I IONALIZED PARTIAL-DURATION BALANCED-HYDROGRAPH MODEL

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ABSTRACT: For design problems where annual-event analyses are inappropriate, such as in the case of channel scour due to floods above a threshold, multiple-event analyses with partial duration series should be used. A procedure for calibrating a *partial-duration, balanced-hydrograph model for regional design work is presented.* The model can be used either as a series of synthetic storm events or as part of a simulation model. Data from Los Angeles were analyzed to develop a model for ungaged urban watersheds in southern California.

INTRODUCTION

Although the literature contains many classification systems of hydrologic design models, a simple classification system can be formed using four primary model types: peak discharge models, single-event design hydrograph models, multiple-event design hydrograph models, and continuous hydrograph models. Peak discharge models are typically used for designs where either the maximum flood stage or a maximum discharge rate is the critical design factor; for these design situations, accumulated storage is not relevant to the design. Single-event design hydrograph models are used for designs where both peak discharge and storage during extreme events are critical design factors, such as in designs where spatial routing is necessary or where small retention structures exist within the watershed. Multiple-event design hydrograph models, which, to date, have been used less frequently than the other three model types, are useful for problems where the design involves the accounting of processes that are a function of time-dependent discharge rates above a threshold discharge. A primary example where multiple-event design hydrograph models should be used is channel erosion that occurs when flows are above a critical velocity. Continuous-record models are used in operational hydrology where the continuous accounting of hydrologic processes is important, such as accounting for variation of soil moisture between periods of rainfall.

For design applications where peak discharge estimates are required and a stream gage record exists, a log Pearson type III analysis of the annual maximum series is commonly used ("Guidelines" 1982). The skew of the annual maximum series is believed to vary with location, and the Water Resources Council (WRC) map can be used where the record length of the annual maximum series is inadequate to obtain an accurate estimate of the skew. When a stream gage record of sufficient length is not available, then a regional model, such as the U.S. Geological Survey, state equations, or an empirical formula, such as the rational method, is used.

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For design applications that require a hydrograph, a number of sy etic unit hydrographs are available. These methods commonly use a un. ak-discharge model and make unique assumptions about the distribution of flow rates over the duration of runoff. Unfortunately, many of these unit hydrograph models have been developed from national databases, and they reflect conditions averaged over the country. More accurate designs could be expected within a region if better estimates of the time distribution of runoff rates were available for the region. The time distribution of volumes needs to be balanced over the duration of the hydrograph so that the design hydrograph reflects average storm runoff conditions. For example, the proportion of the volume under the rising limb of the hydrograph is a major characteristic of the balance of a design hydrograph.

For design work where intermittent accounting of processes is required, the multiple-event modeling approach can be used. This would be appropriate where accounting of discharge rates above a threshold is required, such as in cases involving channel degradation or local scour at abutments or bridge piers. In these cases, noticeable scour will not occur during periods of low flow, so continuous accounting would not be needed. However, accurate estimation of the scour will require accounting for all flows above a threshold, not just the annual maximum series or a single-event design hydrograph. This is essentially a partial duration series analysis. Traditionally, the analysis of a partial duration series has been limited to peak discharge rates, but for processes where the duration of discharges above a threshold influences the criterion (e.g. scour), then the traditional concept of a partial duration analysis for flood peaks needs to be extended to flood hydrographs.

A methodology for developing a regional partial-duration, balanced-hydrograph model has not previously been presented. Partial-duration analyses have been conducted on peak-discharge flood series (Langbein 1949). Balanced hydrographs have been applied to individual design discharges (*Hydrologic* 1975), but the two have not been combined. The combined approach is developed herein and applied to a regional sampling of streamgage data from southern California. The resulting model could be used to generate synthetic traces of partial-duration balanced hydrographs.

