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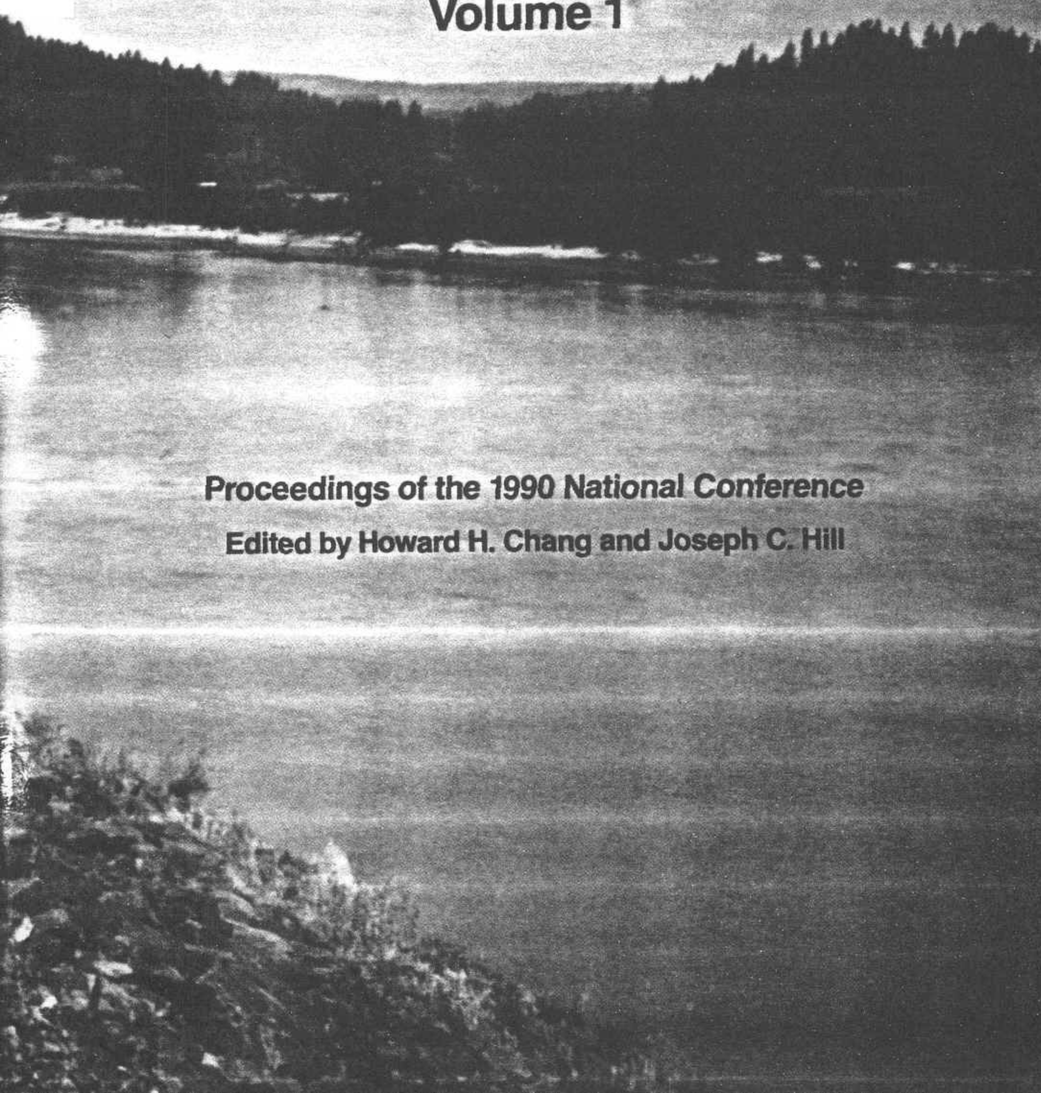

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A REGIONALIZED RAINFALL-RUNOFF STOCHASTIC MODEL

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Abstract

The single area unit hydrograph model is used to develop a distribution model which accommodates the uncertainty in rainfall over the catchment. By categorizing the available rainfall data into storm classes, the model is then calibrated to the rainfall-runoff data to derive a distribution of unit hydrograph correlations on a storm class basis. The unit hydrograph distributions reflect the unknown variations in the effective rainfall over the catchment, among other factors.

