Computer Techniques in Environmental Studies IV

Editor: P. Zannetti, Failure Analysis Associates, Inc., California, USA

Computational Mechanics Publications
Southampton Boston

Co-published with

Elsevier Applied Science
London New York

Unit Hydrograph Models and Uncertainty Distributions

T.V. Hromadka II

Boyle Engineering Corporation, Newport Beach, California and California State University, Fullerton, California 92634, USA

ABSTRACT

In this paper, the uncertainty in the effective rainfall distribution (i.e., rainfall less losses) over the catchment (R) is considered to be a dominating influence in causing flood flow modeling errors in discretized models (i.e., subdivision of R into subareas, linked by channel routing). A single area unit hydrograph (UH) model is also used to represent the uncertainty not only in the effective rainfall distribution over R, but also the uncertainty in the catchment hydraulic responses. Both modeling approaches are focused towards the typical case where only one stream gauge and one rain gauge are available for data analysis. It is shown that due to the limited data available, the simple single area UH model includes several uncertainties that the discretized model misrepresents.

INTRODUCTION

A review of the literature which raises questions as to the development, application, and calibration, of hydrologic models is contained in Hromadka and Whitley[1]. In that literature review, it appears that the unknown distribution of effective rainfall (i.e., rainfall less losses) over the catchment, R, is a barrier to the success in the use of hydrologic models for predicting hydrologic responses.

For example, Schilling and Fuchs[2] write "that the spatial resolution of rain data input is of paramount importance to the accuracy of the simulated hydrograph" due to "the high spatial variability of storms" and "the amplification of rainfall sampling errors by the nonlinear transformation" of rainfall into runoff by hydrologic models. In their study, Schilling and Fuchs analyzed an 1,800-acre catchment with three rain gauge densities (all equally spaced) of 81-, 9-, and a single located at the downstream end of R. In each subarea, R_j, a unit hydrograph (UH) is defined for each storm event i such that the subarea runoff hydrograph, $Q_j{}^i(t)$ is given by

$$Q_j{}^i(t) = \int_{s=0}^{t} e_j{}^i(t - s) \quad {}_j{}^i(s)\, ds \qquad (1)$$

where $e_j{}^i(t)$ is the effective rainfall uniformly distributed over subarea R_j. Because the subarea UH's, $\phi_j{}^i(s)$, may differ between storm events, i, and the channel link travel times also vary on a storm by storm basis, the resulting m-subarea link node model, $Q_m{}^i(t)$, is a reasonable approximation of any hydrologic model. The global model $Q_m{}^i(t)$ for R and storm event i is given by

$$Q_m{}^i(t) = \sum_{j=1}^{m} Q_j{}^i(t - \tau_j{}^i) \qquad (2)$$

where $Q_j{}^i(t - \tau_j{}^i)$ is the runoff hydrograph from R_j offset in time by $\tau_j{}^i$ from the stream gauge; and $\tau_j{}^i$ is the sum of link travel times from R_j to the stream gauge, and is variable between storms, i. For storm event i, the stream gauge runoff hydrograph is given by $Q_g{}^i(t)$.

The available single rain gauge is associated with a runoff measuring system such that the effective rainfall distribution is measured at the rain gauge site, noted as $e_g{}^i(t)$ for storm i. Each subarea R_j is assumed to be sufficiently small such that the subarea effective rainfall, $e_j{}^i(t)$, applies uniformly in R_j. The $e_j{}^i(t)$ are assumed to be linear with respect to $e_g{}^i(t)$ such that (Hromadka[3])

$$e_j{}^i(t) = \sum_{k=1}^{n_j{}^i} \lambda_{jk}{}^i\, e_g{}^i(t - \theta_{jk}{}^i) \qquad (3)$$

where $\lambda_{jk}{}^i$ are positive coefficients $k = 1,2,\ldots,n_j{}^i$; and the $\theta_{jk}{}^i$ are timing offsets. These several constants apply only to R_j and storm i, and enable $e_j{}^i(t)$ to be written as a finite sum of proportions of the data $e_g{}^i(t)$, each offset in time by a timestep, $\theta_{jk}{}^i$.

