\Omega$$

But $\nabla^2\phi = 0$ in Ω. Thus

$$\int_\Gamma \phi \frac{\partial \phi}{\partial n}\,d\Gamma = \int_\Omega \left[\phi_x^2 + \phi_y^2\right] d\Omega$$

But $(\omega,\omega) = 0$ imples $\phi = 0$ on Γ_ϕ and $\psi = 0$ on Γ_ψ (hence $\frac{\partial \psi}{\partial s} = 0 \Rightarrow \frac{\partial \phi}{\partial n} = 0$), and

$$\underbrace{\int_{\Gamma_\phi} \phi\,\phi_n\,d\Gamma}_{0} + \underbrace{\int_{\Gamma_\psi} \phi\,\phi_n\,d\Gamma}_{0} = \int_\Omega \left(\phi_x^2 + \phi_y^2\right) d\Omega$$

Thus $(\omega,\omega) = 0 \Rightarrow \phi_x = 0 = \phi_y$ on Ω.

Thus $\phi(x,y)$ is a constant in Ω. But

$\lim_{z\to\zeta\in\Gamma_\phi} \phi = 0 \Rightarrow \phi = 0$. Similarly, $\psi = 0$. Thus, $\omega = [0]$.

2.5. Theorem (W_Ω is vector space)

W_Ω is a linear vector space over the field of real numbers.

Proof

This follows directly from the character of analytic functions. The sum of analytic functions is analytic, and scalar multiplication of analytic functions is analytic. The zero element has already been noted by [0] in Theorem 2.4.

2.6. Theorem (definition of the inner-product)

Let x, y, z, $\in W_\Omega$. Define a real-valued function (x,y) by

$$(x,y) = \int_{\Gamma_\Omega} \mathrm{Re}\,x\ \mathrm{Re}\,y\,d\mu + \int_{\Gamma_\psi} \mathrm{Im}\,x\ \mathrm{Im}\,y\,d\mu$$

Then $(,)$ is an inner-product over W_Ω.

Proof

It is obvious that $(x,y) = (y,x)$; $(kx,y) = k(x,y)$ for k real; $(x+y,z) = (x,z)+(y,z)$; and $(x,x) = \|x\| \geq 0$. By theorem 2.4, $(x,x) = 0$ implies $\operatorname{Re} x = 0$ ae on Γ_ϕ and $\operatorname{Im} x = 0$ ae on Γ_ψ and $x = [0] \in W_\Omega$.

Three theorems follow immediately from the above, and hence no proof is given.

2.7. Theorem (W_Ω is an inner-product space)

For the defined inner-product, W_Ω is an inner-product space over the field of real numbers.

3. THE CVBEM AND W_Ω

3.1. Definition

Let the number of angle points of Γ be noted as Λ. By a nodal partition of Γ, nodes $\{P_j\}$ with coordinates $\{z_j\}$ are defined on Γ such that a node is located at each vertex of Γ and the remaining nodes are distributed on Γ. Nodes are numbered sequentially in a counterclockwise direction along Γ. The scale of the partition is indicated by l where $l = \max|z_{j+1} - z_j|$. Note that no two nodal points have the same coordinates in Γ.

3.2. Definition

A boundary element Γ_j is the line segment joining nodes z_j and z_{j+1}; $\Gamma_j = \{z\colon z = z(t) = z_j(1-t) + z_{j+1}t,\ 0 \leq t \leq 1\}$. (Note for m nodes on Γ, that $z_{m+1} = z_1$.)

3.3. Discretization of Γ into CVBE

Let a nodal partition be defined on Γ. Then

$$\Gamma = \bigcup_{j=1}^{m} \Gamma_j$$

where m is the number of complex variable boundary elements (CVBE).

3.4. Definition

A linear basis function $N_j(\zeta)$ is defined for $\zeta \in \Gamma$ by

$$N_j(\zeta) = \begin{cases} (\zeta - z_{j-1})/(z_j - z_{j-1}), & \zeta \in \Gamma_{j-1} \\ (z_{j+1} - \zeta)/(z_{j+1} - z_j), & \zeta \in \Gamma_j \\ 0, & \zeta \notin \Gamma_{j-1} \cup \Gamma_j \end{cases}$$

The values of $N_j(\zeta)$ is found to be real and bounded as indicated by the next theorem.

3.5. Theorem

Let $N_j(\zeta)$ be defined for node $P_j \in \Gamma$. Then $0 \le N_j(\zeta) \le 1$.

3.6. Definition

Let a nodal partition of m nodes $\{P_j\}$ be defined on Γ with $m \ge \Lambda$ and with scale l. At each node P_j, define nodal values $\bar{\omega}_j = \bar{\phi}_j + i\bar{\psi}_j$ where $\bar{\phi}_j$ and $\bar{\psi}_j$ are real numbers. A global trial function $G_m(\zeta)$ is defined on Γ for $\zeta \in \Gamma$ by

$$G_m(\zeta) = \sum_{j=1}^{m} N_j(\zeta)\bar{\omega}_j$$

3.7. Theorem

From 3.6, $G_m(\zeta)$ is the sum of integrable continuous functions, and hence

(i) $G_m(\zeta)$ is continuous on Γ.

