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UNIT HYDROGRAPH METHOD

Rainfall-Runoff Model Errors

Let M be a deterministic rainfall-runoff model which transforms gauged rainfall data for some storm event, \( i \), noted by \( P_g^i(t) \), into an estimate of runoff, \( M^i(t) \), by

\[
M^i(t) = M^i(t) \quad \text{(1)}
\]

where \( t \) is time.

Let \( P_g^i(t) \) be rainfall measured at a single gauge, and \( Q_g^i(t) \) be the runoff measured at the stream gauge. Various error (or uncertainty) terms may be combined by the relationship,

\[
Q_g^i(t) = M^i(t) + E_g^i(t) \quad \text{(2)}
\]

where \( E_g^i(t) \) is a conditional notation for total error, given model type \( M \), and storm event \( i \).

The terms in Eq. (2) are each a realization of a stochastic process. And for a future storm event \( D \), the \( E_g^D(t) \) is not known, but is a realization of a stochastic process distributed as \( [E_g^D(t)] \).

\[
[E_g^D(t)] = M^D(t) + [E_g^D(t)] \quad \text{(3)}
\]

In Eq. (5), \( [E_g^D(t)] \) and \( [E_m^D(t)] \) are the stochastic processes associated to the catchment runoff and total modeling errors, respectively, associated with model \( M \), for hypothetical storm event \( D \). In prediction, the model output of Eq. (3) is a stochastic distribution of outcomes, \( [Q_g^D(t)] \). Should \( \mathcal{A} \) be a functional operator on the outcome (e.g., detention basin volume; peak flow rate, median flow velocity, etc.) of storm event \( D \), then the value of \( \mathcal{A} \), noted as \( \mathcal{A}_D^D \), is a random variable distributed as \( [\mathcal{A}_D^D] \), where

\[
[\mathcal{A}_D^D] = \mathcal{A}[Q_g^D(t)] \quad \text{(4)}
\]

Developing Distributions for Model Estimates

The distribution for \( [E_m^D(t)] \) may be estimated by using the available sampling of realizations:

\[
[E_m^D(t)] = [Q_g^D(t) - M^D(t)] , i = 1,2,\ldots \quad \text{(5)}
\]

Assuming elements in \( [E_m^D(t)] \) to be dependent upon the "severity" of \( Q_g^D(t) \), one may partition \( [E_m^D(t)] \) into classes of storms such as mild, major, flood, or others.
The second assumption involved is to assume each $E_M(t)$ is a function of precipitation, $P(t) = F(T_p(t))$. $E_M(t)$ is now expressed as a causal linear filter (for only mild conditions imposed on $P(t)$), given by the stochastic integral equation (see Tsokos and Padgett, 1974)

$$E_M(t_o) = \int_{s=0}^{t_o} F(t_o-s) h_M(s) ds$$  \hspace{1cm} (6)

where $h_M(t)$ is the transfer function between $(E_M(t), P(t))$. Instead of $P(t)$, one may use in (6) the storm rainfall, $P_E(t)$, and the model estimates itself, $M(t)$.

Given a significant set of storm data, an underlying distribution $[h_M(t)]$ of the $h_M(t)$ may be identified, or the $[h_M(t)]$ may be used directly as a discrete distribution of equally-likely realizations. The predicted response from $M$ for future storm event $D$ is modeled as

$$[Q_MD(t)] = M_D(t) + [E_MD(t)]$$  \hspace{1cm} (7)

Combining Eqs. (8) and (9),

$$[Q_MD(t)] = M_D(t) + \int_{s=0}^{t} F_D(t-s) [h_M(s)] ds$$  \hspace{1cm} (8)

and for the functional operation $A$, Eq. (4) is rewritten as

$$[A_MD(t)] = A[Q_MD(t)] = A[M_D(t) + \int_{s=0}^{t} F_D(t-s) h_M(s) ds]$$  \hspace{1cm} (9)

Confidence interval estimates for the chosen criterion variable can now be obtained from the frequency-distribution, $[A_MD]$.  

**A Translation Unsteady Flow Routing Rainfall-Runoff Model**

The catchment $R$ is subdivided into $m$ homogeneous subareas, $R = U R_j$ such that in each $R_j$, the effective rainfall, $e_j(t)$, is assumed given by

$$e_j(t) = \lambda_j(1 + X_j f) P(t)$$  \hspace{1cm} (10)

where $\lambda_j$ is a constant; $X_j$ is a sample from $[X_j]$.

