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# Evaluating Uncertainty in Design Storm Runoff Predictions

ROBERT J. WHITLEY

*Department of Mathematics, University of California, Irvine*

THEODORE V. HROMADKA II[1]

*Williamson and Schmid, Irvine, California*

The use of a given effective rainfall and a stochastic integral equation formulation of the well-known unit hydrograph method gives criterion design variables, such as volume or maximum discharge, which are random variables depending on the stochastic variation in the unit hydrographs. When this variation is modeled by means of a multivariate normal distribution, it is possible to compute the distribution of criterion variable values. For example, the total runoff volume can be shown to be normally distributed, and so confidence intervals for this design variable can be directly obtained. Note that this probability distribution is for a fixed design storm and is due to the multivariate normal variation in the unit hydrograph. A computer simulation can be used to obtain confidence intervals for the maximum discharge estimate. Similarly, probabilistic simulation can be used to develop confidence intervals for other criterion variables.

## INTRODUCTION

The unit hydrograph method is a widely used rainfall-runoff modeling technique. For a single subarea model, in general one selects a method of producing effective rainfall (from the assumed storm rainfall), and a transfer function (or unit hydrograph) for the catchment. *Hromadka and Whitley* [1988, 1989] show that multiple subarea models with routing, each subarea having its own transfer function, can also be put into the form of a single subarea model, and so the model studied here is of broad applicability.

There are several situations in which the transfer function for the catchment should be regarded as stochastic. For example, the catchment of interest may be ungauged. In this case one can use unit hydrographs (i.e., transfer functions) for several different catchments which are considered to be hydrologically similar to this catchment, scale them using catchment characteristics such as lag and ultimate discharge, and averaged the transfer functions to obtain a "regionalized" transfer function. Using this averaged transfer function and the selected effective rainfall model, a criterion variable, for example, peak discharge, can be computed for the considered rainfall. But the single number for peak discharge so obtained does not reflect the uncertainty which arises from averaging transfer functions from other catchments. It can be plausibly argued that the "correct" transfer function for the catchment could be any of the transfer functions used to develop the average. From this point of view, the transfer function is, say, equally likely to be any one of the transfer functions from the other catchments, and this randomness in the transfer function creates a random variation in the calculated peak discharge. For an example of this approach see *Hromadka et al.*, [1987]. This situation can be modeled by supposing the transfer function to be a stochastic process for which a certain number of realizations are known, and that are equally likely to occur.

As another example, even if the catchment is gauged and the transfer functions have been computed for several large storms (i.e., for a given storm class [*Hromadka and Whitley*, 1988, 1989]), variation will still be found among these transfer functions. There are numerous sources of this variation, but a major source is the spatial variation of storms [*Hromadka and Whitley*, 1988]. Since what is known are the rain gauge data, using these as input to the effective rainfall computation in the model amounts to assuming that this rainfall measured at the rain gauge is uniform over the catchment. Because storms are generally far from uniform, the runoff hydrograph obtained represents only a statistical correlation between the rainfall and runoff data. As an analogy, consider the problem of linear regression. It is supposed that $y = mx + b$ holds exactly, but actual observations give $x$ and $y + \varepsilon$, where $\varepsilon$ represents a deviation from the true value due to error. In a similar way, the transfer function for the catchment can be modeled as a stochastic process, which can be thought of as the true transfer function plus a stochastic process expressing the error in any particular realization. Note that linear regression contains the assumption that the form of the true relation between $x$ and $y$ is known (i.e., $y = mx + b$), whereas no such assumption concerning the form of the transfer function has been made. The data of the example which will later be analyzed consist of 12 transfer functions which are based on 25 significant storm events at seven different sites in Los Angeles County, California. From the point of view of the statistics of the process, at a fixed point in time only 12 values of the process are known, from which no accurate statistical conclusion can be drawn. A further analogy with linear regression is appropriate: It is often the case that only a few pairs of data points $(x, y)$ are known, and to discuss the random variation in the linear regression model, for example, to give confidence bands about the regression line, it is assumed that the errors $\varepsilon$ are independent and normally distributed with mean zero and a standard deviation the same for all $x$. In most practical situations

[1]Also at Department of Applied Mathematics, California State University, Fullerton.



