

Results from a Mathematical Model of Frost Heave

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A one-dimensional model for simulation of frost heave in a vertical soil column is presented. The model is based on simultaneous computation of heat and moisture transport in a freezing or thawing soil. Thermal processes at the freezing front are approximated by a lumped isothermal approach. The model accurately simulates frost heave, soil pore-water pressures, and temperatures when compared with a laboratory freezing column; however, to achieve adequate correlation certain model parameters must be determined by calibration. Because the model, like the frost-heave process itself, is highly sensitive to environmental and soil parameters that are variable in both time and space, purely deterministic simulations will not provide sufficiently accurate predictions. Consequently, further development of the model is required in order to include a statistical-probabilistic approach for estimating frost heave within specified confidence limits.

Since July 1975, the Federal Highway Administration (FHWA), the Federal Aviation Administration (FAA), and the U.S. Army Cold Regions Research and Engineering Laboratory (CRREL) have been engaged in a jointly funded project to develop a better understanding of freezing and thawing processes in soils. The main thrust of the research has been the development of more suitable methods for analyzing and simulating potential frost heave and thaw weakening in prototype embankments. The component of the research reported here is the development of a mathematical model of frost heave. The other components of the research are the development of a one-dimensional laboratory freezing and thawing soil column (1) and a paper by Ingersoll and Berg elsewhere in this Record), the collection of field data on frost heave and thaw weakening (2), an evaluation of soil frost-susceptibility index tests (see the paper by Chamberlain elsewhere in this Record), and an evaluation of thaw weakening of soils (see the paper by Cole, Irwin, and Johnson elsewhere in this Record).

It has been recognized for some time (2) that frost-heave-susceptibility criteria for soils and associated laboratory test methods in current use are unreliable. Soils that meet established criteria may experience frost heave while other soils that do not meet the criteria may be free from frost heave (3). The original purpose of the mathematical modeling research was to develop a tool to correlate results of laboratory tests of frost susceptibility with pavement frost heaves measured in the field. The development of a mathematical model was seen as a practical way to integrate most of the complex soil thermal and hydraulic characteristics with the environmental factors that influence a given soil profile. Part of our initial objective was to identify the most significant parameters that influence frost heave in a given soil in order to assess and develop improved laboratory tests of frost-heave susceptibility. As our study progressed, it became apparent that the mathematical model has usefulness beyond the original objective and can probably be applied directly as a design aid or tool. It is emphasized, however, that laboratory tests will still be required to characterize the parameters of the mathematical model.

BASIS OF MATHEMATICAL MODEL

Guymon and others (4) briefly review the current literature related to modeling frost heave and heat and moisture transport in freezing soils and present most of the details of the development of the mathematical model. Although it is recognized that in

many situations a two-dimensional or even a three-dimensional process is involved, a one-dimensional approach was taken in order to simplify the computational problem and concentrate on demonstrating the validity of the modeling concept. In fact, at the time the research was begun in 1975, the modeling approach was regarded as a relatively high-risk research effort. At that time, only Harlan (5) and Guymon and Luthin (6) had shown modest success in modeling coupled heat and moisture movement in freezing soils. They assumed moisture movement in freezing soils could be approximated by unsaturated flow theory, and they did not consider ice segregation and associated frost heave.

Our model is based on the one-dimensional equations of moisture and heat transport in a vertical soil column. Assuming Darcy's law applies and inserting it into the continuity equation yield

$$(\partial/\partial x)(K_H \partial \phi/\partial x) = (\partial \theta_u/\partial t) + M_s \quad (1)$$

where

- x = vertical coordinate (positive downward);
- K_H = hydraulic conductivity = $K_H(\psi, \theta_i)$;
- ϕ = total hydraulic head = $\psi - x$, where ψ equals the pore-water-pressure head;
- θ_u = volumetric water content;
- t = time; and
- M_s = a moisture sink due to ice formation = $\rho_i \partial \theta_i/\partial t$, where ρ is ice and water density and θ_i is ice content.

The coupled heat transport equation for a freezing soil column is given by

$$(\partial/\partial x)(K_T \partial T/\partial x) - C_w v(\partial T/\partial x) = C_m (\partial T/\partial t) - LM_f \quad (2)$$

where

- K_T = thermal conductivity of the soil-ice-water mixture,
- T = temperature,
- C_w = volumetric heat capacity of water,
- v = liquid water velocity flux,
- C_m = volumetric heat capacity of the soil-water-ice mixture, and
- L = volumetric latent heat of fusion of water.