Introduction

A single area unit hydrograph (UH) stochastic model, $[Q_1(t)]$, is

$$[Q_1(t)] = \int_{s=0}^{t} e_g(t)\,[\eta(s)]\,ds \tag{1}$$

where $e_g(t)$ is the effective rainfall distribution measured at the rain gauge site; and $[\eta(s)]$ is a distribution of transfer functions between measured rainfall and runoff realizations. In the problem setting, only one rain gauge and stream gauge is available for data synthesis purposes. Additionally, the catchment R is assumed to drain freely to the stream gauge, with negligible detention and backwater effects.

The mathematical underpinnings in the use of Eq. (1) is given in the equation for $\eta^i(s)$ for storm event i,

$$\eta^i(s) = \sum_{j=1}^{m} \sum_{k=1}^{n_j^i} \lambda_{jk}{}^i\, \phi_j{}^i\,(s - \tau_j{}^i - \theta_{jk}{}^i) \tag{2}$$

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where λ_{jk}^i are coefficients; $\phi_j^i(s)$ are subarea UH's for an m-subarea link node model; τ_j^i is the travel time for flow to travel from subarea R_j to the stream gauge; θ_{jk}^i are timesteps; n_j^i is an index number; and all parameters are evaluated on a storm by storm basis. (Equation (2) can be extended to include the effects of channel storage by using a convolution routing technique (Hromadka and Yen, 1988).

Because the rainfall distribution over the catchment R is unknown, any rainfall-runoff model output must necessarily be a function of at least the random variables $[\lambda_{jk}]$, $[\theta_{jk}]$ used in the effective rainfall distribution in R_j given by (for the linear assumption in storm rainfall)

$$e_j^i(t) = \sum_{k=1}^{m} \lambda_{jk}^i \, e_g^i(t-\theta_{jk}^i) \tag{3}$$

Thus in R_j, the unknown effective rainfall is the random variable

$$[e_j(t)] = \sum_{k=1}^{[n_j^i]} [\lambda_{jk}] \, e_g^i(t-[\theta_{jk}]) \tag{4}$$

where brackets are notation for random variables.

Model Application

To apply $[Q_1(t)]$, an effective rainfall model is needed to modify the rain gauge data, $P_g^i(t)$. Such a model is given by $F(P_g^i(t), \{X_i\})$ where the $\{X_i\}$ are parameters to be selected. To proceed, the $P_g^i(t)$ record is categorized into storm classes, $<\xi_q>$, assumed to result in similar effective rainfall distributions, $e_g^i(t)$, at the rain gauge site.

Given a specific class, $<\xi_o>$, there are several associated data pairs $\{e_g^o(t), Q_g^i(t)\}$ where the $Q_g^i(t)$ may differ even though the $e_g^o(t)$ are nearly identical.

The $Q_1^i(t)$ model is now cast in terms of the UH, $\psi^i(s)$, by

$$Q_1^i(t) = \int_{s=0}^{t} W^i \, e_g^i(t-s) \, \psi^i(s) \, ds \tag{5}$$

where $\psi^i(s) = \eta^i(s)/W^i$, and W^i = ratio of measured stream gauge runoff to effective rainfalls.

The stochastic model now is

$$[Q_1(t)] = \int_{s=0}^{t} F(P_g^i(t), \{X_i\}) \, [W^o \psi^o(s)] \, ds \tag{6}$$

where the correlations $W^o \psi^o(s)$ correspond to the fixed $\{X_i^o\}$. It is assumed that the $e_j^i(t)$ are linear in $F(P_g^i(t), \{X_i^o\})$, and the $[W^o\psi^o(s)]$ and $P_g^i(t)$ are in the apppropriate $<\xi_q>$. If not enough data, the $<\xi_q>$ are grouped together to get the single global distribution, $[W\psi(s)]$.

