# THE DESIGN STORM IN FLOOD CONTROL DESIGN AND PLANNING

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Abstract

The design storm approach, where the subject criterion variable is evaluated by using a synthetic storm pattern composed of identical return pattern input, is shown to be an effective approximation of a considerably more complex probabilistic model. The single area unit hydrograph technique is shown to be an accurate mathematical model of a highly discretized catchment with linear routing for channel flow approximation, and effective rainfalls in subareas which are linear with respect to effective rainfall output for a selected "loss" function. The use of simple "loss" function which directly equates to the distribution of rainfall depth-duration statistics (such as a constant fraction of rainfall, or a $\delta$-index model) is shown to allow the pooling of data and thereby provide a higher level of statistical significance (in estimating T-year outputs for a hydrologic criterion variable) than use of an arbitrary "loss" function. The design storm unit hydrograph approach is shown to provide the T-year estimate of a criterion variable when using rainfall data to estimate runoff.

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## RAINFALL-RUNOFF MODEL OPERATOR FREQUENCY DISTRIBUTIONS

### Probabilistic Distribution Concept

In a Volterra-integral rainfall-runoff model, the correlation distribution for storm event i, $\eta^i(s)$, includes all the uncertainty in the effective rainfall distribution over R, as well as the uncertainty in the several flow routing processes, for the given assumptions about the catchment's runoff response. That is, $\eta^i(\cdot)$ is a realization of the stochastic process, $[\eta(\cdot)]$, where

$$\eta^i(s) = \sum_{j=1}^{m} \sum_{<k>} a^i{}_{<k>j} \; \Sigma \; \lambda_{jk}(1+X_{jk}{}^i) \; \phi_j{}^i \left\{ s - \theta_{jk}{}^i - \alpha^i{}_{<k>j} \right\} \quad (1)$$

for storm event i, which is an element of some storm class $[\xi_z]$.

Although use of the $[\eta(\cdot)]$ realizations combines the uncertainties of both the effective rainfalls and also the channel routing and other processes, Eq. (1) is useful in motivating the use of the probabilistic distribution concept in design and planning studies for all hydrologic models, based on just the magnitude of the uncertainties in the effective rainfall distribution over R. That is, although one may argue that a particular model is "physically based" and represents the "true" hydraulic response distributed throughout the catchment, the uncertainty in model input still remains and is not reduced by increasing hydraulic routing modeling complexity. Rather, the uncertainty in input is reduced only by the use of additional rainfall-runoff data. In Eq. (1), the use of mean value parameters for the routing effects implicitly assumes that the variations in storm parameters of $[X_{jk}]$ and $[\theta_{jk}]$ are not so large such as to develop runoff hydrographs which cannot be modeled by a single set of linear routing parameters on a channel link-by-link basis, for a given storm class.

The Distribution of The Criterion Variable

Let R be a free-draining urban catchment without significant detention effects (e.g., dams, etc.), nor baseflow, with a single stream gauge and rain gauge for data analysis purposes. The goal is to develop estimates of rare occurrence values of a runoff criterion variable (or operator), $A$, evaluated at the stream gauge site. Examples of $A$ are the peak flow rate, or a detention basin peak volume for a given outlet structure located at the stream gauge. Thus, $A$ is the peak demand value of a hydrologic variable from a given runoff hydrograph, evaluated at the stream gauge site.

For simplicity, let all the effects of one year's precipitation be identified with an annual storm event $P_i(\cdot)$; the underlying probability space is then the space of all such annual storms. Event $P_i(\cdot)$ may have a duration of a few hours or a few weeks in order to include all the precipitation assumed to be of importance in correlating the event to the stream gauge measured runoff, $Q_i(\cdot)$.

The criterion variable of interest is noted by $A_i$ for annual event i where

$$A_i = A(Q_i(\cdot)) \tag{2}$$

where $A(Q_i(\cdot))$ is notation for finding the peak value of the demand resulting from the entire runoff hydrograph $Q_i(\cdot)$, and where each $A$ is evaluated at the stream gauge site; for example, peak discharge is max $(Q_i(t)$: t real) and volume of discharge is $\int Q_i(t)dt$.