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these assumptions, including that of normality, are beyond testing. (There is a useful discussion by *Breiman* [1973] of some possible tests of the underlying assumptions in linear regression. To give an idea of the numbers involved in testing distributions, to distinguish between a normal $N(0, 1)$ distribution and a distribution uniform on $(-\sqrt{3}, \sqrt{3})$, at merely the 10% confidence level, requires at least 130 points using the good Kolmogorov-Smirnov test [*Breiman*, 1973]. To distinguish between distributions closer in shape to the normal or at higher levels of confidence requires many more points.) As in the case of linear regression, one may argue heuristically, noting the central limit theorem, that errors tend to be approximately normally distributed. But finally, one proceeds in the hope that statistical insight based on uncertain assumptions is better than no insight.

In comparison with linear regression, modeling the transfer stochastic process is much more difficult. First, the form of the transfer function is not known. Second, that portion of the error process which is caused by nonuniform rainfall will change slowly with time, and so the errors at nearby times will be correlated and not independent. A multivariate normal model allows for these dependencies and is one of the few multivariate models in which basic computations can be made, and as such it is the first model to explore. Therefore it is with a mixed sense of desperation and hope that the multivariate normal model is adopted in this paper.

## Discussion of the Model (Stochastic Integral Equation Method)

We consider a variant of the unit hydrograph method which relates the effective rainfall realization $e(\ )$ and the discharge $Q(\ )$ via the stochastic integral equation,

$$Q(t) = \int_0^t e(t-s)\eta(s)\, ds \tag{1}$$

where $\eta(\ )$ is a realization of a stochastic process distributed as $[\eta(\ )]$ [*Hromadka and Whitley*, 1985, 1989].

Our analysis begins by dividing the study time interval $[0, T]$ into $N$ equal subintervals $I_n = [t_{n-1}, t_n)$, with $t_n = nT/N$ for $n = 0, 1, \cdots, N$, and approximating $e(\ )$ by a step function with the constant value $e_n$ on the interval $I_n$. Letting $\chi[a, b)$ be the characteristic function of the interval $[a, b)$ defined by

$$\chi_{[a,b)}(t) = 1 \quad a \le t < b$$

$$\chi_{[a,b)}(t) = 0 \quad \text{otherwise}$$

$e(\ )$ can be written

$$e(t) = \sum_{n=1}^{N} e_n \chi_{I_n}(t). \tag{2}$$

In the same fashion, approximate the realization $\eta(\ )$ from $[\eta(\ )]$ by a function with constant value $\eta_n$ on the interval $I_n$. Substituting these approximations for $e(\ )$ and $\eta(\ )$ into (1) gives

$$Q(t) = \sum_{n=1}^{N} e_n[S(t-t_{n-1}) - S(t-t_n)] \tag{3}$$

where $S(t)$ is the $S$ graph

$$S(t) = \int_0^t \eta(s)\, ds.$$

Thus $Q(t)$ can be seen to be continuous and piecewise linear, with the derivative $Q'(t)$ taking on a constant value, say $q'_n$, on the interval $I_n$.

To determine the values $\{q'_n\}$, differentiate (3) and choose $t$ to be a point in $I_n$ for $n = 1, 2, \cdots, N$, giving $N$ equations:

$$\begin{aligned} q'_1 &= e_1(\eta_1 - \eta_0) \\ q'_2 &= e_1(\eta_2 - \eta_1) + e_2(\eta_1 - \eta_0) \\ &\cdots \\ q'_N &= e_1(\eta_N - \eta_{N-1}) + e_2(\eta_{N-1} - \eta_{N-2}) \\ &\quad + \cdots + e_N(\eta_1 - \eta_0) \end{aligned} \tag{4}$$

where $\eta_0 = 0$ is used in the formulas for symmetry. These equations can also be rewritten in the form:

$$\begin{aligned} q'_1 &= (e_1 - e_0)\eta_1 \\ q'_2 &= (e_2 - e_1)\eta_1 + (e_1 - e_0)\eta_2 \\ &\cdots \\ q'_N &= (e_N - e_{N-1})\eta_1 + (e_{N-1} - e_{N-2})\eta_2 \\ &\quad + \cdots + (e_1 - e_0)\eta_N \end{aligned} \tag{5}$$

with $e_0 = 0$.