The linearized moisture transport and heat transport equations were independently verified for simple equations (2). The equations of moisture transport and heat transport are coupled through the parameters that arise in their derivation, and these coupled nonlinear equations can only be verified by comparison with laboratory or field data. Parameters such as thermal conductivity and volumetric heat capacity of soil-water-ice mixtures are computed by using the DeVries relations (7) or similar methods. Solution of the moisture transport equation requires a functional relation between pore-water pressure and water content [e.g., Gardner's relation (8)].

A significant aspect of this model is the manner in which phase-change effects are handled and ice segregation is assumed to occur. We have assumed an isothermal freezing process in which a heat budget for a finite volume of freezing soil is established (9). The soil is considered entirely frozen only

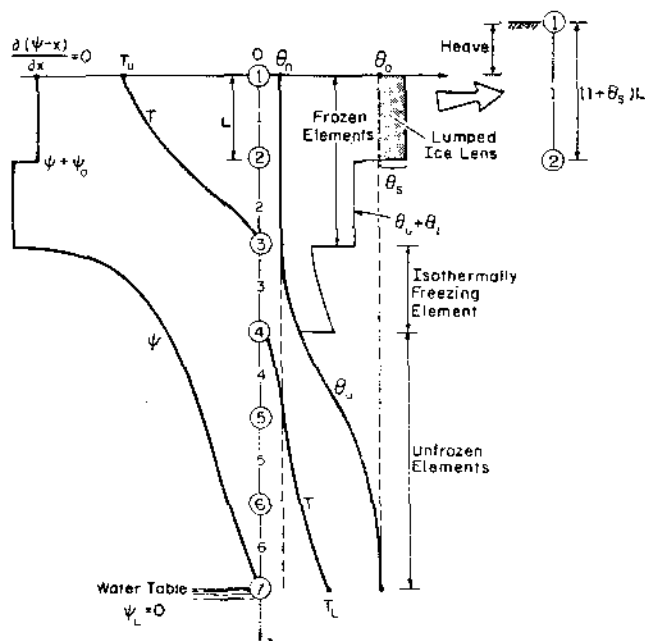
when latent heat effects are satisfied for the amount of water present in excess of an unfrozen water content factor (θ_n); i.e., the water that would remain unfrozen at a preselected subfreezing temperature (10). This method of handling phase change is tantamount to computing small changes in heat evolution rather than solving the heat transport equation for small changes in temperature. Because phase-change processes dominate the thermal process, this manner of computing phase-change effects provides certain numerical advantages by permitting large spatial discretization and large computational time steps. Ice segregation is assumed to occur if, within a finite volume of soil, the frozen water plus the unfrozen water content factor exceed the original porosity of the soil. Figure 1 shows the computational algorithm and the formation of segregated ice (θ_s) during a certain time level.

The θ_n parameter establishes the pore-water pressure at the freezing front for the solution of the moisture transport equation. The lower hydraulic boundary condition is usually the water table. Overburden effects (V_0), which tend to restrain frost heave, are modeled by adjusting the pore-water pressure at the ice segregation front by the weight of the overlying material and surcharge, thereby limiting the amount of water drawn toward the freezing front.

This model of the complex freezing and ice segregation process is a macroscopic lumped thermodynamic model that ignores the complicated and poorly understood microprocesses that occur in the freezing zone. Such processes as regelation and problems associated with water-ice film pressures are not included in our model. In summary, the model is based on macrophysics equations. Specifically, it is assumed that:

1. Moisture transport in the unfrozen zone is governed by the unsaturated flow equation based on Darcy's law.
2. Moisture flow is via liquid movement and vapor flow is negligible.
3. Moisture flow in the frozen zone is negligible.

Figure 1. Typical model simulation result at given simulation time level.



4. The unfrozen zone is nondeformable.
5. Soil pore-water pressures in the freezing zone are governed by an unfrozen water content factor.
6. All processes are single valued; i.e., there is no hysteresis.
7. Heat transport in the entire soil column is governed by the sensible heat transport equation, including a convective term.
8. The frozen zone is deformable for determining thermal parameters.
9. Salt exclusion processes are negligible; i.e., the freezing-point depression of water is constant.
10. Phase-change effects and moisture effects can be modeled as decoupled processes.
11. Freezing is an isothermal process.
12. Constant parameters are constant with respect to time.