The distribution $[A]$ can be estimated from a finite sample $A_1$, $A_2$, $A_3$, and this empirical distribution can be used to obtain the desired T-year return frequency estimates, $A_T$, of the criterion variable where by definition of exceedance probability,

$$P(A_i \geq A_T) = \frac{1}{T}, \text{ for } T > 1 \tag{3}$$

It is noted that $A_i$ is the peak demand value of the criterion variable, $A$, for year i; and $A^i$ is the peak demand of $A$ from arbitrary storm event i.

<u>Sequence of Annual Base Inputs</u>

With only a single rain gauge available, all rainfall-runoff models must operate on the annual precipitation events $P_i(\cdot)$. The notation of "effective rainfall" will be generated in the following.

Let F be a function on the precipitation measured at the rain gauge:

$$F: P_i(\cdot) \longrightarrow F_i(\cdot) \tag{4}$$

such that $F_i(\cdot)$ is a nonnegative, bounded piecewise continuous function of time t. For example,

$$F: P_i(t) \longrightarrow P_i^2(t); \; F: P_i(t) \longrightarrow \int_{s=0}^{t} P_i(s)\, ds. \tag{5}$$

The rainfall-runoff model, M, is used to correlate the synthetic "effective rainfall" $F_i(\cdot)$ to the measured runoff, $Q_i(\cdot)$. Note that $F_i(\cdot)$ depends very strongly on the mapping F chosen.

Thus for the multilinear rainfall-runoff model, M, the base input, $F_i(\cdot)$, and the correlation distribution, $\eta_i(\cdot)$, are used to equate with $Q_i(t)$ by

$$M: \; < F_i(\cdot), \; \eta_i(\cdot) > \; \longrightarrow Q_i(\cdot) \tag{6}$$

where $F_i(\cdot)$ must not be strictly zero where $Q_i(\cdot)$ is not strictly zero.

Letting $\{ P_i(\cdot), \; i = 1,2,\cdots \}$ be the sequence of annual rainfall events measured at the rain gauge, then the function F transforms the rainfall data into the sequence of annual base inputs,

$$F: \{ P_i(\cdot), \; i = 2,2,\cdots \} \longrightarrow \{F_i(\cdot); \; i = 1,2,\cdots\} \tag{7}$$

## Base Input Peak Duration Analysis

Given the base input, $F_i(\cdot)$, let $I_\delta$ be the operation of locating the $\delta$-time interval of peak area in $F_i(\cdot)$. Then (see Fig. 1)

$$I_\delta : F_i(\cdot) \longrightarrow F_i^{\delta}(\cdot) \tag{8}$$

where $F_i^{\delta}(t) \equiv 0$ for all $t \notin I_\delta$ ; $F_i(t) = F_i(t)$ for $t \in I_\delta$ ; and where $\delta > 0$. It is noted that $I_\delta$ is also used as the notation for the peak interval itself.

The contribution to $Q_i(\cdot)$ from $F_i^{\delta}(\cdot)$ is determined by

$$Q_i^{\delta}(t) = \int_{s=0}^{t} F_i^{\delta}(t - s) \, \eta_i(s) \, ds \tag{9}$$

And the contribution of $A_i$ from $F_i^{\delta}(\cdot)$ is

$$A_i^{\delta} = \quad (Q_i^{\delta}(\cdot)) \tag{10}$$

Criterion Variable Distribution

From the above

$$A_i = A\left\{\int_{s=0}^{t} F_i(t-s)\ \eta_i(s)\,ds\right\} \qquad (11)$$

and

$$A_i^{\delta} = A\left\{\int_{s=0}^{t} F_i^{\delta}(t-s)\ \eta_i(s)\,ds\right\} \qquad (12)$$

where $F_i^{\delta}(\cdot) \longrightarrow F_i(\cdot)$ as $\delta\uparrow$ (i.e., as $\delta$ increases from zero). Then $A_i^{\delta} \longrightarrow A_i$ as $\delta\uparrow$ where reasonable assumptions of continuity of $A$ are assumed. The fact that $A_i^{\delta} \longrightarrow A_i$ as $F_i^{\delta}(\cdot) \longrightarrow F_i(\cdot)$ will be used in the following to identify the properties of the operator, F, which are involved in the estimates of T-year values of the distribution of annual outcomes, [A].

