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# Ground-Water Contaminant Transport Modeling Using Complex Variable Element Method

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**Abstract**

The Complex Variable Boundary Element Method (CVBEM) is used to develop a simple but powerful numerical model of contaminant transport of a conservative species in a saturated, confined groundwater aquifer. In this paper, only the steady-state, two-dimensional, advection transport flow problem is considered. Applications include background flows, sources and sinks, and flows introduced by boundary conditions. The numerical model produces locations of streamlines and the contaminant front locations as it changes in time. Because the CVBEM exactly solves the governing mathematical PDE, there is only error in matching prescribed boundary conditions.

## Introduction

Potential flow theory may be used to depict streamlines of groundwater flow for analyzing the extent of subsurface contaminant movement. In preliminary studies, potential flow theory can be used to determine if a more sophisticated study based on a long period of observation and an expensive data collection program is required.

## CVBEM Technique

Many flow field problems are not easily solvable due to limitations of readily available analytic functions. The CVBEM, however, provides an immediate extension. In this approach potential flow theory is used to solve analytically the ground water flow field as governed by sources and sinks (pumping wells and recharge wells), while the background flow conditions are modeled by means of a Cauchy integral allocated at nodal points specified along the problem boundary.

The CVBEM technique accommodates nonhomogeneity on a regional scale (i.e., homogeneous in large subdomains of the problem), and can include spatially distributed sources and sinks such as those mathematically described by Poisson's equation. Detailed development of the CVBEM numerical techniques is given by Hromadka (1984a and 1984b).

For steady state, two-dimensional, homogeneous-domain problems, the CVBEM is used to develop an approximation function which combines an exact solution of the governing groundwater flow equation Laplace equation) and approximate solutions of the boundary conditions. For unsteady flow problems, the CVBEM can be used to give approximate solution to the time advancement of groundwater contaminants by implicit finite difference time-stepping procedures analogous to domain models.

Application of the CVBEM Technique

This application of the CVBEM contaminant transport model is restricted to steady-state flow cases in which solute transport is by advection only. When time-dependent boundary conditions are present and dispersion-diffusion effects are significant, a steady state modeling approach becomes inappropriate. A limitation of this technique is that it does not accommodate nonhomogeneity and anisotropy within the aquifer. These complexities rapidly exceed the modeling capability of the analytic function technique.

Applications

Application 1A

Figure 1 shows a completely penetrating well pumping 50 $m^3$/hr from a homogeneous isotropic aquifer 10 m thick. Contaminated water is being recharged at a rate of 50 $m^3$/hr at a second well (injection well) located 848.5 m from the pumping well. Effective porosity is 0.25, saturated hydraulic conductivity is 1 m/hr, and negligible background groundwater flow is assumed. Figure 1 shows the limits of groundwater contamination corresponding to elapsed times of 0.5, 2, and 4 years. The CVBEM model predicts a first arrival of contamination 4.3 years after beginning of the process.

Application 1B

Two discharge wells are added as shown in Figure 2 to the system of Application 1A. The contaminant front is shown for 0.5, 2, and 4 years. It takes 4.3 years for the contaminated water to reach the middle well, and about 5.6 years for the contaminated water to reach the other two wells.

Application 2A

Here we will consider the steady flow pattern produced by a single pumping well (50 $m^3$/hr) near a landfill site with an equipotential boundary ($\phi$ = 2 m) along the coordinate y = 1000 m. As shown in Figure 3, it takes the contaminant front produced by the landfill 9.0 years to reach the pumping well.

Application 2B

When two injection wells are installed between the landfill and the pumping well, their influence on retarding the contaminant movement can be assessed. When 10 $m^3$/hr is injected at each well it takes more than 13 years for the contaminant front to reach the pumping well (See Figure 4).

Summary

The CVBEM was used to develop a model of steady-state, advective, contaminant transport in groundwater. In the CVBEM approach the Laplace and Poisson partial differential equations are solved exactly, so all modeling error occurs in matching the prescribed boundary conditions.

Because the modeling technique is based upon a boundary integral equation approach, domain mesh generators or control-volume (finite element) discretizations are not required. Nodal points are required only along the problem boundary rather than in the interior of the domain. Consequently, the computer-coding requirements are modest and programs based on the CVBEM are easily run on personal computers.