ANALYSIS PROCEDURE

The generation of a synthetic trace of a partial-duration flood series requires generators for the number of storms and the probability distribution of the flood series. Experience has shown that the number of floods per year can be generated with a Poisson distribution. Experience has also shown that the magnitudes of floods above a threshold in a partial duration series can be represented by an exponential distribution. A balanced-hydrograph model, which uses the flood magnitude from the exponential distribution, requires volume-duration-frequency characteristics to define the ordinates of the hydrograph. Thus, the analysis requires determination of the number of storms, the threshold discharge, the flood distribution, and the ordinates of the balanced hydrograph.

Flood Count and Threshold Determination

To simulate a partial-duration balanced-hydrograph flood series, the base level or threshold discharge must be selected. The number of storms that occur within a certain period of time, usually a year, can then be determined.

TABLE 1. Characteristics of Watersheds and Partial Duration Series

Watershed number (1)	Watershed (2)	Area (sq. mi.) (3)	Period of Record: Peak data (4)	Period of Record: Volume data (5)	Partial Duration Series: Time lag (hrs) (6)	Log standard deviation (7)	Log skew (8)	Mean discharge (cfs) (9)	Standard deviation of discharge (cfs) (10)	Years of record (11)	Average number of storms per year (12)
1	Alhambra Wash near Klingerman St.	15.2	1930–83	1930–37, 40–75, 78, 83	0.6	0.225	−0.3	1,148	917	47	3.89
2	Arcadia Wash below Grand Avenue	8.5	1956–84	1956–67, 74, 75, 78, 83	0.4	0.248	−0.3	1,267	865	16	2.13
3	Compton Creek near Greenleaf Dr.	22.6	1928–83	1929–37, 39–75, 78	1.6	0.247	−0.4	1,033	709	48	3.23
4	Compton Creek at 120th St.	14.5	1952–78	1953–57, 59–68, 73, 78	1.4	0.175	−0.1	1,030	770	22	2.64
5	Dominiquez Channel at Vermont St.	37.1	1967–82	1968–75, 78, 83	1.5	0.188	−0.7	1,280	793	10	2.40
6	Eaton Wash at Loftus Dr.	22.8	1957–89	1957–67, 78, 83	1.0	0.208	0.0	1,255	1,026	14	2.50
7	Rubio Wash at Glendon Way	10.9	1930–83	1930, 33–69, 75, 78, 83	0.5	0.210	−0.4	1,405	922	42	2.67
Mean	—	—	—	—	—	0.21	−0.3	—	—	—	—

known to have occurred during the period of record. These data represent runoff conditions from catchments that have been in a fully developed condition for most of their record lengths. Table 1 provides a summary of the periods of record for both instantaneous peak discharges and volumes of flood hydrographs. The seven long-term stream-flow records were analyzed to develop the regionalized partial-duration balanced-hydrograph model for Southern California. The data were used to model the number of storms to be expected each year, as well as the distribution of storms.

Flood Count and Threshold Estimation

The data set for each watershed was based on a preliminary threshold discharge above which all storms are assumed to be independent; this threshold was set in the collection of the data. The initial thresholds are given in Table 2. The Kolmogorov-Smirnov test was then used to determine whether the annual flood count above these thresholds followed a Poisson distribution. The Poisson parameter, λ, was estimated using the mean number of storms, given in Table 2. Based on the Kolmogorov-Smirnov test, the flood count of each of the seven data sets, with the exception of watershed 3, followed a Poisson distribution (Table 2). Watershed 3 was retested at a different threshold as discussed in the following section.

One of the characteristics of the Poisson distribution is that the mean is equal to the variance, i.e., $\sigma^2/\mu = 1$. This was not the case for the sample statistics of the flood count for the seven data sets. To test the significance of the observed differences, the Fisher dispersion test was used. The test statistic $R = \sigma^2/\mu$ was compared at a 0.05 significance level with the critical chi-square value of (2). The results, given in Table 2, show that the computed values of the statistic R for all watersheds, except watershed 7, were less than the critical values of R_c. Thus, it was concluded that for the Fisher test the initial threshold provided a Poisson distribution for the flood counts of these six watersheds. For watershed 7, R was greater than R_c at a level of significance of 0.01 using the initial threshold; thus, the hypothesis of a Poisson distribution was rejected for this watershed.

