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Evaluation of rainfall-runoff performance using the stochastic integral equation method

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Abstract: The stochastic integral equation method (S.I.E.M.) is used to evaluate the relative performance of a set of both calibrated and uncalibrated rainfall-runoff models with respect to prediction errors. The S.I.E.M. is also used to estimate confidence (prediction) interval values of a runoff criterion variable, given a prescribed rainfall-runoff model, and a similarity measure used to condition the storms that are utilized for model calibration purposes.

Because of the increasing attention given to the issue of uncertainty in rainfall-runoff modeling estimates, the S.I.E.M. provides a promising tool for the hydrologist to consider in both research and design.

Key words: Stochastic integral equation method, rainfall-runoff models, confidence interval

1 Introduction

A recent development in the analysis of rainfall-runoff modeling uncertainty is the use of stochastic integral equations to represent the total error in runoff estimates. The mathematical details of the stochastic integral equation method (S.I.E.M.) are contained in Hromadka and Whitley (1988). In that paper, a mathematical derivation of the well-known design storm approach to estimating T-year values of a runoff criterion variable (e.g., peak flow rate, pipe size, retention basin peak volume demand, etc.) is presented, with special emphasis on a computational example problem which provides both a test of the design storm approach, and also a feasible problem which has an analytic solution. The S.I.E.M. is based upon the theory of stochastic integral equations such has been successfully applied in the field of life science (e.g. Tsokos and Padgett 1974), which provide the mathematical underpinnings of the stochastic integral equation approach, its validity, and application.

In this paper, the S.I.E.M. is applied towards evaluating the performance, with respect to prediction error, of a set of rainfall-runoff models in the estimation of a specific criterion variable value for a hypothetical (e.g. future, or design) storm event. The criterion variable considered herein is the peak flow rate anticipated at a specified location, resulting from a hypothetical rainfall event assumed to be measured at a given rain gauge(s).

Nine rainfall-runoff model structures are considered. Both calibrated and uncalibrated versions of each of the model structures are evaluated, resulting in the consideration of a total of 18 rainfall-runoff models.

The S.I.E.M. is applied towards approximating confidence (prediction) interval estimates of the prescribed criterion variable value. The S.I.E.M. is also applied towards evaluating the relative performance of the considered rainfall-runoff models with respect to prediction errors, given a set of rainfall-runoff data. The use of the S.I.E.M. to represent the model error is somewhat analogous to regression analysis techniques, except that the S.I.E.M. provides for a distribution of runoff hydrographs to be subsequently used in estimating a distribution of values of the prescribed criterion variable, rather than directly regressing the criterion variable values.

2 Problem setting

For storm event ω, rainfall and runoff information at an arbitrary point p in the catchment, at time t, are given by the realizations $P(t,p;\omega)$ and $Q(t,p;\omega)$, respectively. The typical problem setting is to have rainfall data obtained at a single point, p_r, (i.e. the rain gauge), and runoff data obtained at the single point, p, (i.e. the stream guage). We will focus upon the problem of using a rainfall-runoff model, M to predict the runoff response at the stream guage, given a hypothetical storm even (i.e. the rainfall assumed to be applied at point p_r, including prior rainfall considered of importance). In order to simplify the analysis, it is assumed that the storm events are of relatively short time duration (i.e. durations of less than a few days, including the associated antecedent rainfall), and that the catchment is relatively "free draining" in that significant detention or retention effects are minor. Such catchments frequently occur in urbanized areas where storm channel systems are in place that provide for peak flow rate flood protection, without the use of dams or flood control basins.

We will utilize the available record of rainfall-runoff data to develop estimates of the distributions of the several stochastic processes involved. These distributions may then be used to estimate confidence intervals on prescribed runoff criterion variables (e.g. peak flow rate, peak 1-hour mean flow rate, peak flow velocity, pipe size, etc.), based upon the available rainfall-runoff data and the model's performance record in approximating the catchment's runoff response at the stream gauge.