Numerical solution of the governing equations discussed above, subject to their respective boundary and initial conditions, is by either the sub-domain method or the finite element method (3,11). The one-dimensional solution domain is divided into a number of variable-length "finite elements", where parameters are assumed temporarily constant but may vary from element to element and from time to time. The state variable in each element is assumed to be described by a linear basis function in such a way that the state variable is continuous throughout the solution domain (3). The time domain solution is by the well-known Crank-Nicolson method (12). The governing equations are reduced to a set of explicit algebraic equations that may be solved on the "mini" class of computers (e.g., the PDP 11/34). One of our main objectives is to provide a mathematical model that is efficient and capable of operating on readily available small computers.

Application of the model requires the following soil hydraulic and thermal parameters:

1. Relation between water content and pressure (unfrozen soil),
2. Relation between unsaturated hydraulic conductivity and pore pressure (unfrozen soil),
3. Hydraulic conductivity correction factor for partly frozen soil,
4. Soil porosity,
5. Soil density,
6. Relation between subfreezing temperature and unfrozen water content for the soil,
7. Soil thermal conductivity,
8. Soil heat capacity, and
9. Soil-water freezing-point depression.

Thermal parameters for water and ice are included in the model. The above-required parameters must be determined in the laboratory, or values must be assumed in order to use the computer model. In addition, the model requires the following auxiliary conditions:

1. Initial conditions for pore pressure, ice content, and temperature;
2. Soil-surface boundary conditions for pore pressure and temperatures (may vary with time); and
3. Lower boundary conditions for pore-water pressure and temperature (may vary with time).

To conduct a computer simulation of frost heave, a column length must be specified (e.g., 1 m), and the total length must be divided into sublengths (finite elements) that are on the order of 1 cm. Time solution factors are specified, such as time-step size and total simulation time.

VERIFICATION RESULTS

Preliminary verification of the model is discussed elsewhere (2,4). The model verification work will be expanded on here, and additional results will be presented.

The primary emphasis of our verification has been to compare simulated values of frost heave, soil pore-water pressures, and soil-water-ice temperatures with corresponding actual values measured in a laboratory soil column. Berg and others (1) describe the soil column and present data for Fairbanks silt, which is the primary test soil considered to date.

The test column filled with Fairbanks silt was subjected to various boundary conditions, and frost heave was measured while a 15-cm length was frozen. The lower portions of the column were unfrozen with a water table initially approximately 45 cm below the top of the column. Laboratory analyses of Fairbanks silt were conducted to determine soil hydraulic properties. These parameters and boundary conditions were approximated as input data for the mathematical model simulations. Two cases were used for comparison. The first case was a 110-day simulation for nearly unrestrained heave in which a nominal surcharge of 3.4 kPa was applied to the top of the soil column. This case was also used for an extensive sensitivity analysis of the parameters. The second case considered was a problem of restrained heave in which a 34.5-kPa surcharge was applied to the top of the laboratory column.

Figure 2 shows a comparison of measured and simulated heave for the small-surcharge case. Pore-water pressures and locations of the 0°C isotherm were also compared (4), and good agreement was obtained between the model and the soil column (see Table 1). The simulated frost heave shown in Figure 2 was achieved before incorporation of a correction factor for the hydraulic conductivity in the freezing zone. Slight (within 10 percent) adjustment of the laboratory-derived soil moisture characteristic curve (soil water pressure versus water content) was required in order to achieve the results shown in Figure 2 and given in Table 1. As Figure 2 shows, the rate of heave during the initial 25-day period is accurately computed. Total heave is simulated relatively well for the entire test. The laboratory soil column had essentially ceased heaving when the test ended at 40 days, and the model also exhibited no additional heaving after about 40 days. Parameters in the model were varied singly and in pairs to test their sensitivity for the early 25-day period, which showed the maximum heave rate. For Fairbanks silt, the unfrozen water content factor was the most

Figure 2. Simulated versus measured heave in vertical column of Fairbanks silt with nominal surcharge.

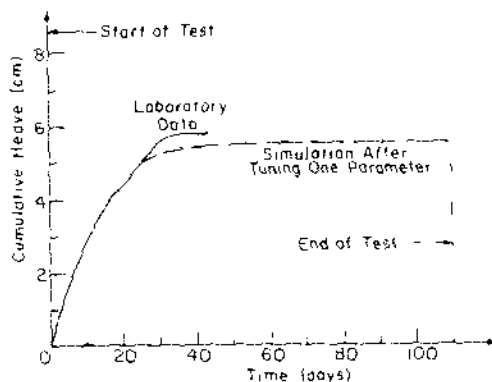


Table 1. Comparison of simulated and measured frost-heave data for 3.4-kPa surcharge.