Since the hypothesis of a Poisson distribution was rejected for watershed 3 using the Kolmogorov-Smirnov test and the initial threshold discharge (see Table 2), the Kolmogorov-Smirnov test was repeated using the threshold discharge previously determined 17 m^3/s (600 cfs). This time the hypothesis of a Poisson distribution was accepted. The test statistic equaled 0.119 and the critical value was 0.196 at a significance level of 0.05.

At the 0.05 level of significance, the hypothesis of a Poisson distribution for watershed 7 could not be accepted at any threshold for which $\mu > 1$. Since the Poisson parameter is estimated using the mean, it does not make sense to use a value less than 1.0, since this would imply that there is less than one storm per year, which means the partial duration series is approximately the same as the annual series. Brubaker and McCuen (1990) showed that the 5% level of significance may not be optimal for the statistical analysis of engineering data. Therefore, the preceding procedure was repeated, this time at a significance level of 0.01. The Poisson hypothesis was accepted for watershed 7 at a threshold of 28.3 m^3/s (1,000 cfs). The resulting thresholds and mean annual flood count, which is the Poisson parameter, for the watersheds are shown in Table 2.

Synthetic Generation of Storm Magnitudes

Having determined the number of storms, the storm magnitudes of the partial duration series may be represented by an exponential distribution

TABLE 2. Results of Kolmogorov-Smirnov and Fisher Tests for Hypotheses of Poisson Distribution

Watershed number (1)	Initial threshold (cfs) (2)	Initial Poisson parameter (3)	Critical Kolmogorov-Smirnov difference at α = 0.05 (4)	Computed Kolmogorov-Smirnov test statistic (5)	Kolmogorov-Smirnov test decision @ 5% (6)	Fisher Test		Final threshold (cfs) (9)	Final Poisson parameter (10)
						R (7)	Decision (8)		
1	350	3.89	0.198	0.107	Accept H_0	1.48	Accept H_0 @ 1%	350	3.89
2	400	2.13	0.328	0.119	Accept H_0	1.15	Accept H_0 @ 5%	400	2.13
3	350	3.23	0.196	0.311	Reject H_0	1.35	Accept H_0 @ 5%	600	2.31
4	300	2.64	0.284	0.083	Accept H_0	1.41	Accept H_0 @ 5%	300	2.64
5	425	2.40	0.410	0.230	Accept H_0	1.39	Accept H_0 @ 5%	425	2.40
6	350	2.50	0.349	0.100	Accept H_0	1.39	Accept H_0 @ 5%	350	2.50
7	400	2.67	0.210	0.109	Accept H_0	1.92	Reject H_0 @ 1%	1,000	1.50

$$q = -\beta \ln(U) + q_b \qquad (3)$$

where β = exponential parameter; U = uniform random number between 0 and 1; and q_b = base level or threshold discharge. The single-parameter exponential distribution is a relatively simple distribution with a closed-form solution, making it easily adaptable in a simulation model.

The mean of the partial duration flood series is used as an estimate of the parameter of the exponential distribution. However, the fit of the flood distribution may be improved by optimizing the parameter, which was done by comparing the resulting theoretical distribution against the measured data using a chi-square goodness-of-fit test. The parameter that provided the lowest value of the chi-square test statistic was chosen as the optimum parameter. The optimum values are given in Table 3.

To test the resulting exponential distributions under the hypothesis that each distribution followed an exponential distribution, the Kolmogorov-Smirnov test was used at a significance level of 0.05. The threshold discharge values for the distributions were those determined previously from the Poisson tests. The optimized parameters for the exponential distributions were those previously determined using the chi-squared goodness-of-fit test. The results of the Kolmogorov-Smirnov test, also given in Table 3, show that the hypothesis of an exponential distribution was accepted in all cases.