Combining Eqs. (1), (2), and (3) gives the global m-subarea model of R for storm i,

$$Q_m{}^i(t) = \sum_{j=1}^{m} \int_{s=0}^{n_j{}^i} e_g{}^i(t - s) \sum_{k=1}^{n_j{}^i} \lambda_{jk}{}^i\, \phi_j{}^i(s - \theta_{jk}{}^i - \tau_j{}^i)\, ds \qquad (4)$$

Equation (4) is immediately reduced to the single area UH model (Hromadka[3])

$$Q_m{}^i(t) = Q_1{}^i(t) = \int_{s=0}^{t} W^i\, e_g{}^i(t - s)\, \psi^i(s)\, ds$$

where W^i is the ratio of stream gauge runoff to the site's effective rainfall; and $\psi^i(s)$ is the unit defined by

$$\psi^i(s) = \sum_{j=1}^{m} \sum_{k=1}^{n_j{}^i} \lambda_{jk}{}^i\, \phi_j{}^i(s - \theta_{jk}{}^i - \tau_j{}^i)/W^i$$

A more convenient representation of Eq. (5) is

$$Q_1{}^i(t) = \int_{s=0}^{t} e_g{}^i(t - s)\, \eta^i(s)\, ds$$

where

$$\eta^i(s) = W^i\, \psi^i(s)$$

From the above development, the single area UH mod includes all the assumptions leading to the m-subare model, $Q_m{}^i(t)$. However, with only one stream gau gauge, the practitioner must use the estimator $\hat{Q}_m{}^i(t$ an associated error due to the need to estimate th parameters, $\{\phi_j{}^i(s), \tau_j{}^i\}$ without the benefit hydrologic data. There is also the discretizatic artificially defining the effective rainfall distri R (i.e., in each subarea R_j). Without subarea rain data, the effective rainfall parameters $\{\lambda_{jk}{}^i, \theta_{jk}$ all incorrectly defined in $\hat{Q}_m{}^i(t)$. The importanc parameters in hydrologic modeling is reflecte literature review (Hromadka[3]).

In comparison to $\hat{Q}_m{}^i(t)$, however, the $Q_1{}^i(t)$ the "correct" values for $\eta_j{}^i(s)$ for storm event i. includes the effects of channel flow routing storage

In other words, with the available data the Q represents the "true" $Q_m{}^i(t)$ model with all paramete defined; but in practice, use of the m-subarea link results in using the estimator, $\hat{Q}_m{}^i(t)$, given by

$$\hat{Q}_m{}^i(t) = \sum_{j=1}^{m} \int_{s=0}^{t} \hat{e}_j{}^i(t - s)\, \hat{\phi}_j{}^i(s - \hat{\tau}_j{}^i)\, ds$$

where the hats are notation for imates. Ea estimates used in (9) are unsupported by subarea measured at the rain gauge. The $Q_1{}^i(t)$ model, in comparison, includes the correct variation in rainfall over R (magnitude, timing, and storm pattern shape) in the unit hydrograph, $\psi^i(s)$, as shown in Eq. (6).

HYDROLOGIC MODELING UNCERTAINTY ANALYSIS: DATA REPRESENTATION

The $Q_1{}^i(t)$ model correlates the data pair $\{e_g{}^i(t), Q_g{}^i(t)\}$, for each storm i, for the given modeling assumptions. This correlation is integrated into the time distribution of the parameter, $\eta^i(s)$. Thus for each storm event i, there is an associated $\eta^i(s)$.

To proceed with the uncertainty analysis, it is assumed that there is "sufficient" data at the rain gauge site such as to develop equivalence classes of effective rainfall distributions measured at the rain gauge site. These storm classes are noted as $<\xi_q>$. Any two events in $<\xi_q>$ would be nearly identical (storm duration, antecedent moisture conditions, and other effects) such that the catchment response from R would be thought to be also nearly identical. It is assumed that there is sufficient effective rainfall data to develop a set of classes $<\xi_q>$ such that a reasonable statistical analysis can be made for each class individually.