The problem of modeling the statistical variation in each of the parameter sets $\{q'_1, \cdots, q'_N\}$, $\{\eta_1, \cdots, \eta_N\}$, and $\{e_1, \cdots, e_N\}$, can be considered for various cases; the one which we will consider here is where $e(\ )$ is a future storm event; e.g., it is a given design storm effective rainfall. Even for an idealized set of effective rainfall events with identical patterns and magnitudes at the rain gauge, there would still be variations in the effective rainfall over the catchment, which would yield observed variations in the associated $Q(\ )$, and thereby variations in $\eta(\ )$. Consequently, there would be one realization, $\eta(\ )$, for each data pair of $\{e(\ ), Q(\ )\}$. Because of the random variations in the effective rainfall over the catchment, and the many random processes occurring in any hydrologic rainfall-runoff model such as errors in measurements and errors in computing runoff, $\eta(\ )$ is a stochastic process.

Each value $\eta_n$ of $\eta(\ )$ on the interval $I_n$ is itself a random variable and so the vector $\mathscr{E} = (\eta_1, \cdots, \eta_N)$ is a multivariate random variable. Moreover, for small time intervals, say, unit periods of 5 min, there will be some dependence between the values of $\eta(\ )$. This important mutual dependency in the set of components of $\mathscr{E}$ makes the problem of probabilistic modeling much more difficult. With no strong evidence to the contrary, an appeal to the central limit theorem for multivariate random variables [*Breiman*, 1968; *Billingsley*, 1986] suggests that $\mathscr{E}$ can be modeled with a multivariate normal distribution. And, in fact, this distribution is one of the few multivariate distributions which is simple enough to allow basic calculations to be made and yet which allows dependence between components.

As (5) indicates, $q'_n$ is a linear combination of components of $\mathscr{E}$. By a known property of multivariate normal distributions, this implies that $\mathbf{Q}' = (q'_1, \cdots, q'_N)$ is also a multivariate normal [*Kendall and Stuart*, 1977]. Conversely, by solving (5), if $\mathbf{Q}'$ is a multivariate normal then so is $\mathscr{E}$.

A useful fact is that a multivariate normal distribution $\mathbf{X} = (X_1, \cdots, X_N)$ is completely determined by its means and its covariance matrix $\gamma_{ij} = E[(X_i - \mu_i)(X_j - \mu_j)]$, where $\mu_j = E(X_j)$. In fact, in the (usual) simplest case where the covariance matrix $\Gamma = [\gamma_{ij}]$ has an inverse $A = [a_{ij}]$, the density function of $\mathbf{X}$ is

$$[(2\pi)^N \det(\Gamma)]^{-1/2} \exp\left(-0.5 \sum_{i,j=1}^{N} a_{ij}(x_i - \mu_i) \cdot (x_j - \mu_j)\right) \quad (6)$$

Consequently, this density can be estimated by estimating the covariances and means.

Thus under the model assumptions that either $\mathscr{E}$ or $\mathbf{Q}'$ is multinormal, the other is also multinormal, and therefore one distribution can be estimated by using (5) and estimating the covariance matrix for the other distribution.

We will use this technique to study the statistical properties of predicted $Q(\ )$, which are a consequence of the statistical properties of $\mathscr{E}$ and the choice of design storm effective rainfall $e(\ )$, and so will be able to study some of the statistics of the stochastic integral equation representation of the unit hydrograph model under assumptions which allow realistic dependencies between random components of the processes.

## Criterion Variables

As an example of a runoff criterion variable, consider the total volume of runoff, $V$. The trapezoidal rule with partition points $t_0, t_1, \cdots, t_N$ is exact for the piecewise linear $Q$ and gives