The base input $F_i^{\delta}(\cdot)$ is written as the sum of components $\bar{F}_i^{\delta}(\cdot)$ and $\Delta F_i^{\delta}(\cdot)$ where (Fig. 2)

$$\bar{I}(F_i^{\delta}(\cdot)) = \frac{1}{\delta}\int_{s=0}^{\infty} F_i^{\delta}(s)\,ds = \frac{1}{\delta}\int_{I_\delta} F_i(s)\,ds, \qquad (13)$$

$$\bar{F}_i^{\delta}(t) = \begin{cases} \bar{I}(F_i^{\delta}(\cdot)), & \text{for } t \in I_\delta \\ 0, & \text{otherwise} \end{cases} \qquad (14)$$

$$\Delta F_i^{\delta}(t) = \begin{cases} F_i^{\delta}(t) - \bar{F}_i^{\delta}(t), & t \in I_{\delta} \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

From Eq. (13), T-year return frequency values of $\bar{I}(F_i^{\delta}(\cdot))$ are denoted by $\bar{I}_T^{\delta}$, where (see Fig. 3)

$$P(\bar{I}(F_i(\cdot)) \geq \bar{I}_T^{\delta}) = \frac{1}{T} \tag{16}$$

We are interested in the "average shape" of the base inputs which have, for a given $I_{\delta}$, the same total mass of input. To proceed, the entire collection of realizations $F_i^{\delta}(\cdot)$ are translated in time to begin at a reference time $t = 0$. Thus each $F_i^{\delta}(\cdot)$ is zero except (possibly) for time $0 \leq t \leq \delta$. Given peak duration time increment $\delta$, the $F_i^{\delta}(\cdot)$ are further categorized according to similar total mass. Thusly, although several $F_i^{\delta}(\cdot)$ realizations have similar total mass, they differ in their time distributions of mass. We want the expected shape of the $\Delta F_i^{\delta}(\cdot)$ in each grouping of similar mass and define

$$E_i^{\delta}(\cdot) = E\left\{\Delta F_j(\cdot) \mid \bar{I}(F_j^{\delta}(\cdot)) = \bar{I}(F_i^{\delta}(\cdot))\right\} \tag{17}$$

It is recalled that in Eq. (17), each $\Delta F_j^{\delta}(\cdot)$ has been translated appropriately in time (Fig. 4), and the expectation is taken with respect to time, t.

Define $\varepsilon_i^{\delta}(t)$ by

$$\varepsilon_i^{\delta}(t) = \Delta F_i^{\delta}(t) - E_i^{\delta}(t) \tag{18}$$

where $\varepsilon_i^{\delta}(t)$ is the variation in base input shape about the expected base input shape of all base inputs with the same mass (approximately) as $F_i^{\delta}(\cdot)$, (Fig. 4).

Then in summary, with all components appropriately translated in time,

$$Q_i^{\delta}(t) = \int \left[\bar{F}_i^{\delta}(s) + E_i^{\delta}(s) + \varepsilon_i^{\delta}(s)\right] \eta_i(t-s)\, ds \tag{19}$$

where $\bar{F}_i^{\delta}(\cdot)$ is the mean intensity of the base input, $F_i^{\delta}(\cdot)$, over the time interval $0 \leq t \leq \delta$ (where $F_i^{\delta}(\cdot)$ has been translated to begin at time $t = 0$); $E_i^{\delta}(\cdot)$ is the expected shape of all possible $\delta$-interval peak durations of base inputs with the same total mass of $F_i^{\delta}(\cdot)$; $\varepsilon_i^{\delta}(\cdot)$ is the variation of $\Delta F_i^{\delta}(\cdot)$ about the expected shape, $E_i^{\delta}(\cdot)$; and $\eta_i(\cdot)$ is the necessary multilinear model correlation distribution for the parent annual event $F_i(\cdot)$, in some storm class $[\xi_z]$.

