ENGINEERING HYDROLOGY

Proceedings of the Symposium

Sponsored by the
Hydraulics Division of the
American Society of Civil Engineers

in cooperation with the
Environmental Engineering Division
Irrigation and Drainage Division
Water Resources Planning and Management Division
Waterway Port Coastal and Ocean Division
of ASCE

San Francisco, California
July 25-30, 1993

Edited by Chin Y. Kuo

Published by the
American Society of Civil Engineers
345 East 47th Street
New York, New York 10017-2398

BACK TO THE UNIT HYDROGRAPH METHOD

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Abstract

In this paper, the basic single area unit hydrograph model is shown to represent a highly complex, rational, link-node model which includes (i) variable effective rainfall distributions over each subarea, including variations in distribution magnitude, timing, and storm pattern shape; (ii) variations in the subarea runoff response on a storm class basis; (iii) variations in channel flow routing response on a storm class basis, including peak attenuation and translation timing effects.

Introduction

Due to the nondeterministic nature of the rainfall-runoff processes occurring over the catchment, the mathematical descriptions of these processes result in stochastic equations. Additionally, the so-called deterministic rainfall-runoff models used to describe the several physical processes contain parameters or coefficients which have well-defined physically-based meanings, but whose exact values are unknown. The boundary conditions of the problem itself are unknown (e.g., the temporal and spatial distribution of rainfall over the catchment for the storm event under study and also for all prior storm events) and often exhibit considerable variations with respect to the assumed boundary conditions, the measured rainfall at a single location (e.g., Nash and Sutcliffe, 1970; Huff, 1970). Thus the physically-based parameters and coefficients, and also the problem boundary conditions, are not the assumed values, but are instead random variables and stochastic processes.

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Rainfall-Runoff Model Errors

Let M be a deterministic rainfall-runoff model which transforms gauged rainfall data for some storm event, i, noted by $P_g{}^i(t)$, into an estimate of runoff, $M^i(t)$, by

$$M: P_g{}^i(t) \rightarrow M^i(t) \tag{1}$$

where t is time.

Let $P_g{}^i(t)$ be rainfall measured at a single gauge, and $Q_g{}^i(t)$ be the runoff measured at the stream gauge. Various error (or uncertainty) terms may be combined by the relationship,

$$Q_g{}^i(t) = M^i(t) + E_M{}^i(t) \tag{2}$$

where $E_M{}^i(t)$ is a conditional notation for total error, given model type M, and storm event i.

The terms in Eq. (2) are each a realization of a stochastic process. And for a future storm event D, the $E_M{}^D(t)$ is not known, but is a realization of a stochastic process distributed as $[E_M{}^D(t)]$ where

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)] \tag{3}$$

In Eq. (5), $[Q_M{}^D(t)]$ and $[E_M{}^D(t)]$ are the stochastic processes associated to the catchment runoff and total modeling errors, respectively, associated with model M, for hypothetical storm event D. In prediction, the model output of Eq. (3) is a stochastic distribution of outcomes, $[Q_M{}^D(t)]$. Should $\mathcal{A}$ be a functional operator on the outcome (e.g., detention basin volume; peak flow rate; median flow velocity, etc.) of storm event D, then the value of $\mathcal{A}$, noted as $A_M{}^D$, is a random variable distributed as $[A_M{}^D]$, where

$$[A_M{}^D] = \mathcal{A}[Q_M{}^D(t)] \tag{4}$$

Developing Distributions for Model Estimates

The distribution for $[E_M{}^D(t)]$ may be estimated by using the available sampling of realizations:

$$\{E_M{}^i(t)\} = \{Q_g{}^i(t) - M^i(t)\},\ i = 1,2,\cdots \tag{5}$$

Assuming elements in $\{E_M{}^i(t)\}$ to be dependent upon the "severity" of $Q_g{}^i(t)$, one may partition $\{E_M{}^i(t)\}$ into classes of storms such as mild, major, flood, or others.