Development of Regional Balanced Hydrographs

For each hydrograph in the flood record, the volume was computed for durations of 5 min, 15 min, and 30 min and 1 hr, 2 hr, 3 hr, 6 hr, 12 hr, and 24 hr. An annual maximum series was created for each duration at each gage and a log Pearson type III analysis was made for each combination. For all stations, the mean of the logarithms decreased, as expected, as the duration increased, with the relationship showing a very consistent decrease. For each watershed the standard deviation and skew showed relatively constant, consistent relationships with duration for durations less than the mean watershed lag, which are given in Table 1. Fig. 1 shows the moments as a function of the duration for the Eaton Wash watershed. For durations longer than the watershed lag, the sample estimates of the standard deviation and skew showed considerable scatter with no consistent trend. Since the scatter of the sample estimates of the standard deviation and skew is small for durations less than the watershed lag, station averages were computed using

TABLE 3. Kolmogorov-Smirnov Testing of Exponential Distribution for Partial-Duration Series

Watershed number (1)	Initial parameter (sample mean) (2)	Critical Kolmogorov-Smirnov			Decision (6)
		Optimized parameter (3)	Difference at α = 0.05 (4)	Kolmogorov-Smirnov test statistic (5)	
1	1,148	752	0.109	0.039	Accept H_0
2	1,267	730	0.232	0.091	Accept H_0
3	1,033	971	0.129	0.106	Accept H_0
4	1,030	909	0.179	0.077	Accept H_0
5	1,280	694	0.275	0.156	Accept H_0
6	1,255	769	0.289	0.156	Accept H_0
7	1,405	1,163	0.171	0.064	Accept H_0