Estimation of T-Year Values of The Criterion Variable

For each peak duration, $I_{\delta}$, the samples of $F_i^{\delta}(\cdot)$ (see Eq.s (14) and (15)) are now analyzed to determine the underlying distribution of the annual outcomes of the values, $\bar{I}(F_i^{\delta}(\cdot))$. From these distributions of mean intensity of I base inputs, T-year values, $\bar{I}_T^{\delta}$ of the $\bar{I}(F^{\delta}(\cdot))$ can be derived (Fig. 3) and the unique T-year $\bar{F}_T^{\delta}(\cdot)$ defined by:

$$\bar{I}(\bar{F}_T^{\delta}(\cdot)) = \bar{I}_T^{\delta} \tag{20}$$

Given $\bar{I}_T^{\delta}$, $\bar{F}_T^{\delta}(\cdot)$ is defined and also both the corresponding $E_T^{\delta}(\cdot)$ and the distribution $[\varepsilon_T^{\delta}(\cdot)]$. The "T-year $I_{\delta}$ base input", $S_T^{\delta}(\cdot)$, is defined as

$$S_T^{\delta}(\cdot) = \bar{F}_T^{\delta}(\cdot)\, E_T^{\delta}(\cdot) \tag{21}$$

Fig. 5 shows a set of $S_T^{\delta}(\cdot)$ for T = 100 years, and various $\delta$, using the data from Southern California. The T-year $I_{\delta}$ base input, $S_T^{\delta}(\cdot)$, varies in both shape and mass as either T or $\delta$ varies. The distribution $[Q^{\delta}(\cdot)]$ of realizations of $Q_i^{\delta}(\cdot)$ is now written

$$[Q^{\delta}(t)] = \int_{s=0}^{t} \left\{ \bar{F}_T^{\delta}(s) + E_T^{\delta}(s) + [\varepsilon_T^{\delta}(s)] \right\} [\eta_Z(t-s)]\, ds \tag{22}$$

where in Eq. (22), return frequency, T, is allowed to vary as a real valued positive (nonzero) random variable; and $[\eta_Z(\cdot)]$ is the distribution of realizations, $\eta_i(\cdot)$, when the parent $F_i(\cdot) \in [\xi_Z]$, (that is, there may be several correlation realizations associated to the single realization of $F_i(\cdot)$). The distribution $[\xi_T^{\delta}(\cdot)]$ follows from Eqs. (17), (18), and (20).

Combining Eqs. (21) and (22),

$$[Q^{\delta}(t)] = \int_{s=0}^{t} \left\{ S_T^{\delta}(s) + [\varepsilon_T^{\delta}(s)] \right\} [\eta_Z(t-s)]\, ds \tag{23}$$

and, for operator $A$, Eq. (23) is used to provide the frequency distribution,

$$[A^{\delta}] = A[Q^{\delta}(\cdot)] \tag{24}$$

Figure 6 shows a flow-chart which implements the procedures leading to Eq. (24). Because $A_i^{\delta} \longrightarrow A_i$ as $\delta\uparrow$, then necessarily $[A^{\delta}] \longrightarrow [A]$ as $\delta\uparrow$.

T-Year Estimate Model Simplifications

Equation (23) can be considerably simplified if it is assumed that

$$[A^{\delta}] \approx A[E(Q^{\delta}(\cdot))] \tag{25}$$

in which case $E[\varepsilon_T^{\delta}(\cdot)] = 0$ and $E[\eta_Z(\cdot)] = \eta_Z(\cdot)$, and Eqs. (23) and (25) can be combined as

$$[A^{\delta}] \approx A\left\{\int_{s=0}^{t} S_T^{\delta}(t-s)\ \eta_Z(s)\,ds\right\} \tag{26}$$