$$V = \sum_{k=1}^{N} [Q(t_{k-1}) + Q(t_k)]/2. \quad (7)$$

Since

$$Q(t_k) = (\delta) \sum_{j=1}^{k} q'_j \quad Q(t_0) = 0 \quad (8)$$

where $\delta$ is the width $T/N$ of the intervals, $I_i$, $V$ is seen to be a linear combination of $q'_1, q'_2, \cdots, q'_N$, and therefore is normally distributed. Hence to find confidence intervals for design values of $V$ (which is a prediction of the random variable $V$ given a future effective rainfall), the ordinary statistical methods for a normally distributed random variable apply. Ordinarily, one would expect the (total) volume $V$ to merely equal the volume of effective rainfall because the usual definition of effective rainfall is that rainfall which does run off the catchment. Here, $V$ is a random variable for several reasons: (1) The average transfer function $\eta_0$ is usually scaled so that it passes through exactly the volume of "effective rainfall"; here we see the results of using $\eta_0 + \varepsilon$ for various error terms $\varepsilon$. (2) "Effective rainfall" usually means rainfall less losses, and in that case the volume of runoff is the volume of effective rainfall. Here, as in the work by *Hromadka and Whitley* [1989], effective rainfall is thought of as some function of precipitation measured at the rain gauge, from which one tries to predict criterion variables, such as peak discharge. From this point of view the "effective rainfall" may best predict peak discharge without necessarily having the volume of rain over the catchment agree with the volume extrapolated from the rain gauge data. (3) The volume $V$ is taken over the time interval $[0, T]$, which does not necessarily exhaust the runoff.

Note that the variation in $V$, which is characterized as normal, is that produced by a fixed given design storm and a variable set of $\eta(\ )$ used in the stochastic integral equation formulation of the unit hydrograph method. This is distinct from the variation in $V$ which would be found in runoff volume data from a specific catchment (should such data be used directly); rather, this observed variation is, to a large extent, due to the spatial variation in the effective rainfall over the catchment with respect to the assumed effective rainfall, among other factors. (As in many applications of the normal distribution, this model is not perfect in that it predicts discharge with negative volumes, but this is only with insignificant probability for typical observed values of $V$ and their standard deviations.)

A criterion design variable of great interest is the peak flow rate. Unlike the case of the total volume of runoff, for the peak flow rate there is no simple derivation of its distribution. To analyze a specific case requires a statistical simulation.

Consider a set of $\eta(\ )$, each approximated by constants on the time intervals $I_n$ as was done following (2). On each interval $I_n$ the values $\eta_n$ are normally distributed, but that information alone is not enough to determine the joint distribution of the $\eta(\ )$ because of the dependence between values on different intervals. The values $\{Q(t_i)\}$, among which the peak flow rate is to be found, are each a linear combination of $\eta_1, \eta_2, \cdots, \eta_N$ since, as was noted in (8), they are linear combinations of $q'_1, \cdots, q'_N$ which, from (5), are in turn linear combinations of $\eta_1, \cdots, \eta_N$:

$$Q(t_i) = \sum_{j=1}^{N} b_{ij}\eta_j \quad (9)$$

where $b_{ij}$ are to be determined. The $\eta_1, \cdots, \eta_N$ will now be regarded as random variables, and we note that

$$E(Q(t_i)) = \sum_{j=1}^{N} b_{ij}E(\eta_j),$$

so that if we subtract the expected value from each $\eta_j$ this will subtract the expected value from each $Q(t_i)$ and

$$Q(t_i) - E(Q(t_i)) = \sum_{j=1}^{N} b_{ij}(\eta_j - E(\eta_j)) \quad (10)$$

Consider the random variables

$$X_i = Q(t_i) - E(Q(t_i)) \quad (11)$$

These have a multivariate normal distribution and each has an expected value of zero. The covariance matrix $C$ for these $X_1, \cdots, X_N$,

$$C = [\mathrm{Cov}\ (X_i, X_j)] \quad (12)$$

is symmetric and semidefinite. If positive definite, it has a Cholesky factorization into

$$C = LL^T \quad (13)$$

where $L$ is lower triangular and $L^T$ is the transpose of $L$; and it also has this factorization after the appropriate interchanges, which we will suppose to have been made, it it is only semidefinite [*Wilkinson*, 1978].