A Regionalized Stochastic Rainfall-Runoff

By developing a regionalized model, studies can be made at ungauged locations assuming that the regionalized distribution of transfer function realizations, $[\eta(s)]$, is transferable.

As a case study, a regionalized stochastic model is developed for the Los Angeles and Orange County area in California. A total of 12 urbanized catchments are considered directly, supplemented by additional data prepared by U.S. Army Corps of Engineers, Los Angeles office. In developing realizations of transfer functions, only severe storms were used for the derivation of the parameters in Eq. (5), (i.e., on a storm class basis). Table 1 depicts the data utilized in this study.

Peak Loss Rate, R_p

The loss function used in Eq. (6) is a simple phi-index which is calibrated on a storm basis with the unit hydrograph. From Table 1, several peak rainfall loss rates, F_p, are tabulated which include two loss rates for double-peak storms. The range of F_p estimates lie between 0.30 and 0.65 inch/hour. Except for Verdugo Wash, $0.20 \le F_p \le 0.60$ which is a variation in values of the order noted for Alhambra Wash along. From Table 1, 92-percent of F_p values are between 0.20 and 0.45 inch/hour, with 80-percent of the values between 0.20 and 0.40 inch/hour. Consequently, a regional mean value of F_p equal to 0.30 inch/hour contains 80-percent of the F_p values, for all watersheds, for all storms, within 0.10 inch/hour.

S-Graph

Each watershed has S-graphs developed for each storm. By averaging the several S-graph ordinates, the average S-graph is obtained. By combining the several watershed average S-graphs (Fig. 1) and weighting the ordinates by the number of storm events, an average of averaged S-graphs is obtained. This regionalized S-graph is an estimate of the expected S-graph for the region. It is noted that the variation in S-graphs for a single watershed for different storms is comparable to the variation between the several catchment averaged S-graphs.

To represent a particular S-graph of the sample set,

$$S(X) = X\,S_1 + (1-X)\,S_2 \tag{7}$$

where S(X) is the S-graph as a function of X, and S_1 and S_2 are two enveloping S-graphs of the data. Figure 2 shows the frequency distribution of X.

TABLE 1.

WATERSHED CHARACTERISTICS

Watershed Name	Watershed Geometry: Area (mi²)	Length (mi)	Length of Centroid (mi)	Slope (ft/mi)	Percent Impervious (%)	Tc (Hrs)	Calibration Results: Storm Date	Peak F_p (inch/hr)	Lag (hrs)	Basin factor
Alhambra Wash[1]	13.67	8.62	4.17	82.4	45	0.89	Feb.78 Mar.78 Feb.80	0.59,0.24 0.35,0.29 0.24	0.62	0.015
Compton2[1]	24.66	12.69	6.63	13.8	55	2.22	Feb.78 Mar.78 Feb.80	0.36 0.29 0.44	0.94	0.015
Verdugo Wash[1]	26.8	10.98	5.49	316.9	20	--	Feb.78	0.65	0.64	0.016
Limekiln[1]	10.3	7.77	3.41	295.7	25	--	Feb.78 Feb.80	0.27 0.27	0.73	0.026
San Jose[2]	83.4	23.00	8.5	60.0	18		Feb.78 Feb.80	0.20 0.39	1.66	0.020
Sepulveda[2]	152.0	19.0	9.0	143.0	24	--	Feb.78 Mar.78 Feb.80	0.22,0.21 0.32 0.42	1.12	0.017
Eaton Wash[1]	11.02[4] (57%)	8.14	3.41	90.9	40	1.05	---	---	---	0.015[7]
Rubio Wash[1]	12.20[5] (3%)	9.47	5.11	125.7	40	0.68	---	---	---	0.015[7]
Arcadia Wash[1]	7.70[6] (14%)	5.87	3.03	156.7	45	0.60	---	---	---	0.015[8]
Compton1[1]	15.08	9.47	3.79	14.3	55	1.92	---	---	---	0.015[8]
Dominguez[1]	37.30	11.36	4.92	7.9	60	2.08	---	---	---	0.015[8]
Santa Ana Delhi[3]	17.6	8.71	4.17	16.0	40	1.73	---	---	---	0.053[9] 0.040[10]
Westminster[3]	6.7	5.65	1.39	13	40		---	---	---	0.079[9] 0.040[10]
El Modena-Irvine[3]	11.9	6.34	2.69	52	40	0.78	---	---	---	0.028[9]
Garden Grove-Wintersberg[1]	20.8	11.74	4.73	10.6	64	1.98	---	---	---	---
San Diego Creek[1]	36.8	14.2	8.52	95.0	20	1.39	---	---	---	---