where T is the annual series random variable. If furthermore it is assumed that the storm classes of base input, $[\![\xi_Z]\!]$, are highly correlated to T-year values of base input mean intensity, then storm classes of T-year base input can be defined, $[\![\xi_T]\!]$, Eq. (26) becomes

$$[A^{\delta}] \approx A\left\{\int_{s=0}^{t} S_T^{\delta}(t-s)\ \eta_T(s)\,ds\right\} \tag{27}$$

where T varies as an independent random variable. Finally, if it is assumed that the T-year value of $[A^{\delta}]$ monotonically increases as T-increases in Eq. (27), then the $T_o$ return frequency value of $A$ is

$$A_{T_o} \approx \max_{\delta} A\left\{\int_{s=0}^{t} S_{T_o}^{\delta}(t-s)\ \eta_{T_o}(s)\,ds\right\}, \text{ as } \delta\uparrow \tag{28}$$

where $\eta_{T_o}(\cdot)$ is the expected realization of a multilinear surface runoff model response $[\eta(\cdot)]$ corresponding to storm class $[\![\xi_T]\!]$. Equation (28) is a form of the well-known design storm single area unit hydrograph procedures (e.g., reference 1).

## References

1. Hromadka II, T.V., McCuen, R.H., and Yen, C.C., Computational Hydrology in Flood Control Design and Planning, Lighthouse Publications, 1987.