Let $<\xi_o>$ be a class of storms, (it is recalled that the measured effective rainfall distributions are used in the classification, not the rainfall). Let $e_o{}^i(t)$ be an element of $<\xi_o>$, for $i = 1,2,\ldots,n_o$ where n_o is the number of elements in $<\xi_o>$. To each $e_o{}^i(t)$ there is an associated $Q_o{}^i(t)$ measured at the stream gauge. Correlating each pair $\{e_o{}^i(t), Q_o{}^i(t)\}$ by the single area UH model results in n_o distributions, $\{\eta_o{}^i(s)\}$, $i = 1,2,\ldots,n_o$.

The $\eta_o{}^i(s)$ can be represented by a summation graph, $\overline{S}_o{}^i(s)$, where

$$\overline{S}_o{}^i(s) = \int_{t=0}^{s} \eta_o{}^i(t)\, dt \qquad (10)$$

Figure 1 shows a plot of $\overline{S}_o{}^i(s)$ developed from storms of similar severity from a basin in Los Angeles County, California. By examining the plots, usually a normalization technique is apparent. In Fig. 1, plotting each $\overline{S}_o{}^i(s)$ divided by its ultimate discharge, $U_o{}^i$ (i.e., $U_o{}^i = \overline{S}_o{}^i(s = \infty)$), normalizes the vertical axis from 0- to 100-percent. Defining lag^i to be the time that $\overline{S}_o{}^i(s)$ reaches 50-percent of ultimate discharge ($U_o{}^i$) normalizes the horizontal axis to be time in percent of lag. Figure 2 shows the resulting S-graphs, noted as $S_o{}^i(s)$ for storm class $<\xi_o>$. The several S-graphs can now be identified by a characteristic parameter. A convenient parameter to use is the linear scaling X between the enveloping curves of the S-graph data set. Usually, two of the correlation S-graphs will bound the entire set (see Fig. 3). By identifying an X to each S-graph,

$$S_o{}^i(X,s) = X\, S_o{}^A(s) + (1-X)\, S_o{}^B(s)$$

where $S_o{}^A$ and $S_o{}^B$ are the enveloping S-graphs, ar scaling parameter with $0 \leq X \leq 1$.

Based on the above normalizations and parame each distribution graph, $\overline{S}_o{}^i(s)$, is identified b point vector $\underset{\sim}{\eta}_o{}^i \equiv \{lag^i, U_o{}^i, X^i\}$. Consequ correlation distribution, $\eta_o{}^i(s)$, is identified by $\underset{\sim}{\eta}_o{}^i$, for $i = 1,2,\ldots,n_o$.

Marginal distributions are developed b frequency-distributions of each point in the vecto Fig. 4).

Based on the marginal distributions, the estimate associated to vector, $\underset{\sim}{\eta}_o{}^i$, is given by $P(\underset{\sim}{\eta}_o$

$$P(\underset{\sim}{\eta}_o{}^i) = P(lag^i, U_o{}^i, X^i)$$

Should more identifying characteristics be used to d $\eta_o{}^i(s)$, Eq. (12) is immediately extended. How should be sufficient storms in $<\xi_o>$ to develop frequency-distribution for each identifying cha From the above, a distribution $[\eta_o(s)]$ of distributions, $\eta_o{}^i(s)$, is derived for storm class, $<\xi$

From the above development, storm class $<\xi_o>$ is with its distribution of vectors, $\underset{\sim}{\eta}_o{}^i$. Each storm can be analyzed to determine their respecti distributions, $\underset{\sim}{\eta}_q{}^i$.

Each distribution $\eta_q{}^i(s)$ associated to represents the correlation of the effective rainfa the stream gauge data, using the single area UH mod It is recalled that the effective rainfall distribut is a random variable, and is reflected through the of available effective-rainfall to runoff data by th distribution of the vectors, $\underset{\sim}{\eta}_q{}^i$, for storm class, $<\xi$,

HYDROLOGIC MODELING UNCERTAINTY ANALYSIS: PREDICTIVE RELATIONSHIPS

The true use of hydrologic models is not in the rec of a runoff event given both the rainfall and runoff in predicting a hydrologic response from R given a hypothetical, storm event.