(ii) For $\omega(\zeta) \in W_\Omega$, $\omega(\zeta) \in L^2(\Gamma)$.

4. TAYLOR SERIES EXPANSIONS ON CVBE

4.1. Construction

Let $\omega \in W_\Omega$. Then ω is analytic on an open domain Ω^A such that $\Omega \cup \Gamma$ is entirely contained in the interior of Ω^A. Let Γ^* be in Ω^A such that Γ^* is a finite length simple closed contour that is exterior of $\Omega \cup \Gamma$. Then ω is analytic on Γ^* and, by the maximum modulus theorem,

$$|\omega(z)| \le M, \text{ for } z \in \Gamma^*, \text{ for some positive constant } M. \quad (2)$$

Also,

$$|\omega(z)| \le M, \text{ for } z \in \Omega \cup \Gamma \quad (3)$$

Define a nodal partition of m nodes on Γ. Complex Variable Boundary Elements (CVBE) are defined to be the straight line segments $\Gamma_j = [z_j, z_{j+1}]$ where, for m nodes, $z_{m+1} \equiv z_1$. At the midpoint $\bar{z}_j = \frac{1}{2}(z_j + z_{j+1})$ of each Γ_j, expand $\omega(z)$ into a Taylor series $T_j(z - z_j)$. Each $T_j(z - \bar{z}_j)$ has a nonzero radius of convergence R_j, and $T_j(z - \bar{z}_j) = \omega(z)$ in the interior of circle $C_j = \{z: |z - \bar{z}_j| = R_j\}$. The C_j all minimally have radii R where $R = \min |\zeta_1 - \zeta_2|$ such that $\zeta_1 \in \Gamma$ and $\zeta_2 \in \Gamma^*$. Discretize Γ into m CVBE, Γ_j, $j = 1,2,\cdots,m$ such that the length of Γ_j, $||\Gamma_j|| \le \frac{2L}{m}$ where $L = \int_\Gamma |d\zeta|$ and $\frac{2L}{m} < R$, and the other conditions regarding placement of nodes at angle points of Γ are satisfied.

4.2. Taylor Series Expansion

For $\zeta \in \Gamma_j$, $T_j(\zeta - \bar{z}_j) = P_j^N(\zeta) + E_j^N(\zeta)$ (4)

where N = polynomial degree, and from Cauchy's theorem,

$$E_j^N(\zeta) = \frac{1}{2\pi i}\int_{C_j}\left(\frac{\zeta - \bar{z}_j}{z - \bar{z}_j}\right)^{N+1}\frac{\omega(z)dz}{z-\zeta} \tag{5}$$

The magnitude of $|E_j^N(\zeta)|$ is, for every j,

$$\left|E_j^N(\zeta)\right| \le \frac{1}{2\pi}\left|\frac{\zeta - \bar{z}_j}{z - \bar{z}_j}\right|^{N+1}\frac{\max|\omega(z)|2\pi R}{\min|z-\zeta|}, \quad z\in C_j,\ \zeta\in\Gamma \tag{6}$$

But $$\left|\frac{\zeta - z_j}{z - \bar{z}_j}\right|^{N+1} \le \left(\frac{L/m}{R}\right)^{N+1}$$

thus $$\left|E_j^N(\zeta)\right| \le \frac{1}{2\pi}\left(\frac{L}{mR}\right)^{N+1}\frac{M2\pi R}{R/2} = 2M\left(\frac{L}{mR}\right)^{N+1}$$

which is a result independent of j. Note that as the partition of Γ into CVBE becomes finer, i.e. $\max||\Gamma_j|| \to 0$, then $m\to\infty$ and $|E_j^N(\zeta)| \to 0$. Also, as the order of the Taylor series polynomial increases, $N\to\infty$, and recalling that $(L/m) < R/2$, then $|E_j^N(\zeta)| \to 0$.

4.3. CVBEM Error Analysis

From Cauchy's theorem, for $z\in\Omega$:

$$\omega(z) = \frac{1}{2\pi i}\int_\Gamma \frac{\omega(\zeta)d\zeta}{\zeta - z} \tag{8}$$

On Γ, let

$$\omega(\zeta) = \sum_{j=1}^{m} \chi_j\, T_j(\zeta), \quad \zeta\in\Gamma \tag{9}$$

where χ_j is the j-element characteristic function (i.e., $\chi_j = 1$ for $\zeta\in\Gamma_j$; 0, otherwise). Then for $z\in\Omega$,

$$\omega(z) = \frac{1}{2\pi i}\int_\Gamma \frac{\sum_{j=1}^{m}\chi_j\, T_j(\zeta)\, d\zeta}{\zeta - z}$$

$$= \sum_{j=1}^{m}\frac{1}{2\pi i}\int_{\Gamma_j}\frac{T_j(\zeta)\, d\zeta}{\zeta - z} \tag{10}$$