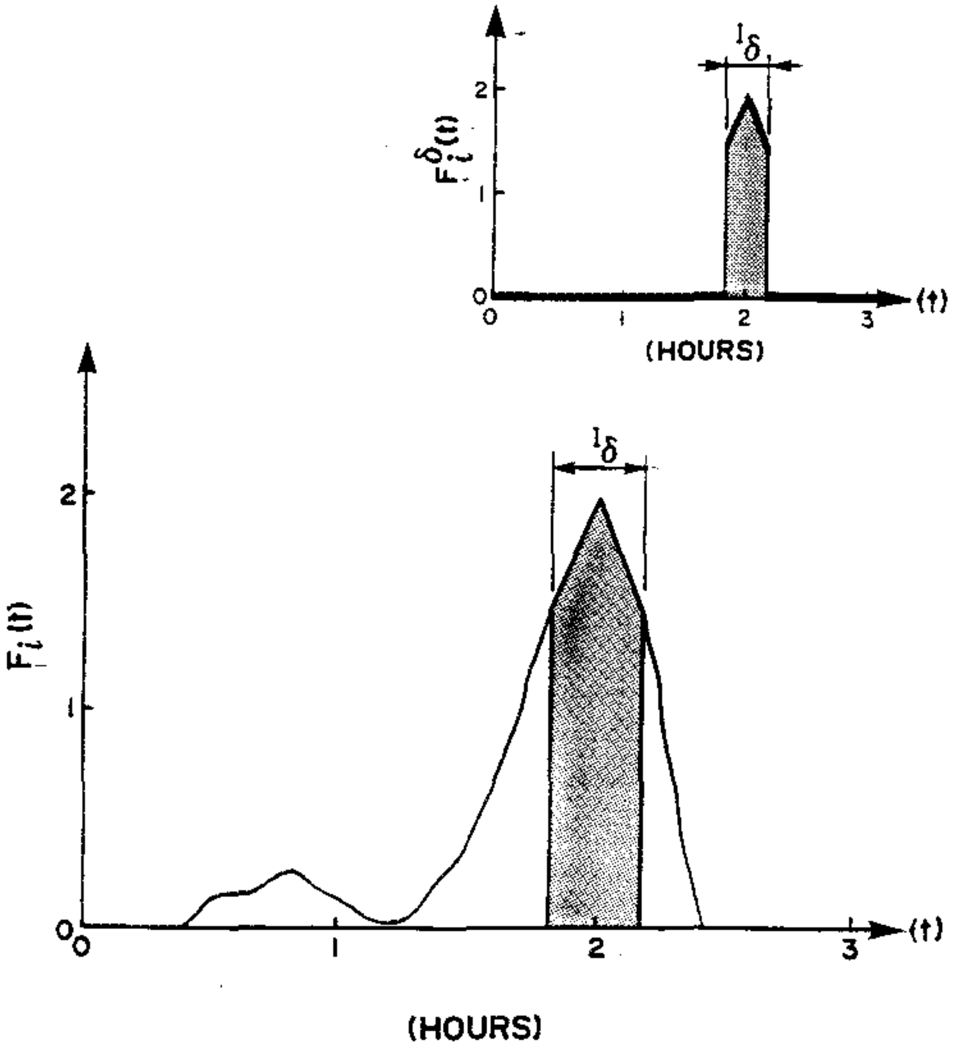


Fig. 1. Locating the Peak Aaea of $F_i(\cdot)$, for Duration, $\delta$.

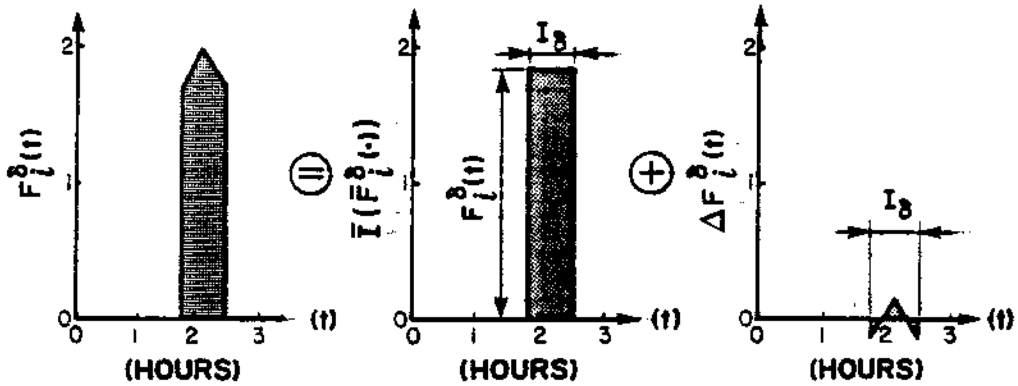


Fig. 2. Resolving $F_i^{\delta}(\cdot)$ into Components for Statistical Analysis.

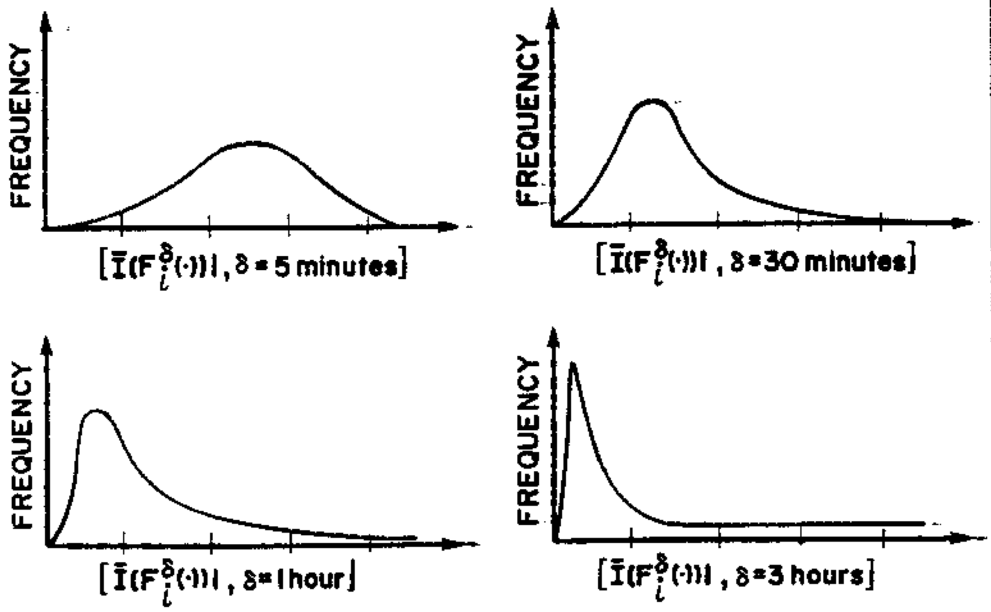


Fig. 3. T-Year Distributions of Annual $\bar{I}(F_i^{\delta}(\cdot))$ Values.