Given a design storm effective rainfall distrib applied at the rain gauge site, $e_g{}^D(t)$, the hydrologi to be used to predict a runoff response from R.

Let $e_g{}^D(t) \in <\xi_o>$.

Then the runoff response is the random variabl where centered gauge. They concluded that variations in runoff volumes and peak flows "is well above 100 percent over the entire range of storms implying that the spatial resolution of rainfall has a dominant influence on the reliability of computed runoff." It is also noted that "errors in the rainfall input are amplified by the rainfall-runoff transformation" so that "a rainfall depth error of 30 percent results in a volume error of 60 percent and a peak flow error of 80 percent." (Other key quotations regarding other studies are contained in Hromadka and Whitley[1].)

In this paper, the uncertainty in the effective rainfall distribution (i.e., rainfall less losses) over the catchment (R) is considered to be a dominating factor in causing hydrologic modeling errors in discretized models (i.e., subdivision of R into subareas, linked by channel routing). A single area unit hydrograph (UH) model is then used which represents the uncertainty not only in the effective rainfall distribution over R, but also the uncertainty in the catchment hydraulic responses. Both modeling approaches are focused towards the situation where only one stream gauge and one rain gauge are available for data analysis and runoff estimates are needed at the stream gauge site.

Because of the uncertainties present in the catchment response, and the uncertainty in effective rainfall over the catchment for each storm, the modeling output is cast as a probability distribution in order to represent the variability in predicted hydrologic estimates given a design storm or hypothetical storm for study purposes. It is shown that the single area UH model can produce such a distribution in output, which represents the natural variance in the correlation of the available rainfall-runoff data; whereas the discretized model typically misrepresents the available data should the discretized model be used for uncertainty analysis.

HYDROLOGIC MODEL DEVELOPMENT

For modeling development purposes, assumptions are made about the catchment, R, and the storm effective rainfall distributions over R.

The catchment, R, is assumed to be sufficiently drained by a free-flowing collector system such that detention or backwater effects are minor throughout R. The catchment is relatively homogeneous such that it can be subdivided into m nearly homogeneous subareas, R_j, $j = 1,2,\ldots,m$. Channel routing along links are assumed to be nearly translation, with a characteristic travel time for each link, for each storm event, i; consequently, a nonlinear response is modeled by using a different travel time for each storm. (Channel flow routing storage effects can be included by assuming a linear routing technique (Hromadka[3]). However, only translation effects will be considered herein in order to simplify the mathematical notation.) Runoff hydrographs are directly summed at confluence points. Finally, a single stream gauge is

$$[Q_1{}^D(t)] = \int_{s=0}^{t} e_g{}^D(t - s)\, [\eta_o(s)]\, ds$$

where brackets indicate a random variable. Note t
(13) the correlation distributions, $\eta_o{}^i(s)$, are now
random variable, as there is no information in e_g
determines a particular distribution from the
$[Q_1{}^D(t)]$ is the collection of runoff hydrographs
possible outcomes associated to the design storm
rainfall, $e_g{}^D(t)$, applied at the rain gauge site.
the collection of correlation distributions associate
class $<\xi_o>$ where $e_g{}^D(t)$ is considered to be s
similar to the elements in $<\xi_o>$. Because Eq.
prediction, any of the elements in $[\eta_o(s)]$, and hence
are candidates as realization of the stochastic pro
model structure is seen to be a causal linear filter.