If we take $Z_1, Z_2, \cdots, Z_N$ to be independent normal $N(0, 1)$ random variables, and $Z$ to be the column vector $(Z_1, \cdots, Z_N)$, then it is easy to compute the covariance matrix of $LZ$ and show that it is the matrix $C$: The covariance matrix of $LZ$ is $E(LZ(LZ)^t) = C$, because $E(Z_iZ_j) = 0$ for $i \neq j$ and $E(Z_i^2) = 1$. (This well-known fact is the basis for the characterization of zero mean multivariate normal distributions as being those whose components are linear combinations of independent $N(0, 1)$ normals [*Breiman*, 1968].) Since the multivariate distribution of $X_1, \cdots, X_N$ is determined by its covariance matrix, the $X$ values can be simulated, if we know their covariance matrix, by simulating the $Z$ values [*Maindonald*, 1984].

For the set of $\eta(\ )$ discussed below it was found that the peak flow rate occurs in only a few unit intervals, and from hydrological and statistical considerations it is unlikely that the maximum falls too far outside these few time intervals in general. So only a small number $X_m, X_{m+1}, \cdots, X_{m+r}$ of $X$ values need be considered, which considerably reduces the complexity of the model.

## Example: Computer Simulation for Peak Flow Rate

In the example case study considered, 12 samples $\eta(\ )$ were obtained from catchment rainfall-runoff data (see Figure 1), each consisting of 25 unit values of flow rate (based on the 5 min time interval). These values of flow rate are assumed to be samples from a multivariate normal distribution. Additionally, all the $\eta(\ )$ were obtained from storms which are considered of similar severity (i.e., in the same storm class; see [*Hromadka and Whitley*, 1989]). The unit flow rates were visually compared with simulated values from a multivariate normal distribution as a rough check; because there are so few sample points, a more discriminating test is not feasible. The design (i.e., future) storm effective rainfall was taken to be linear increasing from 0 to 5 inches/h (0–12.7 cm/h) at 1.5 hours, and then linear back down to zero at 3 hours, and this storm was approximated as piecewise constant in consecutive 5 min time intervals. The criterion variable of interest is the peak flow rate anticipated from the assumed design storm effective rainfall.

From a calculation of the unit flow rate values for each $\eta(\ )$, it can be seen that the peak flow rate falls into one of the three unit time intervals [135, 140], [140, 145], and [145, 150] (time given in minutes).

The computer simulation procedure continues, as dis-

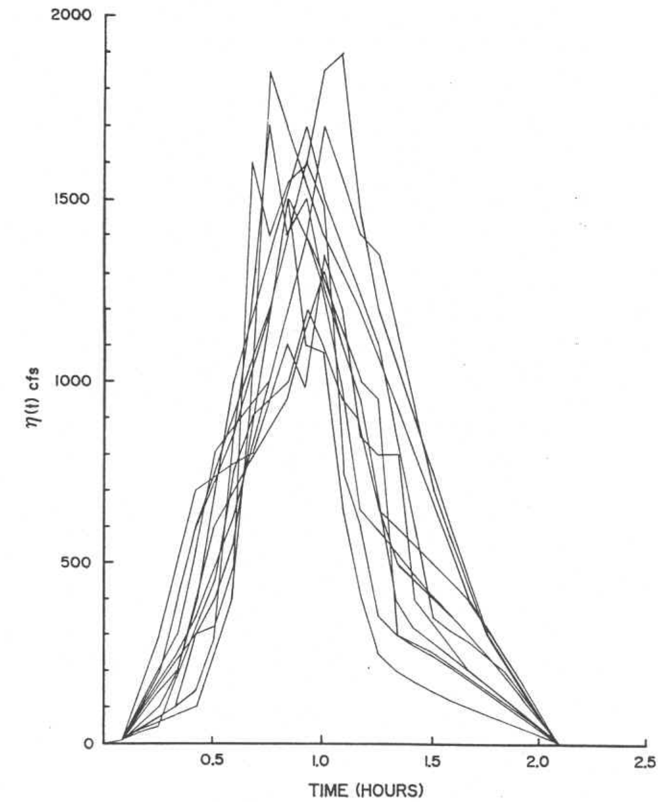


Fig. 1. Plot of $\eta(\ )$ samples using model of (1) (5-min unit periods of flow rate). Flow rates are in cubic feet per second (1 cubic foot equals $2.8317 \times 10^{-2}$ $m^3$).