Type of Data	5 Days	10 Days	15 Days
Laboratory			
Frost heave (cm)	1.6	2.8	4.0
0°C isotherm depth (cm)	6	11	11
Moisture tension at 24-cm depth (cm of water)	≈200	≈200	≈200
Simulated			
Frost heave (cm)	1.5	2.9	3.9
0°C isotherm depth (cm)	4.5-7.5	7-10	10-12.5
Moisture tension at 24-cm depth (cm of water)	100	130	150

critical parameter. Sensitivity analysis disclosed that the model is highly sensitive to pore-water pressure versus water content and hydraulic conductivity parameters and is less sensitive to thermal parameters. A fairly substantial variation in thermal parameters does not significantly affect simulated frost heave because the thermal process is dominated by the phase-change process and sensible heat conduction and convection do not play predominant roles. Processes occurring in the freezing zone dominate the soil-heaving process where the supply of water and the rate of heat extraction are the most significant factors.

In an effort to combine the sensitivity of various hydraulic parameters into one parameter and also to improve the accuracy of model prediction, a correction factor for hydraulic conductivity in the freezing zone was included in the model. This factor accounts for the decreased hydraulic conductivity due to ice formation in soil pores and is defined as

$$K(\Psi, \theta_i) = K(\Psi) \cdot 10^{-E\theta_i} \quad (3)$$

where $E\theta_i \geq 0$ and $K(\Psi)$ is determined from the relation between hydraulic conductivity and pore pressure for unfrozen soil. Currently, the E factor must be determined by calibration for a given soil. When we have more experience with application of the model to a range of soils, it may be possible to select a suitable value without calibration or testing.

Figure 3 shows a plot of frost heave versus time for the small-surcharge Fairbanks silt case (also shown in Figure 2) for the first 25 days. With no tuning of the various parameters, and with E selected equal to 8, the simulated heave almost exactly duplicated the experimentally measured heave. Figure 3 also shows the effect of varying the E parameter. When this parameter is included, other parameters such as $K(\Psi)$ and θ_n show less sensitivity. The sensitivity of thermal parameters remains negligible.

Figure 4 shows the results of a 25-day simulation of restrained heave (34.5-kPa surcharge) compared with laboratory heave data. As can be seen, simulated heave closely approximated the laboratory data. Exact comparison is somewhat difficult because of the need to use approximate boundary conditions in the simulation.

Model simulations were also compared with data from the Tomakomi field site in Japan (13). Soil properties determined from laboratory data, together with approximate boundary conditions, were used to simulate frost heave, which is compared with measured frost heave (Figure 5). The first five days of the heave process were used to calibrate the E parameter to a value of 5. As can be seen, fair agreement was achieved without adjusting the other parameters. Further attempts to more accurately compute frost heave and frost depth were considered

Figure 3. Simulated versus measured frost heave in vertical column of Fairbanks silt with nominal surcharge showing effect of ± 20 percent variation in E parameter (dashed lines).

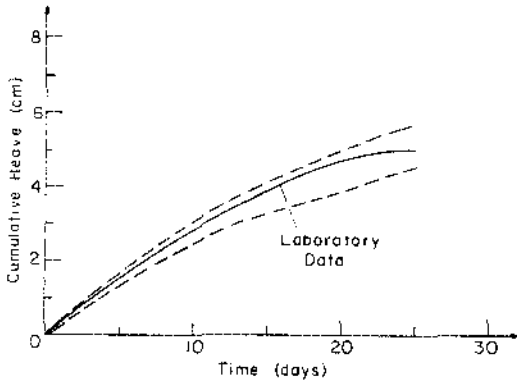
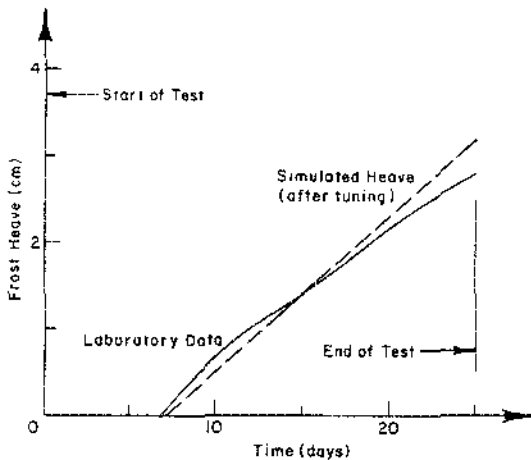