References


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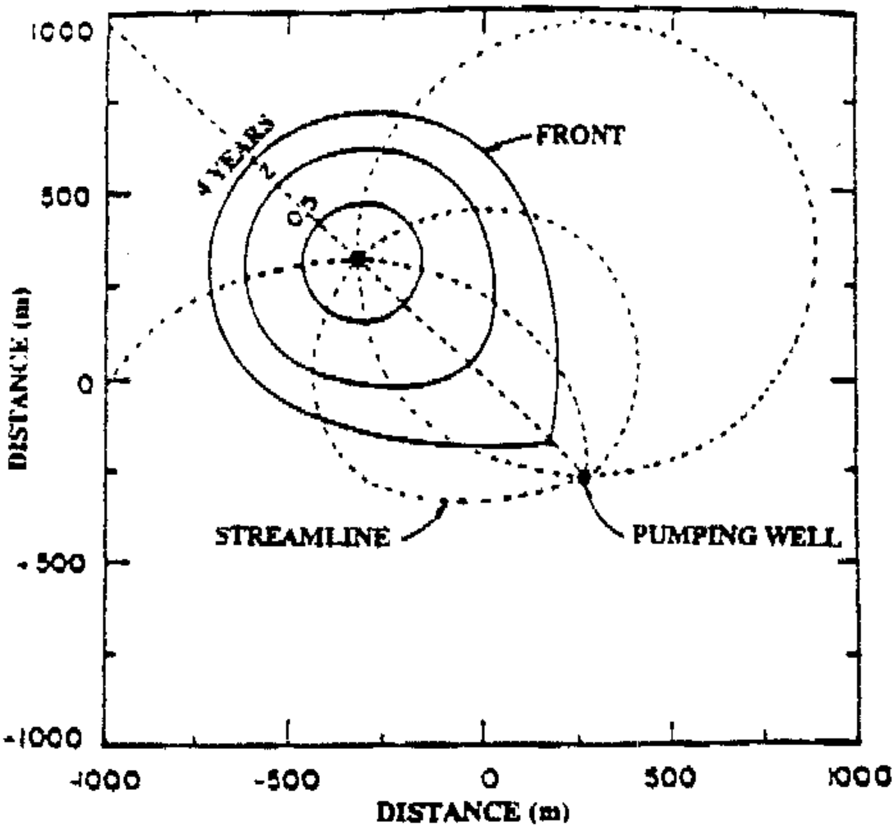