For $T_j(\zeta) = P_j^N(\zeta) + E_j^N(\zeta)$,

$$\omega(z) = \frac{1}{2\pi i}\sum_{j=1}^{m}\int_{\Gamma_j}\frac{P_j^N(\zeta)\,d\zeta}{\zeta - z} + \frac{1}{2\pi i}\sum_{j=1}^{m}\int_{\Gamma_j}\frac{E_j^N(\zeta)\,d\zeta}{\zeta - z}$$

$$= \hat{\omega}_N(z) + E_N(z) \tag{11}$$

The $\hat{\omega}_N(z)$ is the CVBEM approximation based on order N polynomials, where it is understood m nodes are used. The error, $E_N(z)$ is evaluated in magnitude for $z \in \Omega$ and using Eq. (12) to be:

$$\left|E_N(z)\right| = \frac{1}{2\pi}\left|\sum_{j=1}^{m}\int_{\Gamma_j}\frac{E_j^N(\zeta)\,d\zeta}{\zeta - z}\right|$$

$$\leq \frac{1}{2\pi}\ \frac{(m)(\max||\Gamma_j||)(\max|E_j^N(\zeta)|)}{\min|\zeta - z|}$$

$$= \frac{\left(\frac{m}{2\pi}\right)\left(\frac{2L}{m}\right)\left(2M\left[\frac{L}{mR}\right]^{N+1}\right)}{D}$$

$$= \frac{2LM}{\pi D}\left(\frac{L}{mR}\right)^{N+1} \tag{12}$$

where $D = \min|\zeta - z|$, for $\zeta \in \Gamma$.
Recalling that $\left(\frac{L}{m}\right) < R/2$,

$$|E_N(z)| \to 0 \text{ as either } m \to \infty, \text{ or } N \to \infty .$$

Thus, as the number m of CVBE increases, or the order of the interpoliatng polynomial N increases, error $|E_N(z)| \to 0$.

4.4. CVBEM Numerical Analog

As $z \to \zeta \in \Gamma_j$, for $z \in \Omega$, then $\omega(z) \to \omega(\zeta) = T_j(\zeta)$.
The CVBEM procedure is to set in a Cauchy limit sense,

$$T_j(z) = \frac{1}{2\pi i}\sum_{j=1}^{m}\int_{\Gamma_j}\frac{T_j(\zeta)\,d\zeta}{\zeta - z}, \quad \text{as } z \to \Gamma \text{ while } z \in \Omega. \tag{13}$$

For order N Taylor series expansions, the CVBEM sets in the Cauchy limit

$$P_j^N(z) = \frac{1}{2\pi i}\sum_{j=1}^{m}\int_{\Gamma_j}\frac{P_j^N(\zeta)\,d\zeta}{\zeta - z}, \quad \text{as } z \to \Gamma \text{ while } z \in \Omega. \tag{14}$$

If collocation is used, the numerical approach is to set (e.g., Hromadka and Lai, 1987):

$$P_j^N(z_i) = \omega(z_i), \text{ for each nodal coordinate } z_i \in \Gamma_j. \quad (15)$$

If a least-squares approach is used, the numerical approach is to minimize (e.g., Hromadka and Whitley, 1989)

$$||P_j^N(\zeta) - \omega(\zeta)||, \quad \zeta \in \Gamma, \; j = 1,2,\cdots,m \quad (16)$$

Letting

$$G_m(\zeta) = \sum_{j=1}^{m} N_j(\zeta)\, \bar{\omega}_j$$

where it is recalled $\bar{\omega}_j = \omega(z_j)$,

$$\lim_{l \to 0} G(\zeta) = \omega(\zeta)$$

and

$$\omega(\zeta) = \lim_{l \to 0} \frac{1}{2\pi i} \int_{\Gamma} \frac{G(\zeta)\, d\zeta}{\zeta - z}, \quad z \in \Omega \quad (17)$$

where l is the scale of the nodal partition of Γ.

5. IMPLEMENTATION

In general, one does not have both ϕ and ψ values defined on Γ, but instead have ϕ-values defined only on a portion of Γ, specified as Γ_ϕ, and ψ is defined only on the remaining portion of Γ, Γ_ψ where $\Gamma_\phi \cup \Gamma_\psi = \Gamma$. That is, we have a mixed boundary value problem.

The numerical formulation given in the above equations solves for the unknown ψ-values on Γ_ϕ, and the unknown ϕ-values on Γ_ψ. Once the unknown ϕ and ψ values are estimated, denoted as $\hat{\phi}$ and $\hat{\psi}$, then the global trial functions are well defined and can be used in $\hat{\omega}(z)$ estimates for the interior of Ω. The possible variations in such boundary condition issues are addressed in Hromadka and Lai (1987).

In this paper, we focus upon the Taylor series expansions in each Γ_j, as the interpolation polynomial order, N, increases and also as the number of CVBE, m, increases.