The subarea runoff is

$$q_j(t) = \int_{s=0}^{t} e_j(t-s) \phi_j(s) ds$$  \hspace{1cm} (11)

where $\phi_j(s)$ is the unit hydrograph contribution of the subarea $j$.

At this stage of development, unsteady flow routing along channel links is assumed to be pure translation. Thus, each channel link, $L_k$, has the constant travel time, $T_k$. Hence for $m$ links,

$$Q_k(t) = \sum_{j=1}^{m} q_j(t - \tau_j)$$  \hspace{1cm} (12)

where $q_j(t - \tau_j)$ is zero for negative arguments and $\tau_j$ is the sum of link travel times.

For the above assumptions,

$$Q_k(t) = \int_{s=0}^{t} F(t-s) \sum_{j=1}^{m} \lambda_j(1 + X_j f) \phi_j(s - \tau_j) ds$$  \hspace{1cm} (13)

In a final form,

$$Q_k(t) = \int_{s=0}^{t} F(t-s) \sum_{j=1}^{m} \lambda_j X_j \phi_j(s - \tau_j) ds$$  \hspace{1cm} (14)

In the above equations, the samples $[X_j]$ are unknown for any future storm event $i$. From Eq. (14), the model structure $M$, used in design practice is

$$M(t) = \int_{s=0}^{t} F(t-s) \sum_{j=1}^{m} \lambda_j \phi_j(s - \tau_j) ds$$  \hspace{1cm} (15)

Then, $Q_k(t) = M(t) + E_M(t)$ where

$$E_M(t) = \int_{s=0}^{t} F(t-s) h_M(s) ds$$  \hspace{1cm} (16)

where $h_M(s)$ follows directly from Eqs. (14) and (15).

Should the subarea $U$ all be assumed fixed, (i.e., $\phi_j(t) = \phi(t)$),

$$M(t) = \int_{s=0}^{t} F(t-s) \Phi(s) ds$$  \hspace{1cm} (17)
Additionally, the distribution of \( h_M(t) \) is

\[
|h_M(t)| = \sum_{j=1}^{n_2} |X_j| \lambda_j \phi(t - \theta_j)
\]

(18)

The stochastic integral prediction is

\[
|Q_M^D(t)| = \int_{s=0}^{t} F_D(t-s) \Phi(s) \, ds + \int_{s=0}^{t} F_D(t-s) |h_M(s)| \, ds
\]

(19)

A Multilinear Rainfall-Runoff Model

Each subarea's effective rainfall, \( e_j(t) \), is written as a sum of proportions of \( F(t) \) translates, on a storm class basis,

\[
e_j(t) = \sum_k \lambda_{jk} (1 + X_{jk}) F(t - \theta_{jk}) \Phi(t - \theta_{jk}) \epsilon \xi_{D}
\]

(20)

where \( X_{jk} \) and \( \theta_{jk} \) are samples of the random variables distributed as \( \{X_k\} \) and \( \{\theta_k\} \), respectively.

On a storm class basis, each channel link is assumed to be linear (Doyle et al, 1983)

\[
O_L(t) = \sum_i a_{li} I_i(t - \alpha_i)
\]

(21)

where \( O_L(t) \) and \( I_i(t) \) are the outflow and inflow hydrographs for link \( L \), and storm event \( i \); and \( a_i \) and \( \alpha_i \) are constants which are defined on the storm class basis used for the input, \( F(t) \). Thus, the channel link routing algorithm is multilinear with parameters defined by storm class, \( \xi_{D} \) (e.g., Becker and Kundzewicz, 1987).

For \( L \) links, each with their respective stream gauge routing data,

\[
O_L(t) = \sum_{4=1}^{n_L} a_{4.1} \sum_{4=1}^{n_4} a_{4.1} \ldots \sum_{6=1}^{n_6} a_{6.1} \sum_{4=1}^{n_4} a_{6.1} \sum_{4=1}^{n_4} I_1(t - \alpha_4 - \alpha_4 - \ldots - \alpha_4)
\]

(22)

Using an index notation, the above \( O_L(t) \) is written as

\[
O_L(t) = \sum_{i=1}^{n_L} a_{i,n} I_1(t - \alpha_{i,n})
\]

(23)

The predicted runoff response for storm event \( D \) is the stochastic integral formulation

\[
|Q_M^D(t)| = \int_{s=0}^{t} F_D(t-s) \left( \sum_{j=1}^{n_2} \sum_{k=1}^{n_2} a_{jk} \lambda_j (1 + X_{jk}) \Phi(t - \theta_{jk}) \right. \\
\left. \alpha_{jk} \right) \, ds; F_D(t) \epsilon \xi_{D}
\]

(24)

References