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period (1930-1948) resulted in the least runoff estimate; the second, dry period (1948-1966) resulted in a higher runoff estimate, despite reduced precipitation, and the latter wet period (1966-1984), indicated a still higher runoff estimate. The key ingredient in these comparisons is the increasing runoff estimate, regardless of the quantity of the rainfall. As a result the 1965-1984 peak runoff subset was used for evaluation of locations in the Los Angeles River-San Gabriel River where urbanization was an important factor.

(2) Calibrating Rainfall-Runoff Parameters: loss rate parameters were varied for each discrete frequency runoff determination (2-year, 5-year, ... 100-year) in a manner which resulted in the "best composite fit" of subarea discharges to analytical frequency discharges for each frequency. Because of the wide range of relative frequency flows (gauged vs. computed), no general systematic approach could lower the high results, as well as raise the low results. The "best composite fit" concept attempted to produce a normal distribution of relative peak discharges. Ratios of gauged to computed discharges were determined for each discrete frequency for the calibration subareas, and the loss adjustment parameters then modified to produce the most normal distribution of these ratios about 1.0 as the mean. The generalized values of these loss parameters were then input into the HEC-1 rainfall runoff model along with appropriate reconstituted unit graph parameters for each of the 50 LACDA subareas.

Summary

Since the upstream flows are calibrated to observed results, and since simulation of the rainfall-runoff process (including reservoir releases), results in downstream runoff which agrees with observed flows, it is reasonable to expect that the intermediate results are also representative of existing conditions. Furthermore, the modeling process provides these frequency discharges, as well as hydrographs, in a format which allows consistency and the ability to be manipulated while analyzing project alternatives.

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UNCERTAINTY ESTIMATES FOR RAINFALL-RUNOFF MODEL

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Abstract

Almost all rainfall-runoff link-node models in use today involve the subdivision of the catchment into smaller areas, linked together by a system of channel links. These "link-node" hydrologic models represent the flow processes within the channel links by a translation (moving in time) and an attenuation (reduction of the maximum or peak flow rate) of the runoff (floodwater) hydrograph. The runoff in each subarea is based upon the available rainfall data, modified according to an assumed "loss rate" due to soil-infiltration, ponding, evaporation, and other effects. The net effect of all these approximations is a vast spectrum of possible modeling structures. Using a stochastic integral equation, many of these rainfall-runoff models can be represented by a single generalized model that is tractable to analysis of the uncertainty in the model structure.

Introduction

In this paper, the rainfall-runoff model uncertainty problem is addressed by providing a methodology which can be incorporated into almost all rainfall-runoff link-node models. The methodology is based upon the standard theory of stochastic integral equations which has been successfully applied to problems in many scientific areas (e.g., Tsokos and Padgett, 1974, provide a thorough development). The stochastic integral formulation is used to represent the total error between a record of measured rainfall-runoff data and the model estimates.

Stochastic Integgral Equation

Rainfall-Runoff Model Errors

Let M be a deterministic rainfall-runoff model which transforms rainfall data for some event, i, noted by $P_g{}^i(t)$, into an estimate of runoff, $M^i(t)$, by

$$M : P_g{}^i(t) \longrightarrow M^i(t) \quad (1)$$

where t is time. In our problem, rainfall data are obtained from a single rain gauge.

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Let $P_g{}^i(t)$ be the rainfall measured from storm event i, and $Q_g{}^i(t)$ be the runoff measured at the stream gauge. Error terms are defined for an arbitrary storm event i,

$$Q_g{}^i(t) = M^i(t) + E_m{}^i(t) + E_d{}^i(t) + E_r{}^i(t) \quad (2)$$

where $E_m{}^i(t)$ is the modeling error; $E_d{}^i(t)$ is the error in data measurements; and $E_r{}^i(t)$ is the remaining "inexplainable" error.

Let $E^i(t)$ be the total error

$$E^i(t) = E_m{}^i(t) + E_d{}^i(t) + E_r{}^i(t) \quad (3)$$

Because $E^i(t)$ depends on the model M used in Eq. (1),

$$Q_g{}^i(t) = M^i(t) + E_M{}^i(t) \quad (4)$$

where $E_M{}^i(t)$ denotes the conditional dependence of $E^i(t)$, on the given model type M.