The second assumption involved is to assume each $E_M{}^i(t)$ is a function of precipitation, $F^i(t) = F(P_g{}^i(t))$. $E_M{}^i(t)$ is now expressed as a causal linear filter (for only mild conditions imposed on $F^i(t)$), given by the stochastic integral equation (see Tsokos and Padgett, 1974)

$$E_M^i(t_o) = \int_{s=0}^{t_o} F^i(t_o-s)\, h_M^i(s)\, ds \tag{6}$$

where $h_M{}^i(t)$ is the transfer function between $(E_M{}^i(t), F^i(t))$. Instead of $F^i(t)$, one may use in (6) the storm rainfall, $P_g{}^i(t)$, and the model estimates itself, $M^i(t)$.

Given a significant set of storm data, an underlying distribution $[h_M(t)]$ of the $\{h_M{}^i(t)\}$ may be identified, or the $\{h_M{}^i(t)\}$ may be used directly as a discrete distribution of equally-likely realizations. The predicted response from M for future storm event D is modeled as

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)] \tag{7}$$

Combining Eqs. (8) and (9),

$$[Q_M^D(t)] = M^D(t) + \int_{s=0}^{t_o} F^D(t-s)\, [h_M(s)]\, ds \tag{8}$$

and for the functional operation $\mathcal{A}$, Eq. (4) is rewritten as

$$[A_M{}^D] = \mathcal{A}\{Q_M{}^D(t)\} = \mathcal{A}(M^D(t) + \int_{s=0}^{t} F^D(t-s)\, \{h_M(s)\}\, ds) \tag{9}$$

Confidence interval estimates for the chosen criterion variable can now be obtained from the frequency-distribution, $[A_M{}^D]$.

A Translation Unsteady Flow Routing Rainfall-Runoff Model

The catchment R is subdivided into m homogeneous subareas, $R = \cup R_j$, such that in each R_j, the effective rainfall, $e_j{}^i(t)$, is assumed given by

$$e_j{}^i(t) = \lambda_j(1 + X_j{}^i)\, F^i(t) \tag{10}$$

where λ_j is a constant; $X_j{}^i$ is a sample from $[X_j]$.

The subarea runoff is

$$q_j{}^i(t) = \int_{s=0}^{t} e_j{}^i(t-s)\, \phi_j^i(s)\, ds = \int_{s=0}^{t} \lambda_j(i+X_j{}^i)\, F^i(t-s)\, \phi_j^i(s)\, ds \tag{11}$$

At this stage of development, unsteady flow routing along channel links is assumed to be pure translation. Thus, each channel link, L_k, has the constant translation time, T_k. Hence for m links,

$$Q_g^i(t) = \sum_{j=1}^{m} q_j{}^i(t-\tau_j) \tag{12}$$

where $q_j{}^i(t - \tau_j)$ is zero for negative arguments and τ_j is the sum of link travel times.

For the above assumptions,

$$Q_g{}^i(t) = \int_{s=0}^{t} F^i(t-s)\, \left(\sum_{j=1}^{m} \lambda_j\, (1+X_j{}^i)\, \phi_j^i(s-\tau_j)\right) ds \tag{13}$$

In a final form,

$$Q_g^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{9} \lambda_j\, \phi_j^i(s-\tau_j)\, ds + \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{9} \lambda_j X_j{}^i\, \phi_j^i(s-\tau_j)\, ds \tag{14}$$

In the above equations, the samples $\{X_j{}^i\}$ are unknown for any future storm event i. From Eq. (14), the model structure M, used in design practice is

$$M^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{9} \lambda_j\, \phi_j^i(s-\tau_j)\, ds \tag{15}$$

Then, $Q_g{}^i(t) = M^i(t) + E_M{}^i(t)$ where

$$E_M{}^i(t) = \int_{s=0}^{t} F^i(t-s)\, h_M^i(s)\, ds \tag{16}$$

where $h_M{}^i(s)$ follows directly from Eqs. (14) and (15).