2.1 Stochastic integral equation method (S.I.E.M.)

The rainfall-runoff model, M, operates upon the rainfall realization obtained at point p_r, $P(\cdot,p_r;\omega)$, to produce the model estimate (approximation realization of runoff an point p).

$$M:P(\cdot,p_r;\omega) \rightarrow M(\cdot,p;\omega) \quad (1)$$

The total approximation error, $E(t,p;\omega)$ at time t is

$$E(t,p;\omega)=Q(t,p;\omega)-M(t,p;\omega). \quad (2)$$

Here, $Q(t,p;\omega)$ is the true runoff hydrograph. For each storm event, ω, there are the associated realizations $\{P(\cdot,p_r;\omega), M(\cdot,p;\omega), Q(\cdot,p;\omega)\}$.

It is assumed that the model will produce identical model output realizations given identical rainfall input realizations. However, the rainfall-runoff model may be such that several different rainfall events result in identical model output realizations. We define equivalence classes of such model output realizations by $<M(\cdot,p;\omega_i)>$ where, for all storm events defined to begin at storm time t=0,

$$<M(\cdot,p;\omega_i)>=\{M(\cdot,p;\omega):M(t,p;\omega)=M(t,p;\omega_i)\}. \quad (3)$$

In Eq. (3), $<M(\cdot,p;\omega_i)>$ is seen to be the set of all model output realizations that are identical to the model output realization for storm event ω_i.

The total approximation error is now written as a stochastic integral equation (Tsokos and Padgett 1974; Hromadka and Witley 1988) by

$$E(t,p;\omega)=\int_{s=0}^{t} M(t-s,p;\omega)h(s,p;\omega)ds \tag{4}$$

The term $h(\cdot,p;\omega)$ is seen to be a transfer function, to be convoluted with the model output, $M(\cdot,p;\omega)$. (In this paper, a least squares technique is used to evaluated the transfer functions.) Because of the many random variabilities in the storm event, the several hydrologic and hydraulic processes, and other factors, identical model output realizations in $<M(\cdot,p;\omega_i)>$ will generally have associated stream gauge measured runoff realizations which are different. Consequently, for each pair of realizations $\{M(\cdot,p;\omega_j), M(\cdot,p;\omega_k)\}$ in $<M(\cdot,p;\omega_i)>$, there is and associated pair of realizations of approximation error, $\{E(\cdot,p;\omega_j), E(\cdot,p;\omega_k)\}$, and these realizations of approximation error will generally differ. Therefore, for each equivalence class of model output, $<M(\cdot,p;\omega_i)>$, there is an associated distribution of realizations of $h(\cdot,p;\omega)$, noted as $[h(\cdot,p;\omega)|M_{\omega_i}]$, where M_{ω_i} is shorthand notation for $M(\cdot,p;\omega_i)$, developable from use of Eqs. (3) and (4). If there are N events in an equivalence class of model output, $<M(\cdot,p;\omega_i)>$, then it is possible to determine N realizations of $h(\cdot,p;\omega)$ for this equivalence class by solving Eqs. (2) and (4).

Should the rainfall-runoff model be used to predict runoff at the stream gauge, resulting from a hypothetical storm event, ω_D, defined by the hypothetical rainfall event, $P(\cdot,p_r;\omega_D)$, the mode produces the single realization,

$$M:P(\cdot,p_r;\omega_D) \rightarrow M(\cdot,p;\omega_D). \tag{5}$$

We now identify the equivalence class of model output, $<M(\cdot,p;\omega_D)>$, in order to obtain the associated distribution of realizations, $[h(\cdot,p;\omega)|M_{\omega_D}]$. The S.I.E.M. results in a distribution of runoff hydrograph outcomes at the stream gauge, $[Q(t,p;\omega_D)]$, for hypothetical storm event ω_D, by

$$[Q(t,p;\omega_D)]=M(t,p;\omega_D)+\int_{s=0}^{t} M(t-s,p;\omega_D)[h(s,p;\omega)|M_{\omega_D}]ds. \tag{6}$$