For a future storm event D, the $E_M{}^D(t)$ is an unknown realization of a stochastic process distributed as $[E_M{}^D(t)]$ where

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)] \quad (5)$$

Should A be some functional operator on the distribution of possible outcomes (e.g., detention basin volume; peak flow rate; median flow velocity, etc.) of storm event D, then the value of A for storm event D, noted as $A_M{}^D$, is a random variable distributed as $[A_M{}^D]$, where

$$[A_M{}^D] = A[Q_M{}^D(t)] \quad (6)$$

Developing Distributions of Outcomes for Model Estimates

The distribution for $[E_M{}^D(t)]$ may be estimated by using the available samplings of realizations of the various stochastic processes:

$$\{E_M{}^i(t)\} = \{Q_g{}^i(t) - M^i(t)\},\ i = 1,2,\cdots \quad (7)$$

Assuming elements in $\{E_M{}^i(t)\}$ to be dependent upon the "severity" of $Q_g{}^i(t)$, one may partition $\{E_M{}^i(t)\}$ into classes of storms such as mild, major, flood, or others, should ample rainfall-runoff data be available to develop significant distributions for the resulting subclasses.

The second assumption involved is to assume each $E_M{}^i(t)$ is strongly correlated to some function of precipitation, $F^i(t) = F(P_g{}^i(t))$, where F is an operator which includes parameters, memory of prior rainfall, and other factors. Assuming that $E_M{}^i(t_o)$ depends only on the values of $F^i(t)$ for time $t \le t_o$, then $E_M{}^i(t)$ is expressed as a causal linear filter (for only mild conditions imposed on $F^i(t)$), given by the stochastic integral equation (see Tsokos and Padgett, 1974)

$$E_M{}^i(t_o) = \int_{s=0}^{t_0} F^i(t_o - s)\, h_M{}^i(s)\, ds \quad (8)$$

where $h_M{}^i(t)$ is the transfer function between $(E_M{}^i(t),\ F^i(t))$. Other candidates, instead of $F^i(t)$, are $P_g{}^i(t)$, and the model estimates itself, $M^i(t)$.

Given a significant set of storm data, an underlying distribution $[h_M(t)]$ of the $\{h_M{}^i(t)\}$ may be identified, or the $\{h_M{}^i(t)\}$ may be used directly as a discrete distribution of equally-likely realizations,

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)] \quad (9)$$

Combining Eqs. (8) and (9),

$$[Q_M{}^D(t)] = M^D(t) + \int_{s=0}^{t} F^D(t-s)\, [h_M(s)]\, ds \quad (10)$$

For the functional operator A, Eq. (6) is

$$[A_M{}^D] = A[Q_M{}^D(t)] = A\left\{M^D(t) + \int_{s=0}^{t} F^D(t-s)\, [h_M(s)]\, ds\right\} \quad (11)$$

Applications: Development of Total Error Distributions

A Translation Unsteady Flow Routing Rainfall-Runoff Model

Let F be a functional which operates on rainfall data, $P_g{}^i(t)$, to produce the realization, $F^i(t)$, for storm i by

$$F: P_g{}^i(t) \longrightarrow F^i(t) \quad (12)$$

The catchment R is subdivided into m homogeneous subareas, $R = \cup R_j$, such that in each R_j, the effective rainfall, $e_j{}^i(t)$, is simply

$$e_j{}^i(t) = \lambda_j(1 + X_j{}^i)\, F^i(t) \quad (13)$$

where λ_j is a constant proportion factor; and where $X_j{}^i$ is a sample of a random variable.

The subarea runoff is

$$q_j{}^i(t) = \int_{s=0}^{t} e_j{}^i(t-s)\, \phi_j{}^i(s)\, ds = \int_{s=0}^{t} \lambda_j(1 + X_j{}^i)\, F^i(t-s)\, \phi_j{}^i(s)\, ds \quad (14)$$

Unsteady flow routing along channel links is assumed to be pure translation. Thus, each channel link, L_k, has the constant translation time, τ_k. Hence,

$$Q_g{}^i(t) = \sum_{j=1}^{m} q_j{}^i(t - \tau_j)\ ; \quad t \ge \tau_j \quad (15)$$

where $q_j{}^i(t - \tau_j)$ is defined to be zero for negative arguments.