1: Watershed Geometry based on review of quadrangle maps and LACFCD storm drain maps.
2: Watershed Geometry based on COE LACDA Study.
3: Watershed Geometry based on COE Reconstitution Study for Santa Ana Delhi and Westminster Channels (June, 1983).
4: Area reduced 57% due to several debris basins and Eaton Wash Dam reservoir, and groundwater recharge ponds.
5: Area reduced 3% due to debris basin.
6: Area reduced 14% due to several debris basins.
7: 0.013 basin factor reported by COE (subarea characteristics, June, 1984).
8: 0.015 basin factor assumed due to similar watershed values of 0.015.
9: Average basin factor computed from reconstitution studies
10: COE recommended basin factor for flood flows.

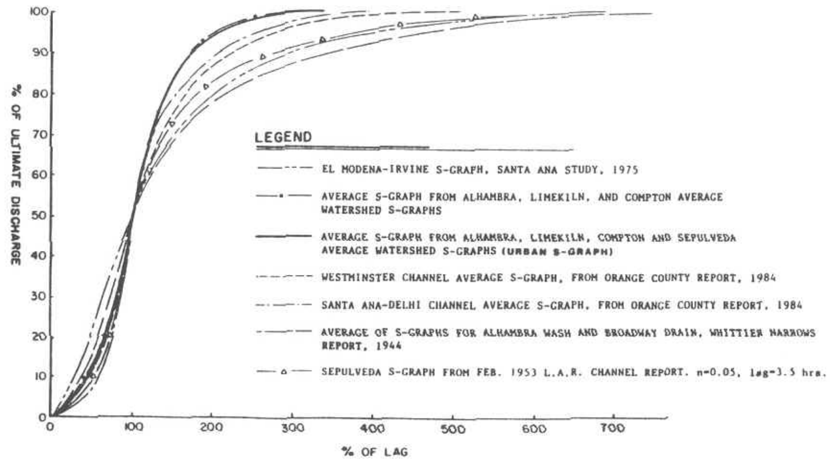

Figure 1. Average S-Graphs.

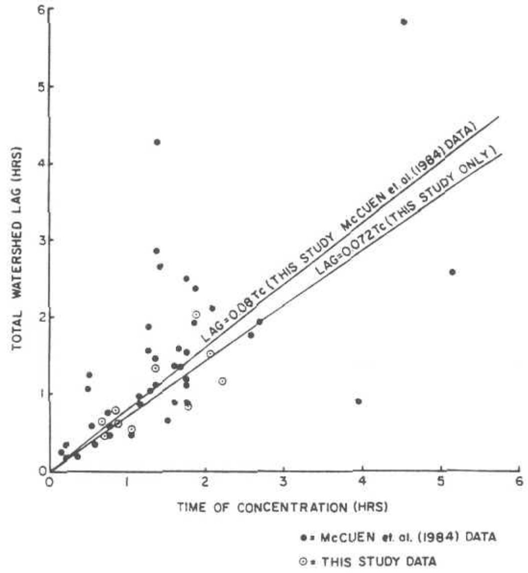

Figure 3. Relationship Between Measured Catchment Lag and Computed Tc.