If there are N events in $<M(\cdot,p;\omega_D)>$, then it is possible to determine N estimates of the runoff hydrograph for event ω_D by computing Eq. (6) for each $h(\cdot,p;\omega_D)$ associated with $<M(\cdot,p;\omega_D)>$. Let A be the criterion variable of interest (e.g., peak flow rate, pipe size, etc.). Then for the hypothetical storm event, ω_D, the value of A is determined to be a random variable distributed as

$$[A(\omega_D)]=A[Q(\cdot,p;\omega_D)]. \tag{7}$$

Here, A is used as notation for operating upon each outcome of the random process, $[Q(\cdot,p;\omega_D)]$, in turn, in order to obtain samples of the criterion variable value. Again, if there are N events in $<M(\cdot,p;\omega_D)>$, it is possible to determine N estimates of the criterion variable, for use in developing a frequency distribution. From $[A(\omega_D)]$, confidence (prediction) intervals can be approximated, and these confidence integral estimates depend upon the rainfall-runoff model used.

It is noted that in the above development, the use of equivalence classes necessarily results in the S.I.E.M. formulation of Eq. (4) being equivalent to the use of Eq. (2) directly. That is, the error determined from Eq. (2) is equivalent to the error from Eq. (4) when one is estimating the error for some future event that produces a model output that fits into an equivalence class. Of key importance, however, is estimating the error for a model output that doesn't fit into an equivalence class. In the next section, the use of equivalence classes is relaxed in order to obtain a more workable extension of the S.I.E.M.

3 Approximation of criterion variable confidence intervals

In practice, we only have a small sample of the total ensemble of realizations associated to the various stochastic processes involved. Consequently, precise definition of equivalence classes, as used in the previous development, is impossible. In oreder to develop meaningful statistics, we need to assemble a reasonable sampling of the realizations. One approach is to use a similarity measure of closeness for comparing realizations of $M(\cdot,p;\omega)$. That is, we will group together model output realizations which are "similar" to each other, rather than being identical. The use of equivalence classes and a similarity measure is analogous to the procedures used in Troutman (1985) to condition the independent variable in regression analysis. A recent use of conditioning for channel flood routing modeling is given in Becker and Kundzewicz (1987). Although one similarity measure is used in the applications considered in a later section of this paper, an overall "best" similarity measure is not proposed herein. Rather, given a prescribed similarity measure, the S.I.E.M. can be used to represent the model error.

Let $\mu(M_{\omega_i},M_{\omega_j})$ be notation for the measure of similarity between two model output realizations of runoff for storm events ω_i and ω_j. For example, the measure of similarity may be based upon a set of characteristic runoff values such as { peak 1-hour mean value of flowrate, peak 2-hour mean value of flowrate }. Then $\mu(M_{\omega_i},M_{\omega_j})<\varepsilon$ indicates that $M(\cdot,p;\omega_i)$ and $M(\cdot,p;\omega_k)$ are considered to be within ε similarity according to the prescribed similarity measure used. Now let $<M(\cdot,p;\omega_i),\mu,\varepsilon>=\{M(\cdot,p;\omega):\mu(M_{\omega_i},M_{\omega})<\varepsilon\}$. That is, $<M(\cdot,p;\omega_i),\mu,\varepsilon>$ is the set of all model output realization which are ε-similar to outcome $M(\cdot,p;\omega_i)$. Then as before, for the selected similarity measure μ, tolerance ε, and model M, given $<M(\cdot,p;\omega_i),\mu,\varepsilon>$ there is an associated distribution $[h(\cdot,p;\omega)|M_{\omega_i},\mu,\varepsilon]$. And in prediction, the runoff estimate for hypothetical storm event ω_D is approximately distributely as

$$[Q(t,p;\omega_D)]=M(t,p;\omega_D)+\int_{s=0}^{t} M(t-s,p;\omega_D)[h(s,p;\omega)|M_{\omega_D},\mu,\varepsilon]ds. \tag{8}$$

And analogous to Eq. (7), confidence intervals for criterion variable values, for hypothetical storm even ω_D, are approximated by use of Eq. (8).