Should the subarea UH all be assumed fixed, (i.e., $\phi_j{}^i(t) = \phi_j(t)$,

$$M^i(t) = \int_{s=0}^{t} F^i(t-s)\, \Phi(s)\, ds \tag{17}$$

Additionally, the distribution of $[h_M(t)]$ is

$$[h_M(t)] = \sum_{j=1}^{m} [X_j]\, \lambda_j\, \phi_j(t-\tau_j) \tag{18}$$

The stochastic integral prediction is

$$[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s)\, \Phi(s)\, ds + \int_{s=0}^{t} F^D(t-s)\, [h_M(s)]\, ds \tag{19}$$

A Multilinear Rainfall-Runoff Model

Each subarea's effective rainfall, $e_j{}^i(t)$, is written as a sum of proportions of $F^i(t)$ translates, on a storm class basis,

$$e_j{}^i(t) = \sum_k \lambda_{jk}\, (1 + X_{jk}{}^i)\, F^i(t - \theta_{jk}{}^i);\ F^i(t) \in [\xi_z] \tag{20}$$

where $X_{jk}{}^i$ and $\theta_{jk}{}^i$ are samples of the random variables distributed as $[X_{jk}]$ and $[\theta_{jk}]$, respectively.

On a storm class basis, each channel link is assumed to be linear (Doyle et al, 1983)

$$O_l{}^i(t) = \sum_{\ell} a_\ell\, I_l{}^i(t - \alpha_\ell) \tag{21}$$

where $O_l{}^i(t)$ and $I_l{}^i(t)$ are the outflow and inflow hydrographs for link l, and storm event i; and $\{a_\ell\}$ and $\{\alpha_\ell\}$ are constants which are defined on the storm class basis used for the input, $F^i(t)$. Thus, the channel link flow routing algorithm is multilinear with parameters defined by storm class, $[\xi_z]$ (e.g., Becker and Kundezewicz, 1987).

For L links, each with their respective stream gauge routing data,

$$O_L(t) = \sum_{\ell_L=1}^{n_L} a_{\ell_L} \sum_{\ell_{L-1}=1}^{n_{L-1}} a_{\ell_{L-1}} \cdots \sum_{\ell_2=1}^{n_2} a_{\ell_2} \sum_{\ell_1=1}^{n_1} a_{\ell_1}\, I_1(t - \alpha_{\ell_1} - \alpha_{\ell_2} - \dots - \alpha_{\ell_{L-1}} - \alpha_{\ell_L}) \tag{22}$$

Using an index notation, the above $O_L(t)$ is written as

$$O_L(t) = \sum_{\langle\ell\rangle} a_{\langle\ell\rangle}\, I_1(t - \alpha_{\langle\ell\rangle}) \tag{23}$$

The predicted runoff response for storm event D is the stochastic integral formulation

$$[Q_M^D(t)] = \int_{-\infty}^{t} F^D(t-s) \Big(\sum_{j=1}^{m} \sum_{\langle\ell\rangle_j} a^i_{\langle\ell\rangle_j} \sum_k \lambda_{jk}(1+[X_{jk}])\, \phi_j(s-[\theta_{jk}] - \alpha^i_{\langle\ell\rangle_j}) \Big)\, ds;\ F^D(t) \in [\xi_D] \tag{24}$$

Back to the Unit Hydrograph Method

A stochastic integral equation that is equivalent to Eq. (24) is

$$[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s)\, [\eta(s)]\, ds;\ F^D(t) \in [\xi_D] \tag{25}$$

where now $[\eta(s)]$ is the distribution of the stochastic process representing the several sets of random variations. In prediction, the expected runoff estimate for storm events that are elements of $[\xi_D]$ is

$$E[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s)\, E[\eta(s)]\, ds;\ F^D(t) \in [\xi_D] \tag{26}$$

which is the well-known unit hydrograph method.

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