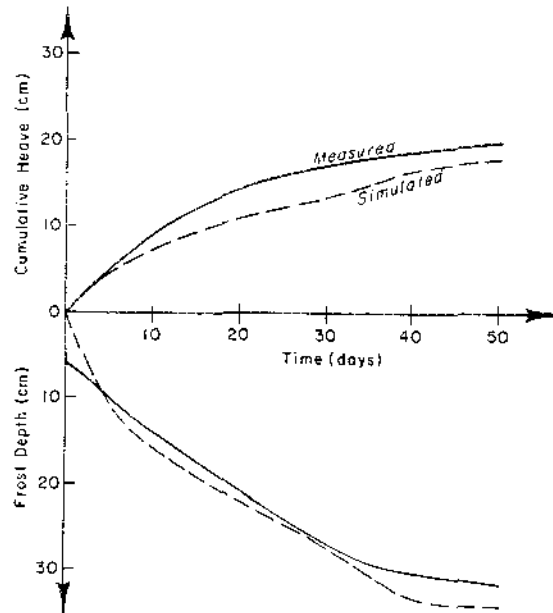
Figure 4. Simulated versus measured restrained heave (34.5-kPa surcharge) in vertical column of Fairbanks silt.



unwarranted because the upper temperature boundary condition was not accurately known. Very limited sensitivity analyses for this case were performed. It appears that the Tomakomi silt is much less sensitive to hydraulic parameters than the Fairbanks silt.

To further verify the model, it is also being applied to other field data. Several sets of field data were obtained from the Winchendon test area in northwestern Massachusetts. Figure 6 shows measured and computed frost heaves and frost depths for Ikalanian silt during the 1978-1979 winter. Laboratory-determined values of hydraulic conductivity and other parameters required by the model were used in the simulation. Boundary conditions and initial conditions for the 1978-1979 winter were estimated or determined from the field data and were applied to the model. Periods from 50 to 70 days were used to calibrate the E parameter. As Figure 6 shows, computed frost heave does not compare with measured frost heave as well as in previous simulations. Simulated frost depths are about the same as those measured in the field except during the spring thaw period (day 70 and beyond), when predicted frost penetration is too shallow. The discrepancy may be due to rather large elements (10 cm) used on the bottom of a 1-m simulation column. Several periods of thaw during the 1978-1979 winter are also approximated by the model. In particular, the spring thaw from day 80 on was approximated accurately in terms

Figure 5. Simulated versus measured frost heave and frost penetration for field tank with Tomakomi silt.



of both thaw consolidation and thaw depth. The difficulty in accurately simulating the entire record is probably due to the fact that there were a number of freeze-thaw cycles during the year. Cyclic freezing and thawing causes parameters to change with respect to time, but the model assumes parameters that are invariant with respect to time.

DISCUSSION OF RESULTS

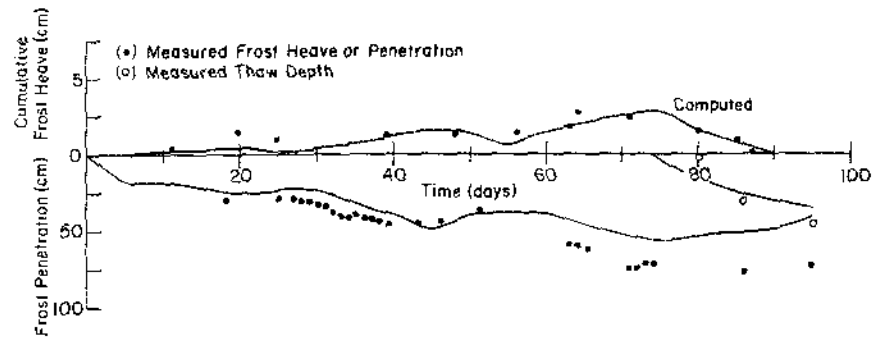
Development of the frost-heave model has progressed to the point where both restrained and unrestrained heave can be simulated in relatively well-controlled cases. Further study is, however, required to improve or extend the model.