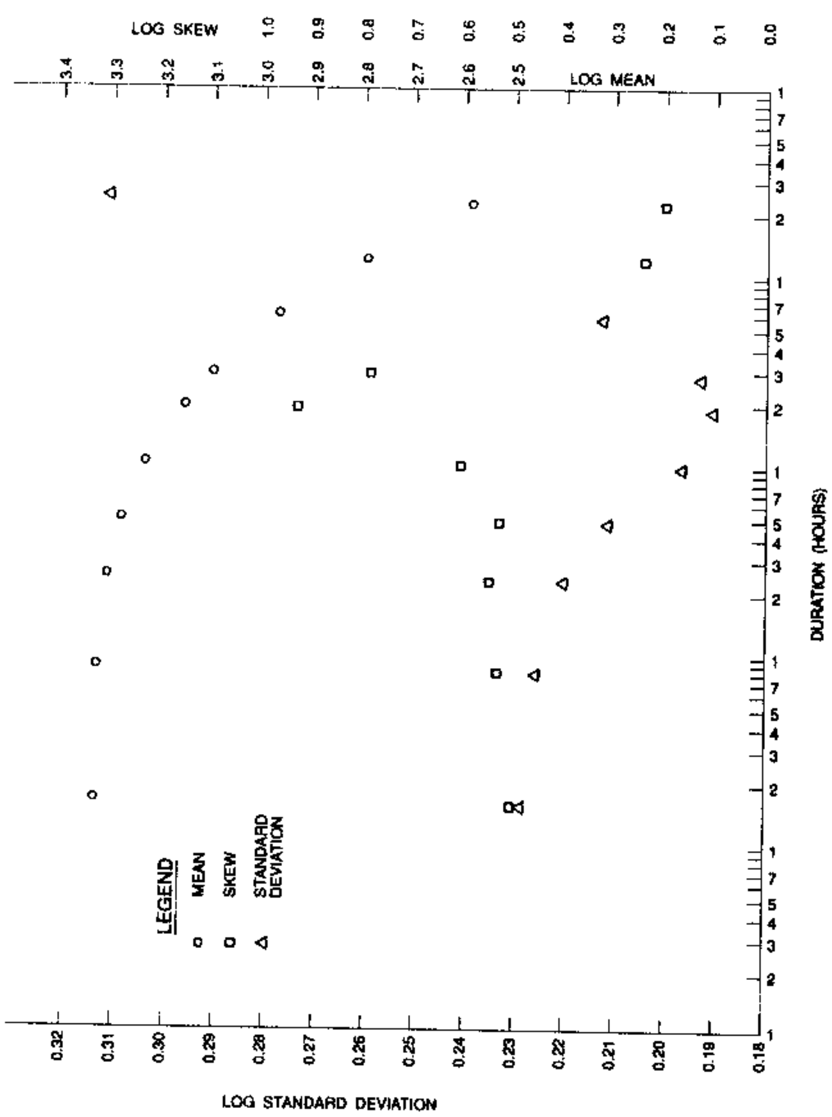

FIG. 1 Variation of Log Mean, Log Standard Deviation, and Log Skew with Storm Duration for Eaton Wash

the computed values for durations less than the lag; these averages also given in Table 1.

When comparing the watershed values for station skews or standard deviations, there was no consistent trend in either the station skews or standard deviations; therefore, average values were computed for the region. Using the sample means and the regionally average standard deviations and skews, frequency curves were developed for each duration. The discharge-frequency curve for Eaton Wash is shown in Fig. 2. The station-average standard deviation and skew were used for durations longer than the watershed lag because of the inconsistency in the sample estimates. Use of the sample estimates would have caused some of the frequency curves for the longer durations to intersect with the frequency curves for shorter durations, which would be irrational.

Balanced design hydrographs can be developed for any storm duration or frequency. Since 24-hr storms are commonly used in design, such storms were developed for each of the seven Los Angeles watersheds for return periods from two years to 200 years. The 24-hr time base is reasonable, since this exceeds the mean watershed lag for all of the sample watersheds as well as for ungaged watersheds where designs are needed. Fig. 3 shows the 10-year and 100-year balanced design hydrographs for Eaton Wash. The time-to-peak of the balanced hydrograph was assumed to occur at a storm time of 8 hr, with one-third of the volume under the rising limb of the hydrograph. The 8-hour time-to-peak was used because the SCS type I and IA rainfall patterns for California are front-loaded; however, other times-to-peak, such as 12 hr, could be used. Unit hydrographs computed from measured storms often have about one-third of the volume under the rising limb, so this fraction was also used for the design hydrograph. The ordinates of the balanced hydrographs of the seven watersheds were relatively constant percentages of the instantaneous peak discharge for each duration; the proportions are given in Table 4 for each duration and each watershed. The mean of the watershed proportions are also given in Table 4. The ordinates for Eaton Wash closely matched the mean proportions. These values could be used for all frequencies not computed in the preceding analysis, such as in the development of synthetic traces of partial-duration balanced-flood hydrographs.

ESTIMATION AT UNGAGED SITES

Estimation of Threshold and Poisson Parameter

To apply the regionalized partial-duration balanced-hydrograph model at ungaged sites in the study region, it is necessary to estimate both the threshold discharge and the Poisson parameter of (3), which is the mean annual flood count. Given the small sample size of long-term continuous records for the region, it was not possible to develop regression relationships to predict the threshold and the Poisson parameter; therefore, mean values were used.

For the Los Angeles County data the threshold appeared to increase with the drainage size. For each of the seven watersheds, the adjusted threshold was divided by the drainage area, which gives a unit threshold in $m^3/s/km^2$ (cfs/mi^2). The mean of the resulting unit thresholds was 0.406 $m^3/s/km^2$ (36.7 cfs/mi^2). Therefore, at ungaged sites in the region the drainage area could be multiplied by the mean unit threshold of 36.7 to obtain an estimate of the threshold discharge for the ungaged watershed.