The variation in any hydrologic quantity is re
use of Eq. (13). For example, the variation in
estimates at storm time t_o is given by

$$[Q_1{}^D(t_o)] = \int_{s=0}^{t_o} e_g{}^D(t_o - s)\, [\eta_o(s)]\, ds$$

Letting t_p be the time of the peak flow rate (wher
function of the random variable, $[\eta_o(s)]$), the unce
peak flow rate estimates, q_p, is given by

$$[q_p] = Q_1{}^D(t_p) = \int_{s=0}^{t_p} e_g{}^D(t_p - s)\, [\eta_o(s)]\, ds$$

Figure 5 shows the distribution of q_p for a catchme
Angeles, California. The frequency distribution
Fig. 5 is determined by evaluating Eq. (15) using th
distributions shown in Figs. 3 and 4, according to th
dependent probability of occurrence given by Eq. (
scanning the entire distribution of $[\eta_o(s)]$ availabl
class $<\xi_o>$, the q_p frequency distribution is construct

Generally, the available data are insufficient t
highly specific classes of storms, $<\xi_q>$, nor devel
statistical estimates within storm classes, should
class categorization be made. Consequently to procee
uncertainty analysis, another assumption must be inv
approach is to transfer the distribution informati
correlation distributions, $[\eta_q(s)]$, from another
considered hydrologically similar with respect
correlation of rainfall-runoff data, (i.e., regiona
Another approach is to assume the $[\eta_q(s)]$ to be d
identically for "similar" storm classes $<\xi_q>$, and co
several storm rainfall-runoff correlations, $\eta^i(s)$,
same distribution. The first technique may be utilize

catchment, but the second technique obviously requires local rainfall-runoff data.

CONCLUSIONS

Given a catchment and rainfall-runoff data, a flood flow model can only serve as a means to statistically correlate the two forms of data. In a single area UH model, the unit hydrograph serves as the link which correlates the effective rainfall data to runoff data. The single area UH model actually represents a complex link-node model, had subarea hydrologic data been available to evaluate subarea runoff, and had stream gauge data been available to evaluate all link-node hydraulic parameters. Without subarea data, however, the link-node model representation becomes an estimator which is in error due to the approximation of hydraulic parameters and the misrepresentation of the effective rainfall distribution over the entire catchment (i.e., discretization error).

REFERENCES

1. Hromadka II, T.V. and Whitley, R.J., 1988. The Design Storm Concept in Flood Control Design and Planning, Stochastic Hydrology and Hydraulics, in-press.

2. Schilling, W. and Fuchs, L., 1986. Errors in Stormwater Modeling - A Quantitative Assessment, ASCE Journal of Hydraulic Engineering, Vol. 112, No. 2.

3. Hromadka II, T.V., 1988. Back to the Unit Hydrograph Method, PROCEEDINGS: Envirosoft 88 Conference, Computational Mechanics, Porto Carras, Greece.