cussed above, in that the covariance matrix of the $Q(t_i)$ is computed. Then a subset of the $X_j$ is chosen; for example, the subset $\{X_{28}, X_{29}, X_{30}\}$ corresponds to the three intervals in which the peak flow rate occurs. Then the covariance matrix for this subset of $X$ is factored into a product of a lower triangular matrix $L$ and its transpose. It is now only necessary to generate independent $N(0, 1)$ random variables, use $L$, and add on the estimated means of each $X$, in order to develop one vector of flow rate values for the time intervals chosen. From the vector of flow rate ($Q$) values, the maximum value of $Q$ is obtained, resulting in one sample point in the simulation of maximum $Q$ values. The program does this repeatedly, and keeps track of the empirical distribution of the maximum $Q$ (i.e., the criterion variable). As a final result, one obtains an estimated distribution of percentiles 5%(5%)95% for the maximum $Q$ based on the subset of unit time intervals chosen.

For the given data set, this calculation was performed for the single unit value $X_{28}$, for $\{X_{27}, X_{28}, X_{29}\}$, and on up to $\{X_{24}, \cdots, X_{34}\}$. The outcome was that all the percentiles were the same for these different subsets of $X$ values to within a few cubic feet per second (see Table 1).

There are two reasons for this simple outcome. The first reason is that, as a cursory inspection of the data show, the maxima tend to fall in a narrow range of time intervals and for those intervals the $Q$ values have approximately the same means and standard deviations. The second reason depends on a less obvious property of this multinormal $Q$ distribution, namely, that when the covariance matrix is factored into $LL^T$, $L$ puts most of its weight into one $Z$

TABLE 1. Peak Flow Rate Percentile Estimates for Various Unit Period Sets

| Unit Period 29 | | Unit Period 28–30 | | Unit Period 27–31 | | Unit Period 24–34 | |
|---|---|---|---|---|---|---|---|
| Percentile | Max $Q$, cfs | Percentile | Max $Q$, cfs | Percentile | Max $Q$, cfs | Percentile | Max $Q$, cfs |
| 5 | 299.66 | 5 | 302.51 | 5 | 301.37 | 5 | 300.70 |
| 10 | 323.41 | 10 | 325.00 | 10 | 324.66 | 10 | 323.78 |
| 15 | 339.77 | 15 | 339.83 | 15 | 339.92 | 15 | 341.29 |
| 20 | 352.68 | 20 | 353.80 | 20 | 353.25 | 20 | 354.36 |
| 25 | 364.69 | 25 | 365.16 | 25 | 364.03 | 25 | 365.34 |
| 30 | 374.81 | 30 | 375.17 | 30 | 374.32 | 30 | 375.89 |
| 35 | 384.21 | 35 | 384.45 | 35 | 383.86 | 35 | 385.36 |
| 40 | 393.20 | 40 | 393.79 | 40 | 392.84 | 40 | 394.44 |
| 45 | 401.69 | 45 | 402.98 | 45 | 401.73 | 45 | 403.70 |
| 50 | 410.50 | 50 | 411.40 | 50 | 410.28 | 50 | 412.25 |
| 55 | 419.52 | 55 | 419.89 | 55 | 419.05 | 55 | 420.45 |
| 60 | 428.26 | 60 | 428.58 | 60 | 427.92 | 60 | 429.59 |
| 65 | 437.20 | 65 | 438.17 | 65 | 436.55 | 65 | 438.05 |
| 70 | 446.64 | 70 | 447.74 | 70 | 445.97 | 70 | 448.12 |
| 75 | 457.00 | 75 | 458.44 | 75 | 456.41 | 75 | 458.21 |
| 80 | 467.99 | 80 | 470.73 | 80 | 467.01 | 80 | 469.53 |
| 85 | 481.56 | 85 | 484.81 | 85 | 480.73 | 85 | 483.07 |
| 90 | 498.22 | 90 | 501.35 | 90 | 496.99 | 90 | 498.99 |
| 95 | 522.11 | 95 | 526.59 | 95 | 522.59 | 95 | 525.46 |

1 cfs (cubic foot per second) equals $2.8317 \times 10^{-2}$ m$^3$.

variable. For example, Table 2 provides the factorization for the subset $\{X_{27}, \cdots, X_{31}\}$.