Thus, the numerical approach used in the CVBEM computer program formulation is outlined by the following steps:

(1) Discretize the problem boundary Γ (which is a finite union of straight line segments) into m CVBE by use of nodal points distributed on Γ where minimally a node is placed at each corner of Γ; i.e. $m \geq \Lambda$.

(2) For $N = 1$, a linear interpolating polynomial is defined on each Γ_j. For $N > 1$, a higher order polynomial expansion is used and, consequently, additional interpolation nodes are defined in each Γ_j. For example, for

$N=2$ a midpoint node is added to each Γ_j; for $N=3$, two additional nodes are defined in the interior of each Γ_j.

(3) Given N, a matrix solution provides the coefficients needed to define interpolating polynomials for each CVBE, using splines.

(4) The unknown nodal values are estimated by means of collocation or least-squares error minimization.

(5) Using the estimates for the unknown nodal values, a CVBEM approximation $\hat{\omega}(z)$ is well-defined for estimating $\omega(z)$ values in the interior of Ω.

(6) CVBEM error is evaluated by comparing $\hat{\omega}(z)$ and $\omega(z)$ with respect to the known boundary values of $\omega(z)$ on Γ; that is, compare $\hat{\phi}$ to ϕ on Γ_ϕ, and compare $\hat{\psi}$ to ψ on Γ_ψ. (From the previous mathematical development, if $\hat{\phi}=\phi$ on Γ_ϕ and $\hat{\psi}=\psi$ on Γ_ψ, then $\hat{\omega}(z)=\omega(z)$ for all $z\in\Omega$, if $\omega\in W_\Omega$.)

(7) After $\hat{\omega}$ and ω are compared as to boundary condition values, then the CVBEM program user can decrease the partition scale (i.e., increase the number of nodes uniformly) and/or increase the CVBE interpolating polynomial order, N. The modeling goal is to increase (m,N) until the boundary conditions are well-approximated by the CVBEM $\hat{\omega}(z)$.

It is recalled that regardless the goodness of fit of $\hat{\omega}(z)$ to the problem boundary conditions, the components of $\hat{\omega}(z)$, i.e., the functions $\hat{\phi}(z)$ and $\hat{\psi}(z)$ (where $\hat{\omega}(z)=\hat{\phi}(z)+i\psi(z)$) both exactly satisfy the Laplacian $\nabla^2\hat{\phi}=0$ and $\nabla^2\hat{\psi}=0$ for all $z\in\Omega$. Thus, there is no error in satisfying the Laplacian equation in Ω. This feature afforded by the CVBEM is not achieved by use of the usual finite element or finite difference numerical techniques which have errors in satisfying the problem's boundary conditions as well as errors in satisfying the flow field Laplacian in Ω.

(8) A new approach to evaluating CVBEM approximation error is to examine the closeness between values of the interpolating polynomial in each CVBE, and the CVBEM $\hat{\omega}(z)$ function, for z in Γ_j. That is examine in a Cauchy limit $||P_j^N(\zeta)-\hat{\omega}(\zeta)||_2$, $\zeta\in\Gamma_j$, for all CVBE Γ_j. As $||P_j^N(\zeta)-\hat{\omega}(\zeta)||_2\to 0$ (i.e., by increasing m and N) for all j and all $\zeta\in\Gamma_j$, then necessarily $\hat{\omega}(z)\to\omega(z)$ for all $z\in\Omega$, if $\omega(z)\in W_\Omega$.

(9) The choices as to increasing m or N is made by increasing both m and N in those boundary elements that have the most approximation error of $||P_j^N(\zeta)-\hat{\omega}(\zeta)||$ for $\zeta\in\Gamma_j$. In this way, $\hat{\omega}(z)$ approximations improve in accuracy without excessive additional computation. Generally, three or four attempts in developing $\hat{\omega}(z)$ functions may be needed for difficult potential flow problems, each successive CVBEM approximator being based upon the prior attempt but with localized increases in m and N where approximation error was largest.

6. APPLICATION

Application A

Solve $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$ in Ω where Ω is the domain shown in Fig. 1. Stream function values are specified along the horizontal lines of Γ, and potential function values are specified along the horizontal lines of Γ, forming a mixed boundary value problem. The analytic solution used is $\omega(z) = \ln[(z+1)/(z-1]$. Figures 1, 2, and 3 show approximation results versus exact values for 12, 25 and 40-boundary nodes, respectively. The accompanying figures show magnitude and integrated root-mean-square error plots along Γ, for the boundary values.

Application B

Figure 4 solves the Laplace equation for ideal fluid flow over a cylinder on the shown domain, Ω. The CVBEM vs. analytic results are compared in Figs. 4, 5, and 6 for 12, 25, and 40 node placements, respectively. Also shown are corresponding error plots in meeting boundary condition values along Γ. The exact solution is $\omega(z) = z + 1/z$. Stream function values are specified along the arc and also on $x = 0$; otherwise, potential function values are specified along Γ.