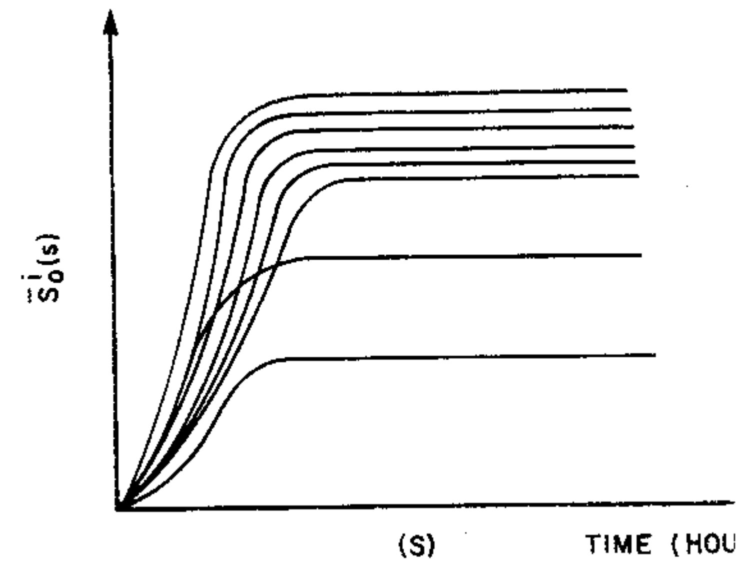

Fig. 1. Example Summation Graphs, $\bar{S}_o^i($ for Storms in Class $<\xi_o>$

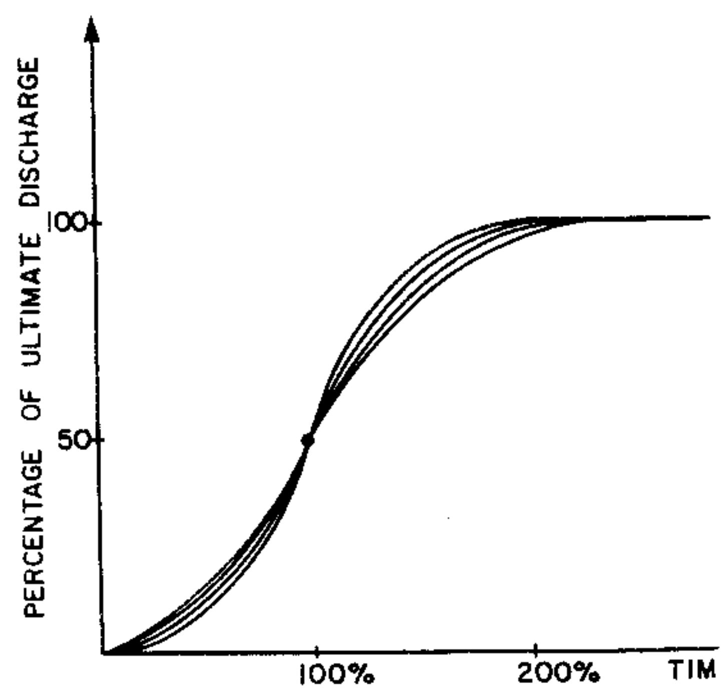

Fig. 2. S-Graphs, $S_o^i(s)$, ~r Storm Cl

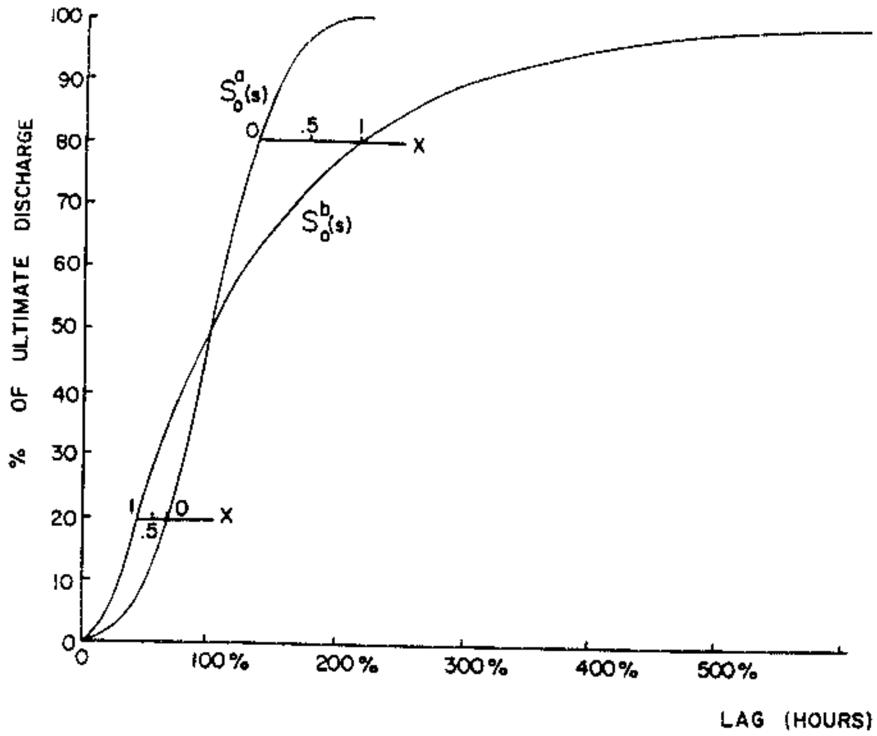