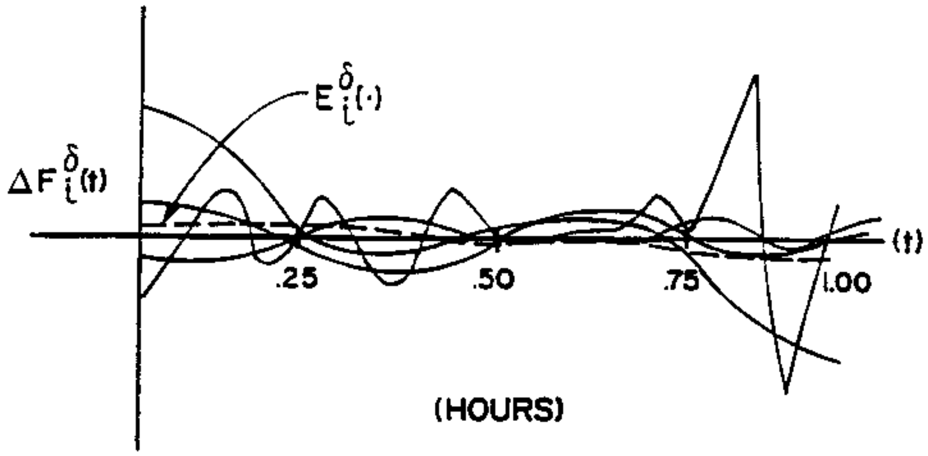


Fig. 4. Plots of $\Delta F_i^\delta(\cdot)$ Translated in Time ($\delta$ = 1 - Hour).

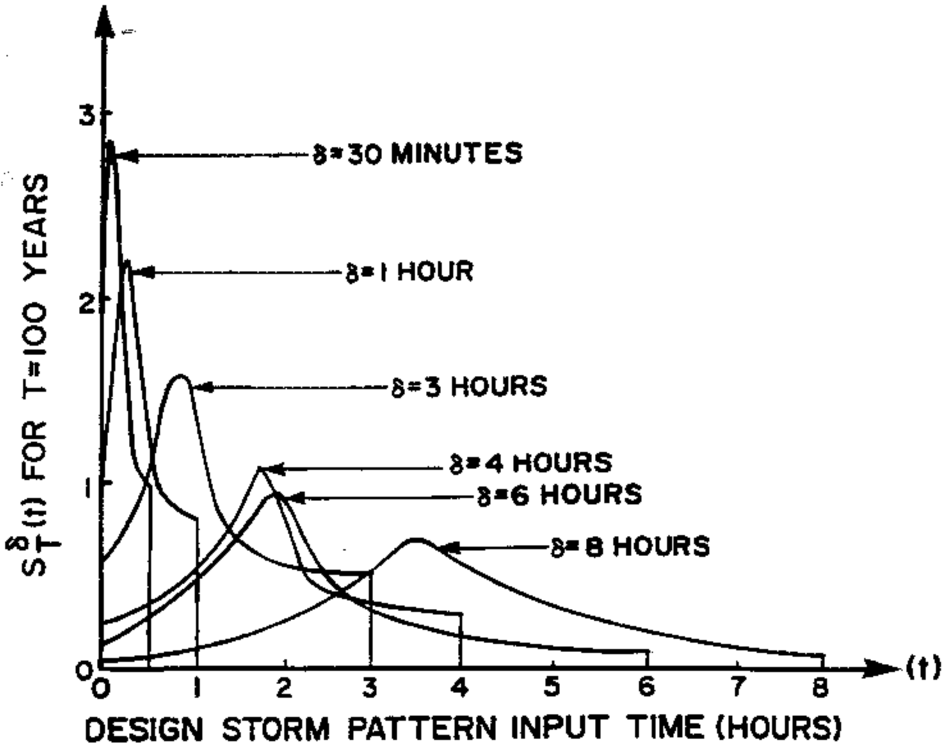


Fig. 5. Plots of $S_T^\delta(\cdot)$, For T = 100, - Years, and Various Values of $\delta$.