7. CONCLUSIONS

The CVBEM is a numerical approach to solving boundary value problems of two-dimensional Laplace and Poisson equations. The CVBEM estimator exactly solves the governing partial differential equations in the problem domain, but only approximately satisfies the problem boundary conditions. The CVBEM approximator can be improved by developing a better fit to the problem boundary conditions. In this paper, a new CVBEM error measure is used in aiding in the development of improved CVBEM approximators. The new approach utilizes Taylor series theory, and can be readily programmed into computer software form. Based on numerous test applications, it appears that use of this new CVBEM error measure leads to the development of significantly improved CVBEM approximation functions.

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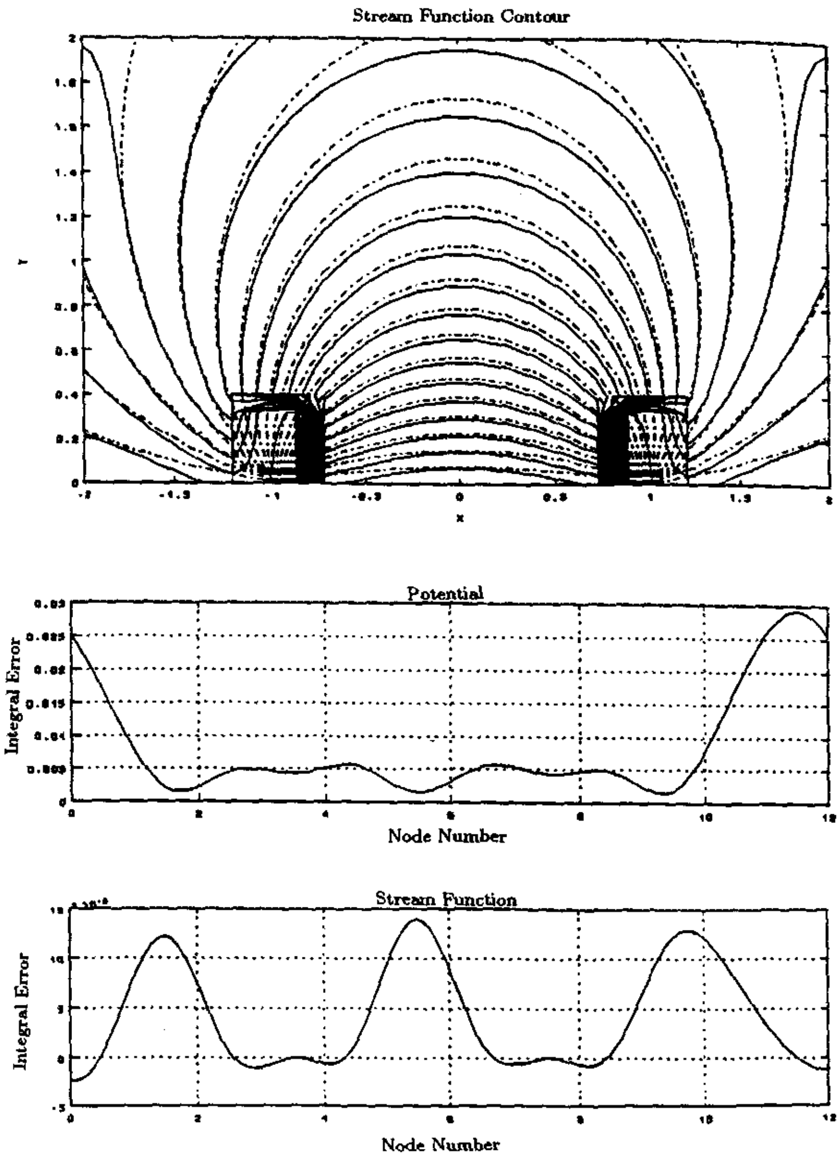

Figure 1. Application A with 12 nodes.