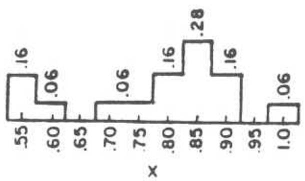

Figure 2. Frequency-Distribution for S-Graph Parameter, X.

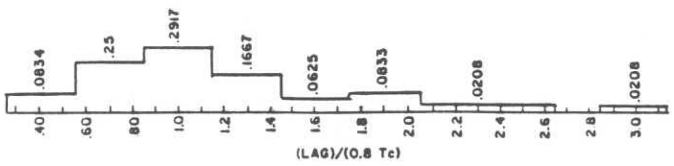

Figure 4. Frequency-Distribution for (LAG)/(0.8 Tc).

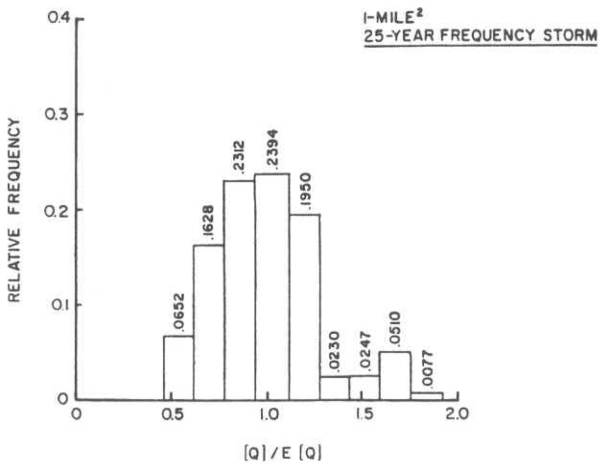

Figure 5. [Q]/E[Q] Distribution for Tc = 1-hour, Area = 1-mi.^{2}.

Catchment Lag

In Figure 1, the Urban S-graph, which represents a regionalized expected S-graph for urbanized watersheds in valley type topography, has an associated X value of 0.85. When the Urban S-graph is compared to the standard SCS S-graph, a striking similarity is evident. Consequently, it was assumed that catchment lag (COE definition) is related to the catchment time of concentration, Tc, as is typically assumed in the SCS mixed-velocity approach.

McCuen et al (1984) provide measured lag values and mixed velocity Tc estimates which, when lag is modified according to the COE definition, can be plotted with the local data such as shown in Figure 3. A least-squares best fit results in:

$$\text{Lag} = 0.80\text{Tc} \tag{8}$$

Adopting a log of 0.80 Tc as the expected value estimator for lag, the distribution of (lag/Tc) values is shown in Figure 4.

The Regionalized Distribution Model, $[Q_1(t)]$

Each of the model parameters are assumed to have probability distribution functions developed from the data.

To evaluate the model distribution, $[Q_1(t)]$, a simulation that exhausts all combinations of parameter values shown in the pdf distribution can be prepared. Because the lag/Tc plot is a function of Tc, several Tc values must be assumed and lag values varied probabilisticaly. An important hydrologic output is the peak flow rate, Q. The distribution of [Q]/E[Q] is shown in Figure 5 for the case of Tc equal to 1 hour and a watershed area of one square mile (hence, depth-area adjustments are not involved). In the figure, [Q] is the distribution of possible model peak flow rate estimates, and $E[Q_m]$ is the peak flow rate obtained from the model using the expected parameters of lag equal to 0.8 Tc, F_p equal to 0.30 inch/hour, and X equal to 0.85 (Urban S-graph).

Conclusions

The single area unit hydrograph model is used to develop a stochastic rainfall-runoff model which accommodates the uncertainty in rainfall over the catchment. By categorizing the available rainfall data into storm classes, the model is then calibrated to the rainfall-runoff data to derive a distribution of unit hydrograph correlations on a storm class basis. The unit hydrograph distributions reflect the unknown variations in the effective rainfall over the catchment, among other factors.

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