Figure 1. Case 1A

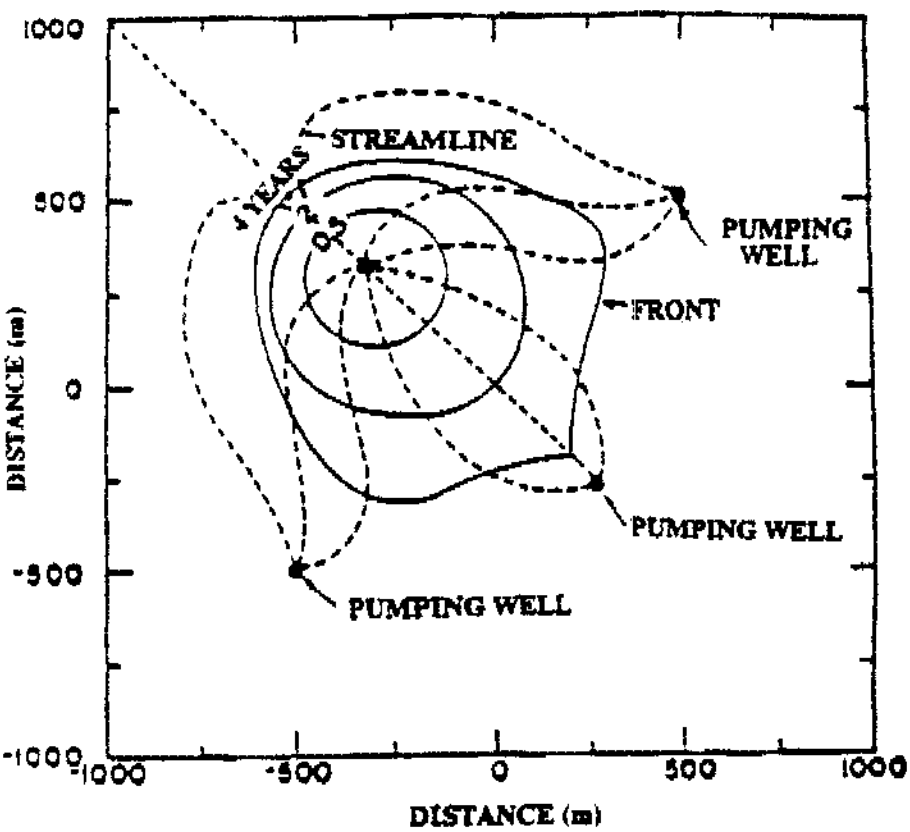


Figure 2. Case 1B

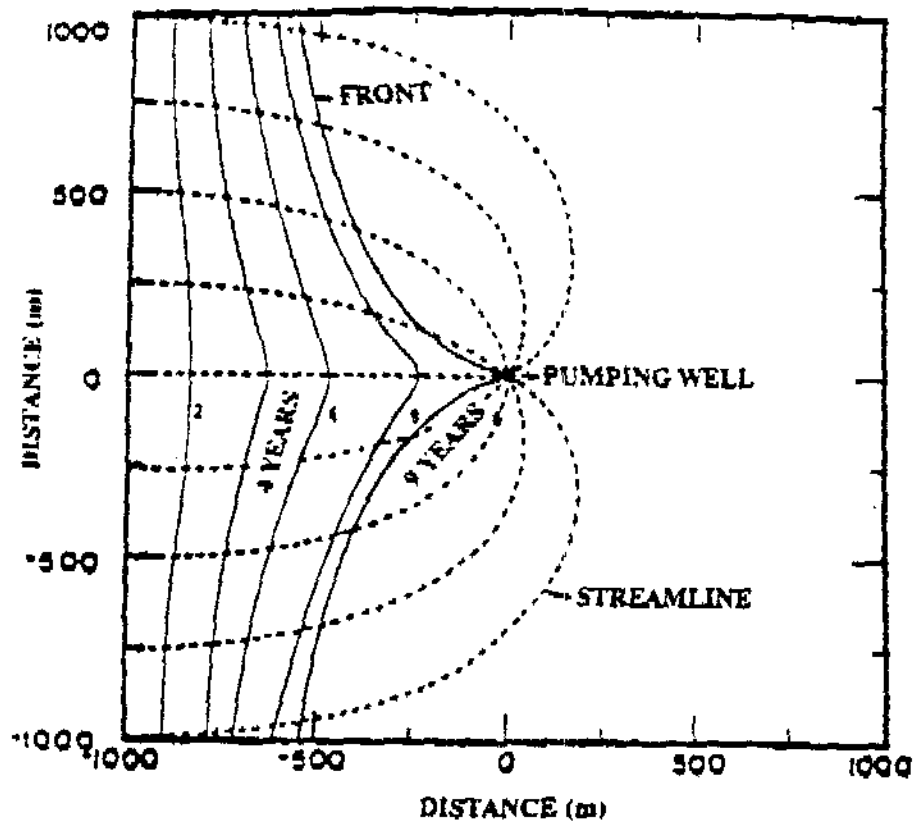


Figure 3. Case 2A

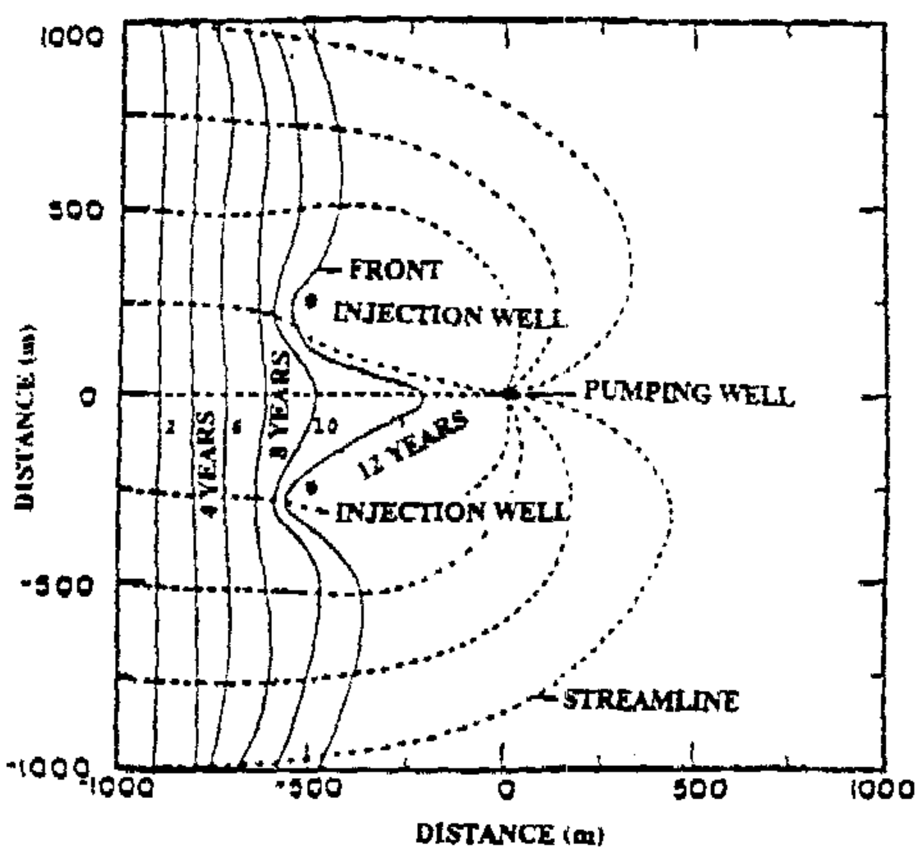


Figure 4. Case 2B