The significance of this result is that, for these data, satisfactory confidence intervals for peak flow rate can be obtained merely by choosing the most common interval in which the 12 data peak flow rates occur, and then supposing those data to come from a (single) normal distribution. Of course, for other criterion variables the resulting distribution need not be even approximately normal, e.g., peak flow rate if the effects of a flowby retarding basin were added to the simulation.

## DISCUSSION

The previous example problem focused upon the runoff criterion variable of peak flow rate. The above methodology can be applied to any criterion variable, $A$, to develop the probability distribution of $[A]$ by $[A] = \mathcal{A}[Q^D(\ )]$ where $[Q^D(\ )]$ is the stochastic process of realizations of possible runoff hydrographs, $Q^D(\ )$, for the assumed design storm effective rainfall, and $\mathcal{A}$ is a functional which operates on each sampled runoff hydrograph realization to develop a sample point of $A$.

The multivariate normal distribution, as applied to the sampled $\eta(\ )$ obtained from rainfall-runoff data using the model of (1), provides an estimate of the underlying probabilistic distribution of that stochastic process, which is distributed as $[\eta(\ )]$. Consequently, even though only 12 samples (realizations) of the $\eta$ are obtained by data analysis using (1), the distribution of the stochastic process, $[\eta(\ )]$, can be estimated using the multivariate normal distribution which is analogous to fitting a probability distribution function to 12 sample points of a random variable. As a result, a continuous probability distribution of the runoff criterion variable, $[A]$, can be obtained rather than developing only a frequency distribution of $m$ sample points of $A$, where $m$ is the number of sample realizations developed from $[\eta(\ )]$. The accuracy of this distribution depends on the accuracy of the multivariate normal assumption.

## CONCLUSIONS AND FURTHER RESEARCH NEEDS

A stochastic integral equation (SIEM) formulation of the well-known unit hydrograph method is used to develop confidence intervals for runoff criterion variables (e.g., peak flow rate, volume, pipe size, etc.). The multivariate normal distribution is used with the SIEM to provide a continuous probability distribution of the selected criterion variable. Example applications to estimating a peak flow rate associated with a future effective rainfall event are considered using measured rainfall-runoff data to develop the underlying probabilistic distributions of the associated stochastic processes. Any runoff criterion variable can be evaluated for confidence interval estimates using the procedures discussed. Extension of the above probabilistic techniques to other rainfall-runoff modeling approaches can be readily achieved by analyzing the rainfall-runoff modeling error as a stochastic process [*Hromadka and Whitley*, 1989].

TABLE 2. Lower Triangular Matrix $L$ (Covariance Matrix $C = LL^T$) for Time Intervals 27, 28, 29, 30, and 31

| | Column 1 | Column 2 | Column 3 | Column 4 | Column 5 |
|---|---|---|---|---|---|
| Row 1 | 61.6 | | | | |
| Row 2 | 65.1 | 2.6 | | | |
| Row 3 | 68.1 | 5.5 | 0.6 | | |
| Row 4 | 70.5 | 8.4 | 1.4 | 0.3 | |
| Row 5 | 72.0 | 11.2 | 2.1 | 0.8 | 0.2 |

Further research is needed on the important topic of developing regionalized multivariate normal distributions of rainfall-runoff modeling error, as well as the extent to which this model fits observed data. Regionalization would provide an estimate of the means and variances in the multivariate normal distribution estimation of modeling error, which could then be transferred to ungauged catchments where the rainfall-runoff model is to be applied. In this fashion, confidence intervals could be estimated for runoff criterion variables of interest, in order to make better design and planning decisions which include uncertainty issues and risk. Further study would also include the effect of estimation errors on the final results.

In this note the distribution for the criterion variable, for example max $Q$, is computed given the effective rainfall $e(\ \ )$, which is the conditional expectation $E(\max Q|e(\ \ ))$. If the distribution for $e(\ \ )$ were known, say, for a class of severe storms, then the (marginal) distribution for max $Q$ could be obtained which would then reflect both the variation in the unit hydrograph and the variation in effective rainfall.

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T. V. Hromadka II, Williamson and Schmid, 15101 Red Hill Avenue, Tustin, CA 92680.

R. J. Whitley, Department of Mathematics, University of California, Irvine, CA 92717.

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