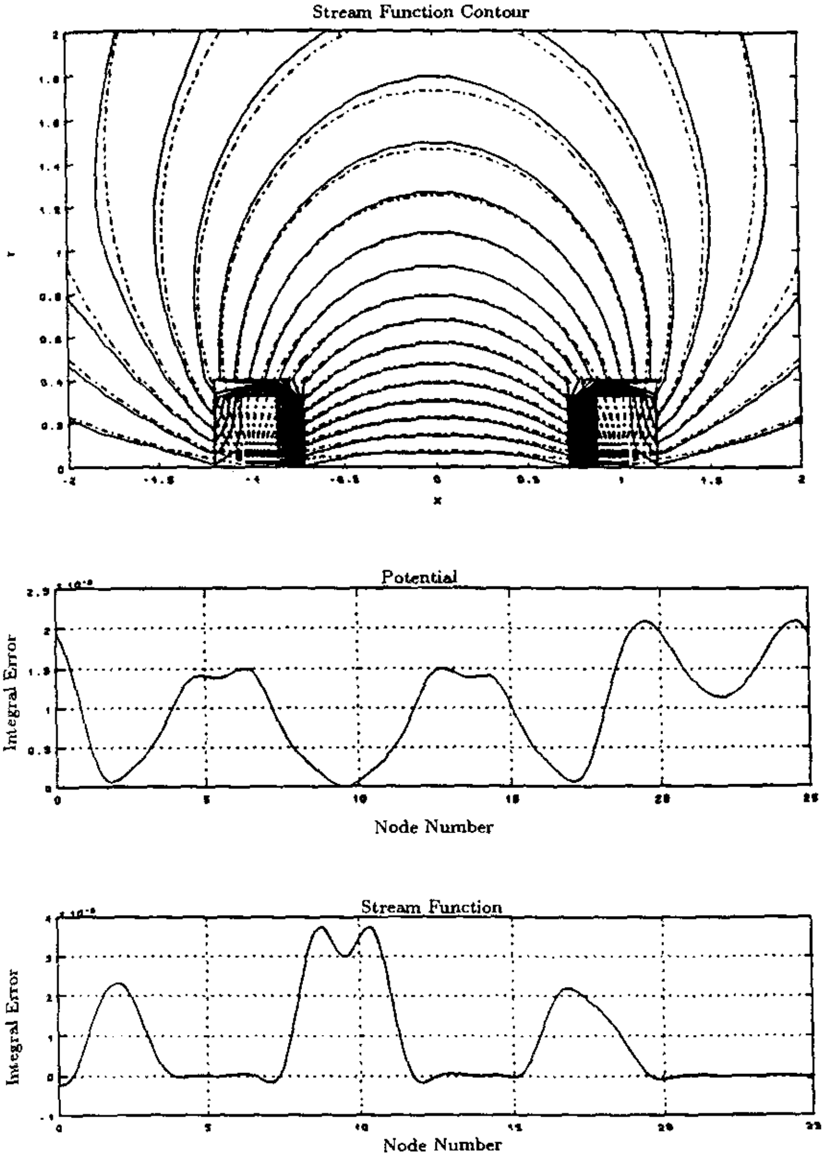

Figure 2. Application A with 25 nodes.

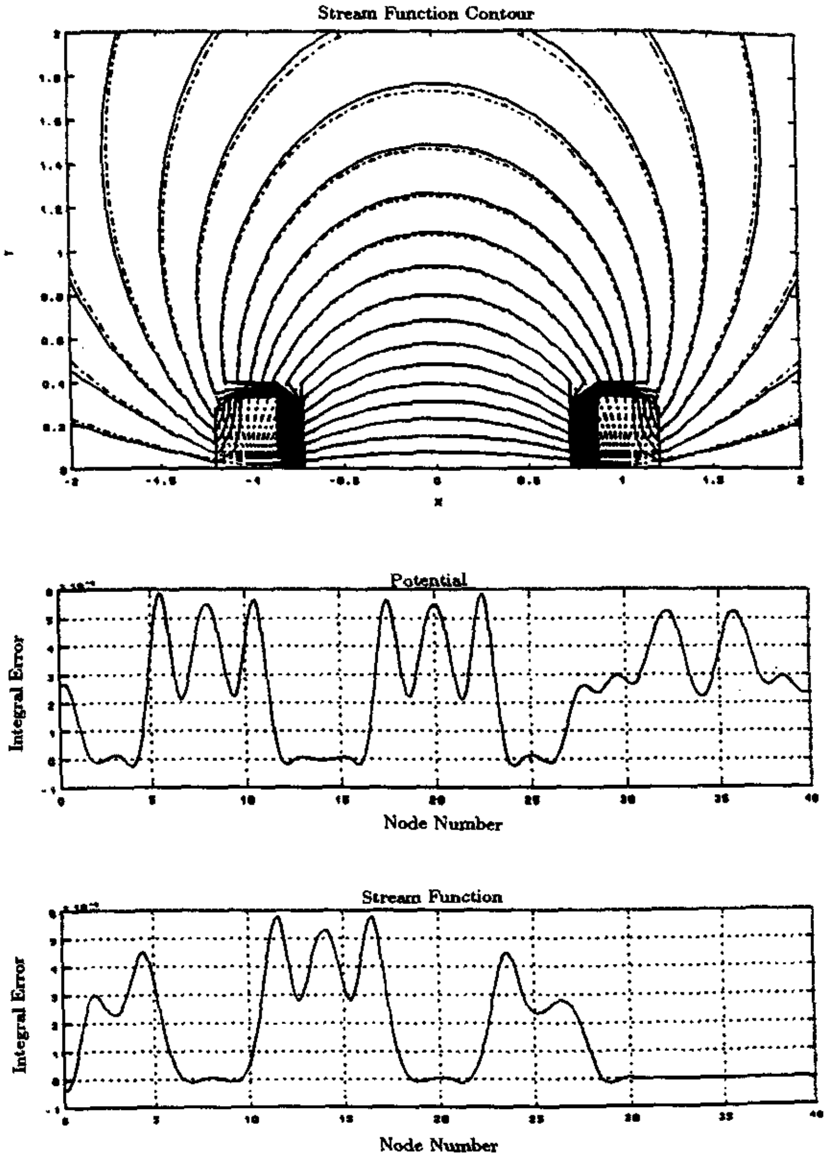

Figure 3. Application A with 40 nodes.