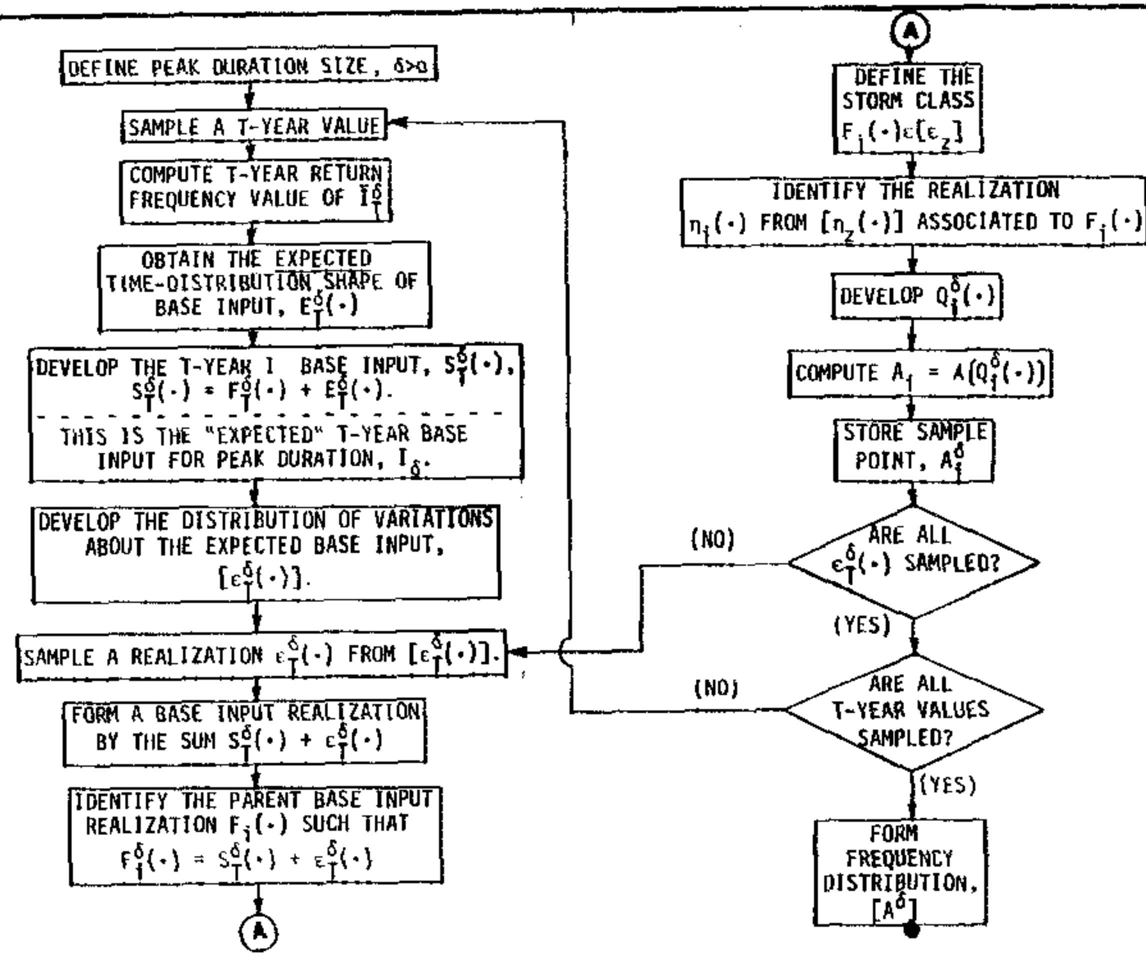


Fig. 6. Flow Chart for Peak Ddration Analysis.