For the above assumptions,

$$Q_g{}^i(t) = \int_{s=0}^{t} F^i(t-s) \left\{ \sum_{j=1}^{m} \lambda_j (1 + X_j{}^i)\, \phi_j{}^i(s - \tau_j) \right\} ds \tag{16}$$

In a final form, the runoff response is

$$Q_g{}^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{m} \lambda_j\, \phi_j{}^i(s-\tau_j)\, ds + \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{m} \lambda_j X_j{}^i \phi_j{}^i(s-\tau_j)\, ds \tag{17}$$

From Eq. (17), the model structure, M, used in design practice is

$$M^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_{j=1}^{m} \lambda_j\, \phi_j{}^i(s-\tau_j)\, ds \tag{18}$$

Then, $Q_g{}^i(t) = M^i(t) + E_M{}^i(t)$ where

$$E_M{}^i(t) = \int_{s=0}^{t} F^i(t-s)\, h_M{}^i(s)\, ds \tag{19}$$

Should the subarea UH all be assumed fixed, (i.e., $\phi_j{}^i(t) = \phi_j(t)$, for all i), as is assumed in practice,

$$M^i(t) = \int_{s=0}^{t} F^i(t-s)\, \phi(s)\, ds \tag{20}$$

where $\phi(s) = \sum_{j=1}^{m} \lambda_j\, \phi_j(s - \tau_j)$. Additionally, the distribution of the stochastic process $[h_M(t)]$ is readily determined for this simple example,

$$[h_M(t)] = \sum_{j=1}^{m} [X_j]\ \lambda_j\, \phi_j(t - \tau_j) \tag{21}$$

where $[h_M(t)]$ is directly equated to the m random variables, $\{X_j, j = 1,2,\cdots,m\}$. Note that the random variables, X_j, may be mutually dependent.

For this example problem, the stochastic integral formulation is

$$[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s)\, \phi(s)\, ds + \int_{s=0}^{t} F^D(t-s)\, [h_M(s)]\, ds \tag{22}$$

Multilinear Unsteady Flow Routing and Storm Classes

The above application is now extended to include the additional assumption that the channel link travel times are strongly correlated to some set of characteristic descriptions of the runoff hydrograph being routed, such as some weighted mean flow rate of the associated hydrograph. For example, the widely used Convex Routing technique (Mockus, 1972) often utilized the 85-percentile of all flows in excess of one-half of the peak flow rate as a statistic used to estimate the routing parameters. Storm classes, $[\xi_z]$, of "equivalent" $F^i(t)$ realizations are defined where all elements of $[\xi_z]$ have the same characteristic parameter set, $C(F^i(t))$, by

$$[\xi_z] = \{F^i(t) \mid C(F^i(t)) = z\} \tag{23}$$

And for all $F^i(t) \in [\xi_z]$, each respective channel link travel time is identical. In the above definition of storm class, z is a characteristic parameter set in vector form.

This extension modifies the previous runoff equations (20) and (21) to be,

$$M^i(t) = \int_{s=0}^{t} F^i(t-s) \sum_j \lambda_j\, \phi_j(s - \tau_j{}^z) ds = \int_{s=0}^{t} F^i(t-s)\, \Phi_z(s) ds;\ F^i(t) \in [\xi_z] \tag{24}$$

where $\Phi_z(s) = \sum_j \lambda_j\, \phi_j(s - \tau_j{}^z)$, and

$$E_M{}^i(t) = \int_{s=0}^{t} F^i(t-s)\, h_{M_z}{}^i(s)\, ds;\ F^i(t) \in [\xi_z] \tag{25}$$

Defining all random processes on a storm class basis,

$$M^i(t) = \int_{s=0}^{t} F^i(t-s)\ \Phi_z(s)\, ds;\ F^i(t) \in [\xi_z] \tag{26}$$

The stochastic process $[h_{M_z}(t)]$ is distributed as

$$[h_{M_z}(t)] = \sum_j [X_j{}^z]\ \lambda_j{}^z\ \phi_j{}^z(s - \tau_j{}^z);\ F^i(t) \in [\xi_z] \tag{27}$$