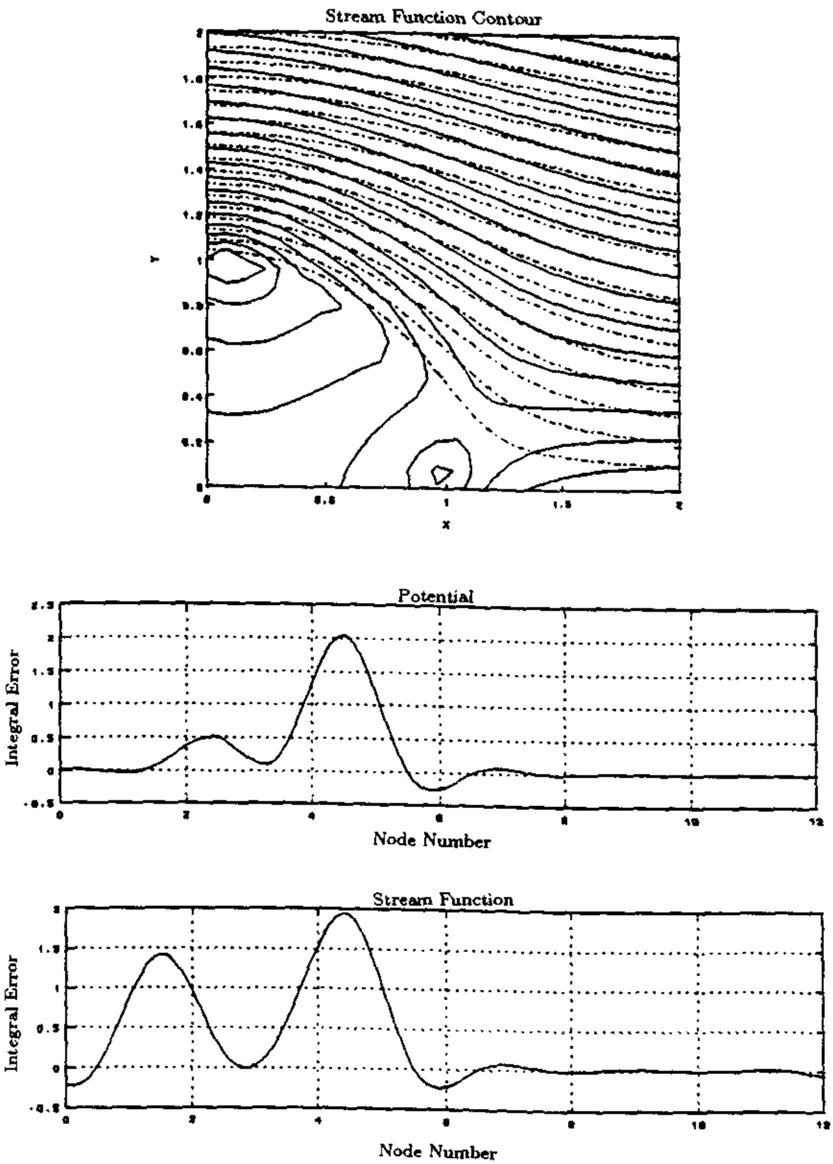

Figure 4. Application B with 12 nodes.