The mean parameter for the gaged areas may be used as an estimate of

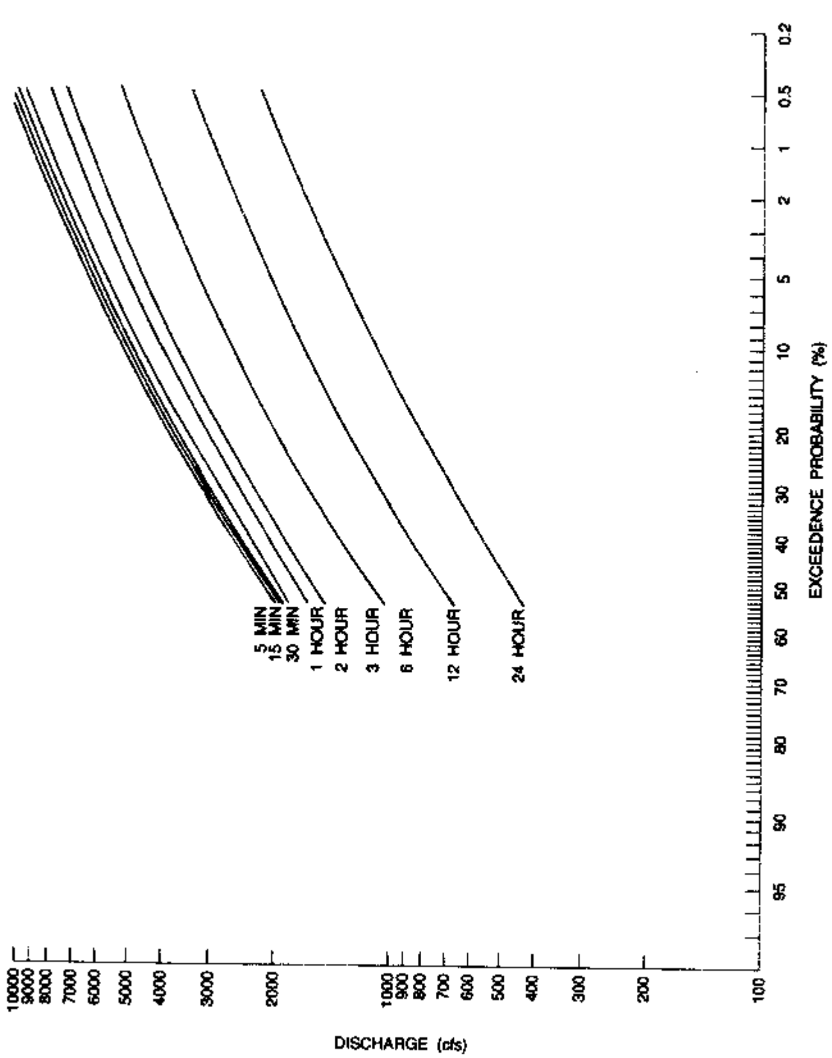

FIG. 2. Discharge-Frequency Curve for Eaton Wash

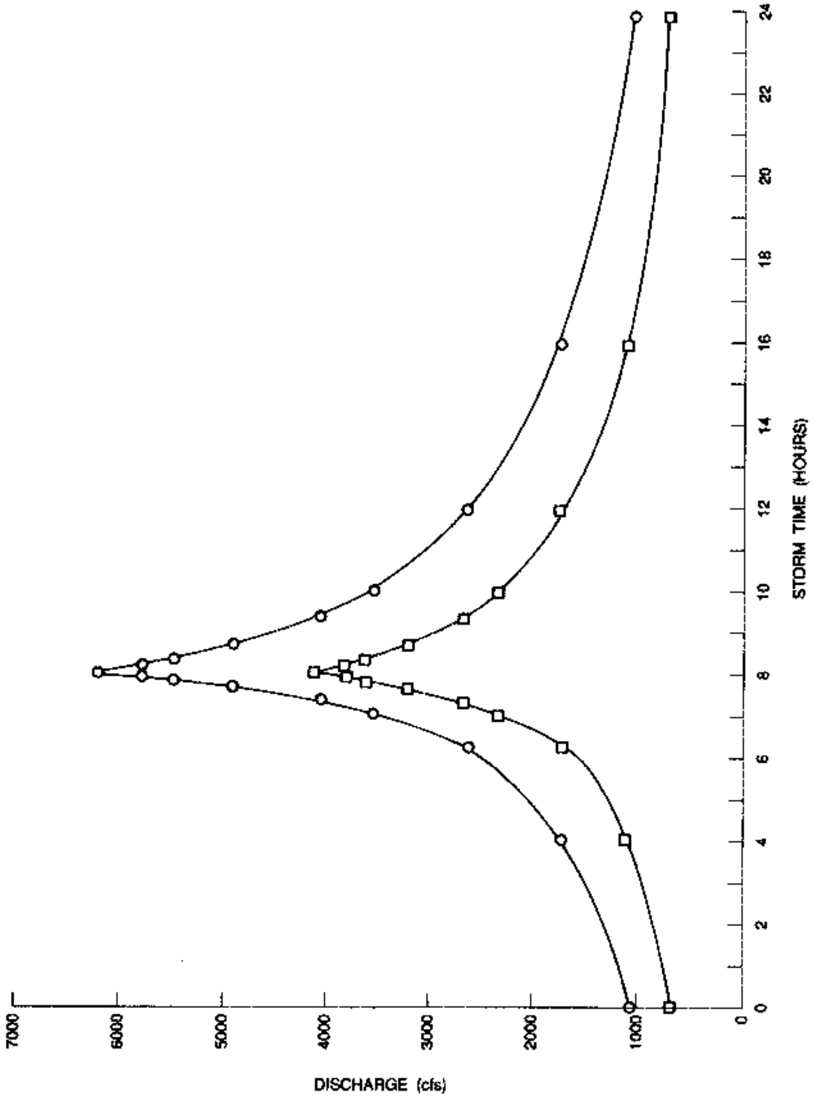

FIG. 3. 10-yr and 100-yr Balanced Hydrographs for Eaton Wash

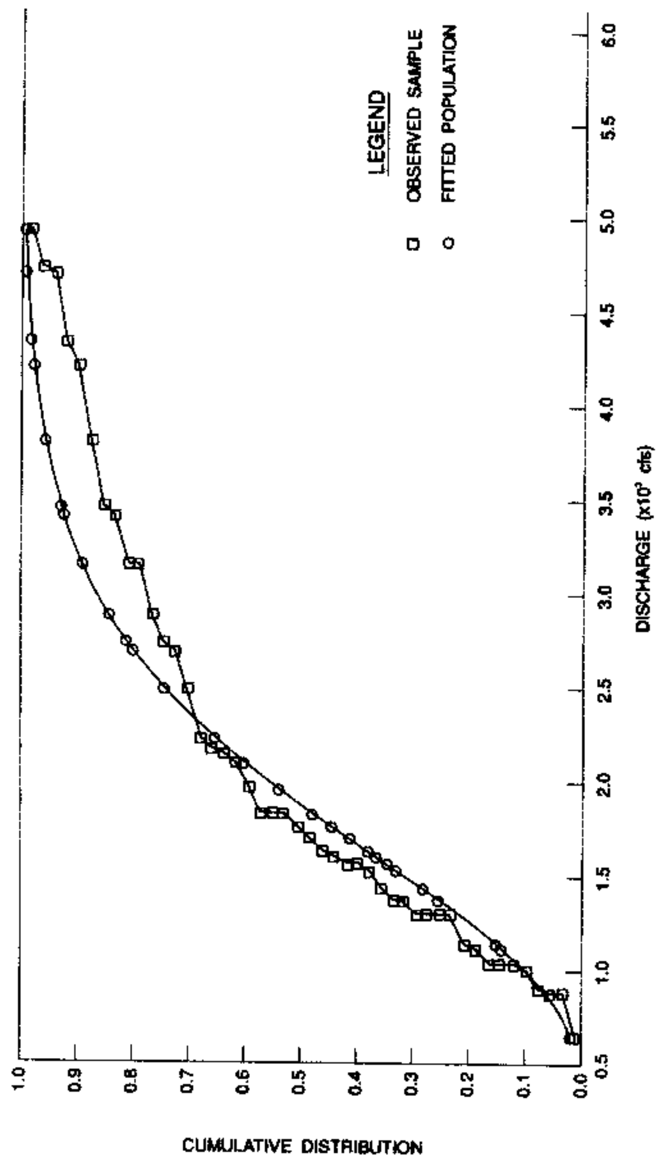

FIG. 4. Sample and Type I Extreme-Value Distributions for Alhambra Wash

TABLE 4. Standardized Volume-Duration Balanced-Hydrograph Ordinates

Duration (1)	Alhambra Wash (2)	Arcadia Wash (3)	Compton Creek 1 (4)	Compton Creek 2 (5)	Dominquez Wash (6)	Eaton Wash (7)	Rubio Wash (8)	Mean (9)
Instant	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5 min	0.97	0.97	0.99	0.99	0.99	0.97	0.97	0.98
15 min	0.92	0.91	0.97	0.97	0.97	0.93	0.92	0.94
30 min	0.86	0.82	0.93	0.94	0.94	0.88	0.85	0.89
1 hr	0.76	0.70	0.86	0.89	0.88	0.79	0.72	0.80
2 hr	0.60	0.58	0.71	0.78	0.76	0.66	0.57	0.67
3 hr	0.51	0.50	0.61	0.69	0.65	0.57	0.49	0.57
6 hr	0.36	0.38	0.43	0.50	0.45	0.42	0.33	0.41
12 hr	0.23	0.24	0.27	0.33	0.27	0.28	0.22	0.26
24 hr	0.14	0.15	0.16	0.20	0.16	0.17	0.13	0.16

the regional mean annual flood count for the Poisson distribution. For the seven watersheds in the region, the average Poisson parameter was 2.3, which is used as the regional value for ungaged sites. Values of 2–3 are commonly used for partial duration series in the United States.