In prediction,

$$[Q_M{}^D(t)] = M^D(t) + [E_M{}^D(t)];\ F^D(t) \in [\xi_D] \tag{28}$$

where

$$[E_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s)\, [h_M(s)]\, ds;\ F^D(t) \in [\xi_D] \tag{29}$$

A Generalized Multilinear Rainfall-Runoff Model

Each subarea's effective rainfall, $e_j^i(t)$, is now defined to be the sum of proportions of $F^i(t)$ translates:

$$e_j^i(t) = \sum_k \lambda_{jk}(1 + X_{jk}^i) F^i(t - \theta_{jk}^i); \; F^i(t) \in [\xi_z] \tag{30}$$

where X_{jk}^i and θ_{jk}^i are samples of the random variables distributed as $[X_{jk}]$ and $[\theta_{jk}]$, respectively.

The subarea runoff is

$$q_j^i(t) = \int_{s=0}^{t} F^i(t - s) \sum_k \lambda_{jk} (1 + X_{jk}^i) \; \phi_j(s - \theta_{jk}^i)ds \tag{31}$$

The channel link flow routing algorithm is now multilinear with routing parameters defined according to the storm class, $[\xi_z]$ (see Becker and Kundezewicz, 1987, for an analogy based on multilinear approximation of nonlinear routing).

For L links, each with their own respective stream gauge routing data, the above linear routing techniques result in the outflow hydrograph for link number L, $O_L(t)$, being given by

$$O_L(t) = \sum_{\ell_L=1}^{n_L} a_{\ell_L} \sum_{\ell_{L-1}=1}^{n_{L-1}} a_{\ell_{L-1}} \cdots$$

$$\cdots \sum_{\ell_2=1}^{n_2} a_{\ell_2} \sum_{\ell_1=1}^{n_1} a_{\ell_1} I_1(t - \alpha_{\ell_1} - \alpha_{\ell_2} - \cdots - \alpha_{\ell_{L-1}} - \alpha_{\ell_L}) \tag{32}$$

Using an index notation, the channel outflow for L links in the reach, $O_L(t)$, is

$$O_L(t) = \sum_{<\ell>} a_{<\ell>} I_1(t - \alpha_{<\ell>}) \tag{33}$$

For subarea R_j, the runoff hydrograph for storm i, $q_j^i(t)$, flows through L_j links before arriving at the stream gauge and contributing to the total modeled runoff hydrograph, $M^i(t)$. All of the parameters $a^i_{<\ell>}$ and $\alpha^i_{<\ell>}$ are constants on a storm class basis,

$$M^i(t) = \sum_{j=1}^{m} \sum_{<\ell>} a^i_{<\ell>_j} \, q_j^i(t - \alpha^i_{<\ell>_j}) \tag{34}$$

The predicted runoff response for storm event D is distributed as

$$[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t-s) \left(\sum_{j=1}^{m} \sum_{<\ell>} a^i_{<\ell>_j} \sum_k \lambda_{jk}(1+[X_{jk}])\phi_j(s-[\theta_{jk}] - \alpha^i_{<\ell>_j}) \right) ds; \; F^D(t) \in [\xi_D] \tag{35}$$

For any operator, A, on the predicted runoff response of Eq. (35), the outcome of A for storm event $P_g{}^D(t)$ is the distribution $[A_M{}^D]$,

$$[A_M{}^D] = A[M^D(t)] = A(\{[X_{jk}], [\theta_{jk}]\}) \tag{36}$$

<u>Conclusions</u>

A stochastic integral equation that is equivalent to Eq. (35) is simply

$$[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t - s) \, [n(s)] \, ds; \; F^D(t) \in [\xi_D] \tag{37}$$

In prediction, the expected runoff estimate for storm events that are elements of $[\xi_D]$ is

$$E[Q_M{}^D(t)] = \int_{s=0}^{t} F^D(t - s) \, E \, [n(s)] \, ds; \; F^D(t) \in [\xi_D] \tag{38}$$

which is a multilinear version of the well-known unit hydrograph method which is perhaps the most widely used rainfall-runoff modeling approach in use today.

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