The sensitivity problem needs further study. The fact that the model is sensitive to certain parameters should not be a surprise, since the frost-heave process itself is highly sensitive. Slight differences in environmental and/or soil conditions cause markedly different results. More complete physics-based information on the ice segregation process may reduce some of the sensitivity. If such information should become available, it can be included in the model; however, it is probable that purely deterministic simulations will never be adequate no matter how complete our knowledge. It therefore seems inevitable that some form of a probabilistic model must be associated with the deterministic model. In other words, a deterministic simulation can be performed and then simulated frost heave would be presented in terms of confidence bands based on given or anticipated variations in parameters and boundary conditions. Research on this aspect of the problem has already commenced and will be reported at a later date.

The model needs to be extended to include thaw weakening. We will develop an algorithm for estimating temporal variations in the resilient modulus (or some other parameter reflective of the strength of the pavement system), especially during periods of thaw weakening.

The third area of concern is the dissemination of the model and its use by engineers. The model is somewhat complex and will not be readily understood

Figure 6. Simulated versus measured frost heave, frost penetration, and thaw depth for Ikalanian silt during 1978-1979 winter at Winchendon, Massachusetts, field site.



by many practicing engineers. We are attempting to simplify the code to provide a model that is versatile and usable on small, readily available computers. During the next year or more, a major effort will be to make the model more readily usable by the practicing engineer.

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REFERENCES

1. R.L. Berg, J.I. Ingersoll, and G.L. Guymon. Frost Heave in an Instrumented Soil Column. *Cold Regions Science and Technology*, Vol. 3, 1980, pp. 211-221.
2. R.L. Berg, G.L. Guymon, and T.C. Johnson. Mathematical Model to Correlate Frost Heave of Pavements with Laboratory Predictions. U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, NH, Rept. 80-10, 1980.
3. D.L. Townsend and T.L. Csathy. Soil Type in Relation to Frost Action. Ontario Joint Highway Program, Queen's Univ., Kingston, Ontario, Canada, Rept. 15, 1963.
4. G.L. Guymon, T.V. Hromadka II, and R.L. Berg. A One-Dimensional Frost Heave Model Based upon Simulation of Simultaneous Heat and Water Flux. *Cold Regions Science and Technology*, Vol. 3, 1980, pp. 253-262.
5. R.L. Harlan. Analysis of Coupled Heat: Fluid Transport in Partially Frozen Soil. *Water Resources Research*, Vol. 9, No. 5, 1973, pp. 1314-1323.
6. G.L. Guymon and J.N. Luthin. A Coupled Heat and Moisture Transport Model for Arctic Soils. *Water Resources Research*, Vol. 10, No. 5, 1974, pp. 995-1001.
7. D.A. DeVries. *Thermal Properties of Soils*. In *Physics of Plant Environment* (W.E. Van Wijk, ed.), North-Holland Publishing Co., Amsterdam, The Netherlands, 1966, pp. 210-235.
8. W.R. Gardner. Some Steady-State Solutions of the Unsaturated Moisture Flow Equation with Application to Evaporation from a Water Table. *Soil Science*, Vol. 85, 1958, pp. 228-232.
9. T.V. Hromadka II, G.L. Guymon, and R.L. Berg. Some Approaches to Modeling Phase Change in Freezing Soils. *Cold Regions Science and Technology* (in preparation).
10. D.M. Anderson, A.R. Tice, and H.L. McKim. The Unfrozen Water and the Apparent Specific Heat Capacity of Frozen Ground. Proc., 2nd International Conference on Permafrost, North American Contribution, National Academy of Sciences, Washington, DC, 1973, pp. 289-294.
11. T.V. Hromadka II and G.L. Guymon. Nodal Domain Integration Model of One-Dimensional Advection-Diffusion. *Advances in Water Resources* (in preparation).
12. C.S. Desai. *Elementary Finite Element Method*. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1979.
13. S. Kinoshita and others. Observations of Frost Heaving Action in the Experimental Site, Tomakomi, Japan. Proc., 3rd International Conference on Permafrost, National Research Council of Canada, Ottawa, Ontario, Vol. 1, 1978.

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Evaluation of a Self-Refrigerated Unit for Frost-Heave Testing

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The British Transport and Road Research Laboratory (TRL) frost-heave test, in which specimens are frozen from the top downward in an open system for 250 h, is specified in terms of a cold room but is more often undertaken in a self-refrigerated unit (SRU). In both units the air temperature

above the specimens is maintained at -17°C and the water bath at $+4^{\circ}\text{C}$. The maximum permitted heave is 13 mm in England and 18 mm in Scotland. A comparative study involving six aggregates of 40-mm maximum size and covering a range of geological types and gradings was undertaken to estab-