To apply the methodology at ungaged sites in other regions, the preceding procedure would have to be applied at gaged sites, with regionalized values of the threshold discharge and the mean annual flood count developed. For areas where the number of long-term continuous stations are available, it may be possible to develop regional regression equations for the two parameters, rather than using mean values as was necessary for southern California.

Estimation of Exponential Parameter for Flood Magnitudes

The process just described may be repeated for any gaged watershed or group of watersheds within a region. In the absence of gaged data, the exponential parameter must be estimated. One method of estimating the parameter follows.

For the seven watersheds, the annual series was found to follow a type I extreme value distribution. As an example, Fig. 4 shows the theoretical type I distribution and the data points for the Alhambra Wash. From the type I distribution, the parameter for the parent exponential distribution may be derived as (Ang and Tang 1984)

$$\beta = \frac{\sigma\sqrt{6}}{\pi} \quad \quad (4)$$

where σ = standard deviation of the annual series. Eq. (4) provides an estimate of the exponential parameter for ungaged sites in southern California. The same procedure may be followed for other areas outside of the region.

CONCLUSIONS

There is a continual need to improve methods used for hydrologic design. Many policies and design manuals require hydrologic design work to be based on a specific computational method, which is often a method that has been calibrated using a national database. Where adequate data exist, we would expect greater design accuracy from design methods that have been developed for a specific region than from methods based on a national database. Stream-gage records from southern California were analyzed to develop regionalized balanced hydrographs. These hydrographs reflect the volume-duration characteristics of the region.

Many design situations use an annual-maximum discharge hydrograph; however, this approach may not be appropriate for all design problems. Other design problems, such as bridge-pier scour and water-quality assessments, may need to consider cumulative storm effects. For such problems, a partial-duration approach would be more appropriate. Although partial-duration peak discharge analyses are common, the concept has not previously been extended to storm hydrographs. A method for developing a regionalized partial-duration hydrograph model was presented herein and applied to data from southern California, an area that is hydrologically unlike the portions of the U.S. for which the standard design methods were developed.

A third problem that can arise in developing regional hydrologic design manuals is that methods based on national databases may not properly characterize the temporal balance of runoff for a region. For example, unit hydrographs developed for average slopes may be inaccurate when used in either coastal areas or mountain environments. The stream-flow data for southern California were analyzed to develop hydrographs that reflect the balance for the region.

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