Fig. 3. Definition of S-Graph Parameter X using $S_0^a(s)$ and $S_0^b(s)$

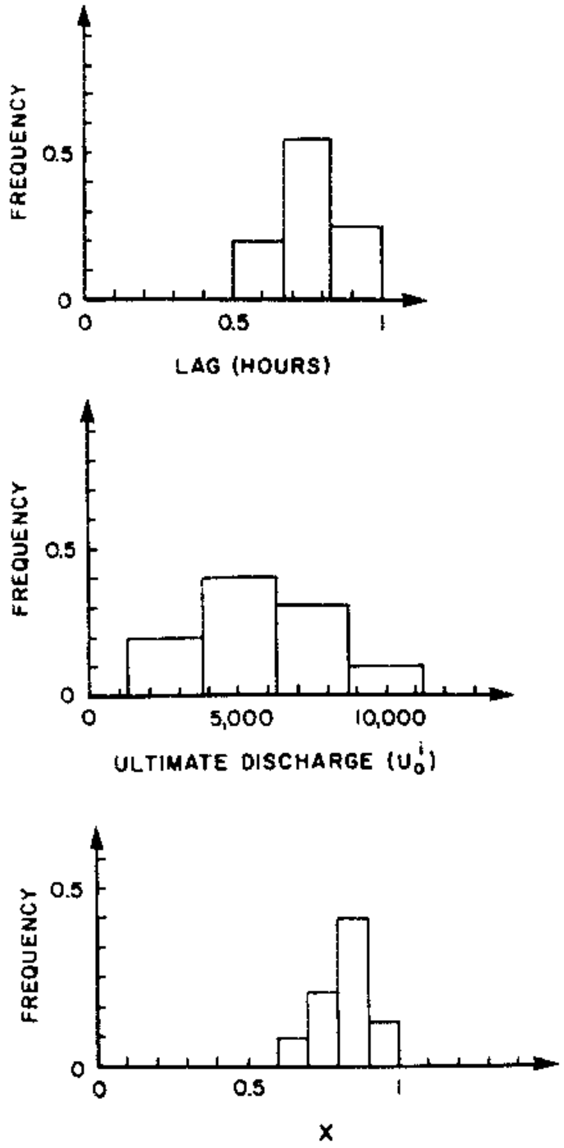

Fig. 4. Marginal Distributions for Vector, η_0^i, co

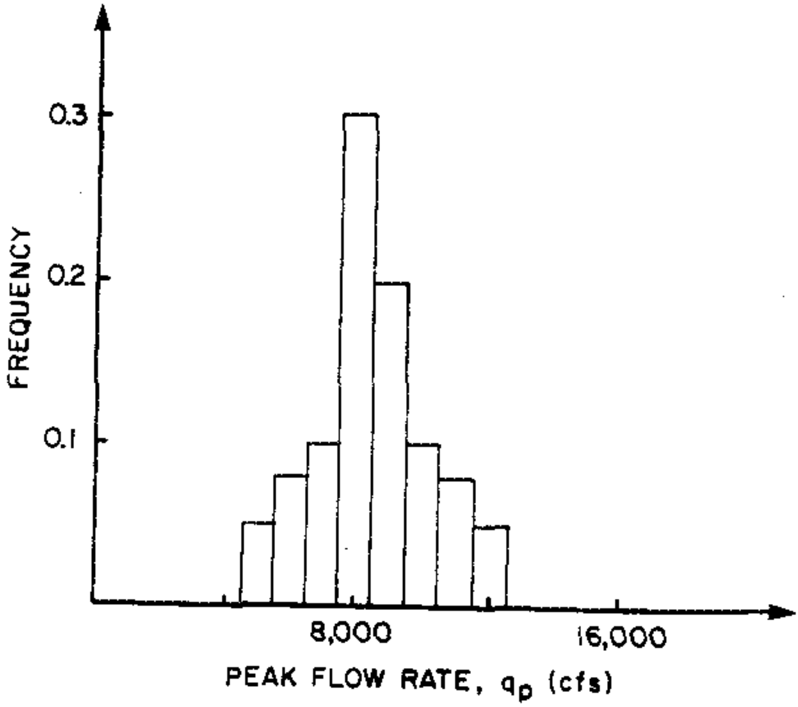

Fig. 5. Uncertainty Distribution for the Estimate of a Peak Flow Rate, q_p.