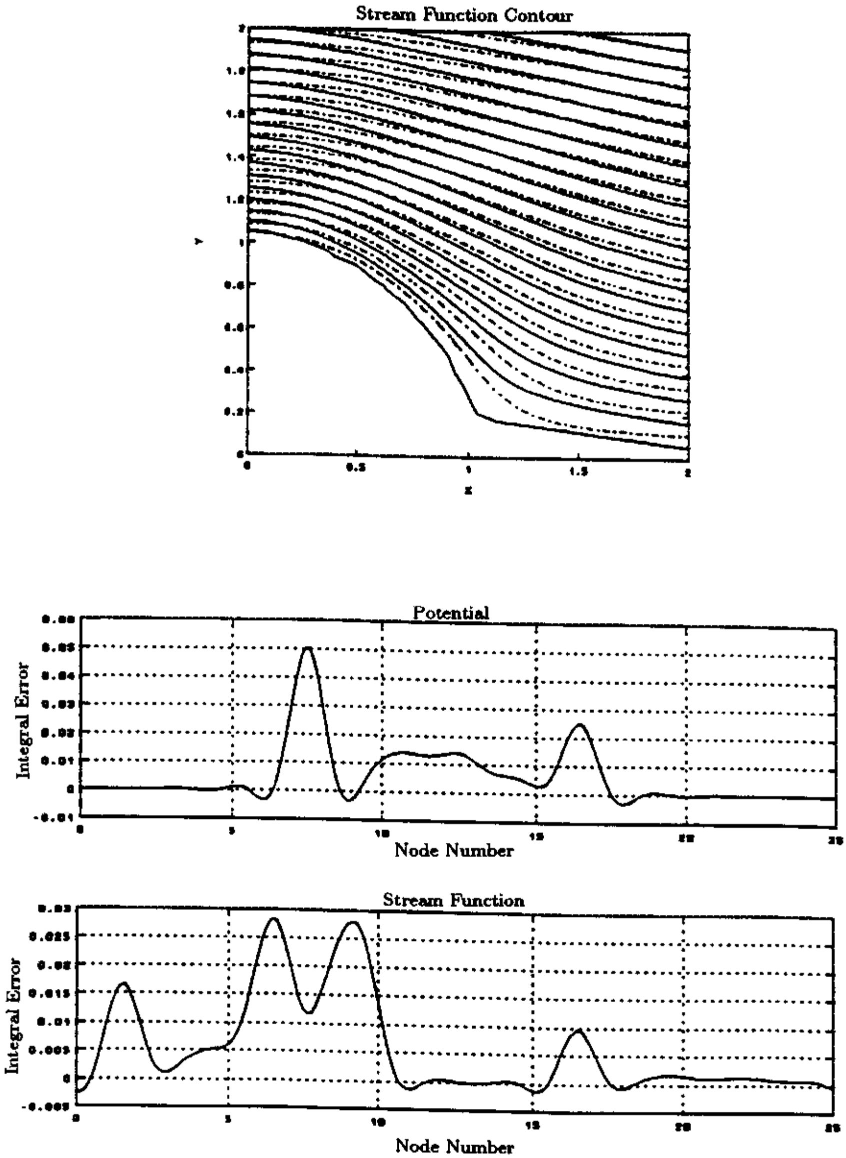

Figure 5. Application B with 25 nodes.

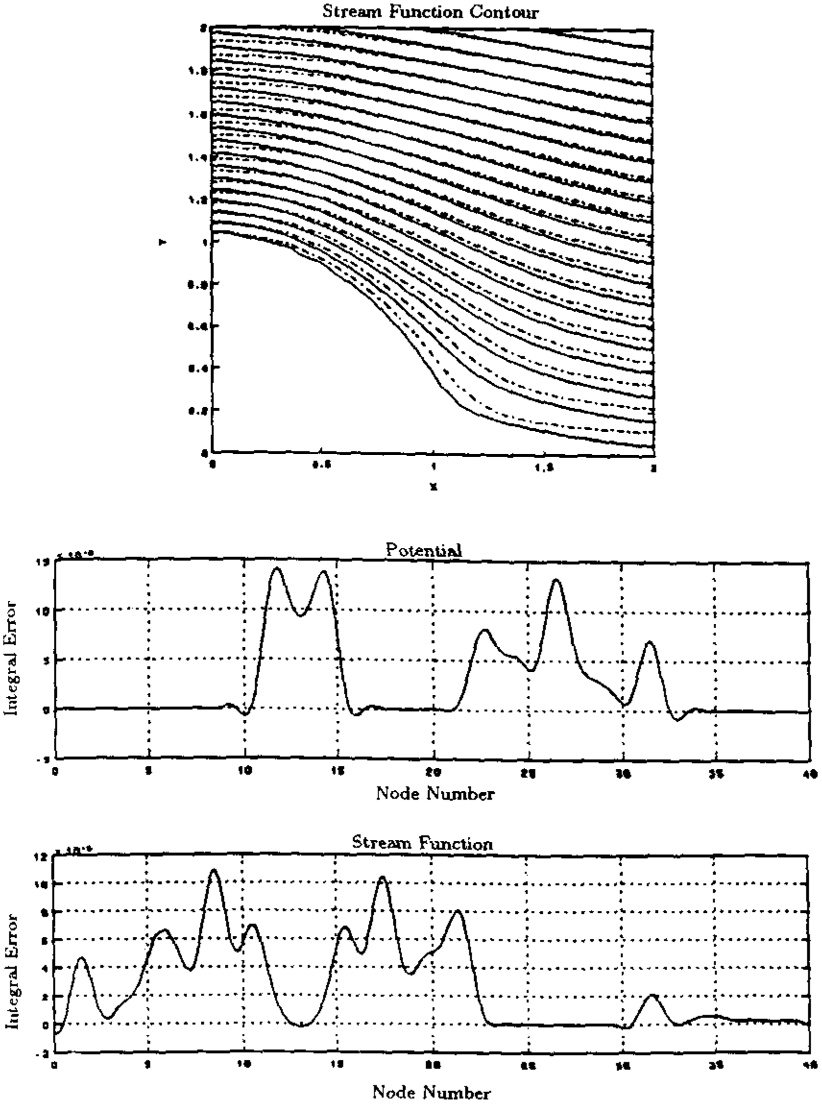

Figure 6. Application B with 40 nodes.