For a given record of rainfall-runoff data, the estimates of the distribution $[Q(\cdot,p;\omega_D)]$ depend on the selected model, M; the selected similarity measure, μ; and the selected tolerance, ε, used to develop the similarity classes of model output, $<M(\cdot,p;\omega_i),\mu,\varepsilon>$. And the estimates of the distribution of the runoff criterion variable also depend on the above selections. And of course, any estimate of the various distributions involved are subject to the usual errors due to statistical sampling. Because of the dependence of estimates of $[Q(\cdot,p;\omega_D)]$ on M, μ, and ε, one may consider the notation,

$$[Q(\cdot,p;\omega_D)]=[Q(\cdot,p;\omega_D)|M,\mu,\varepsilon]. \tag{9}$$

And from the above discussion, it is seen that our estimate of the distribution of runoff criterion variable A is given for even ω_D by

$$[A(\omega_D)] \approx [A(\omega_D) | M, \mu, \varepsilon] \tag{10}$$

where

$$[A(\omega_D) | M, \mu, \varepsilon] = A[Q(\cdot, p; \omega_D) | M, \mu, \varepsilon]. \tag{11}$$

The use of a similarity measure to partition the probability space of model output into classes is analogous to conditioning. In effect, one is claiming that the rainfall-runoff modeling error depends upon the modeling predictions. Perhaps the rainfall-runoff model is though to be more accurate when large runoff events are being handled, than when small runoff events are being studied. Or one may suppose that the modeling error is linear with respect to the model prediction of runoff, and define a single storm class to accommodate all model estimates. One may elect to utilize other approaches to partition the probability space of model output; for example, Hromadka and Whitley (1988) consider storm classes of measured storm rainfalls and assumed basin-averaged effective rainfall for formulating storm classes. The need for similarity measures becomes especially important in situations involving soil-moisture accounting; that is, a simple unit hydrograph model with a fixed phi-index loss function can become an improved estimator of runoff should the phi-index be conditioned based upon prior rainfall conditions, i.e., by use of storm classes of model input.

4 Rainfall-runoff models, and the variance in the criterion variable estimates

There exists a wide range of rainfall-runoff models in use today. Each model applied to a record of rainfall-runoff data will typically result in different estimates of confidence intervals for the specified runoff criterion variable. The S.I.E.M. provides a means for evaluating the performance of the selected rainfall-runoff model. Adding complexity to the rainfall-runoff model (such as subdividing the catchment into smaller subareas, or adding another component to the hydrologic mass balance algorithms) may or may not account for the unexplained phenomena that contribute to the variance in the model runoff predictions. The application of the S.I.E.M. to evaluating rainfall-runoff model performance, with respect to prediction error, is demonstrated in the following example computational problem.

Example problem

The use of the S.I.E.M. to evaluate rainfall-runoff model performance, with respect to prediction error, will be demonstrated using a set of data for a fully urbanized catchment located in Los Angelos, California. The catchment has a fully developed storm channel system that was designed to protect for peak flood flows associated to severe storm events, (i.e., a 50-year design storm event), and any hydraulic effects due to backwater may be assumed negligible. The catchment is subject to coastal rainfall events, with most storms of flood control interest being of durations less than 24-hours. Three rain gauges are available within the approximately 15 square-mile catchment area, resulting in the rainfall data being assumed applicable to nearly equal area sizes of the catchment. Because the rainfall used for model input depends on the location where the runoff is being studied, (due to a linear weighting of more than one rain gauge), we write the assumed rainfall as $P(\cdot, p; \omega)$. A stream gauge, (point p), is located in one of the large concrete channels.

A review of 25-years of rainfall-runoff data indicated that only 15 storm events were of interest for flood control purposes. These storms were assumed to be elements of the same model output similarity class (i.e., storm class). That is, the similarity measure used in this example is simply to partition all storm events into two similarity classes; namely, significant and insignificant storm events. Obviously, a myriad of other similarity measures are possible, and the resulting S.I.E.M. distributions reflect both the similarity measure and the rainfall-runoff model being used. We will consider the use of several rainfall-runoff models.

The first rainfall-runoff model considered, M_1, is a single area unit hydrograph model with effective rainfall estimated by

$$F_1(t,p;\omega)=y_1 P(t,p;\omega) \tag{12}$$

where $F_1(\cdot,p;\omega)$ is the area-averaged effective rainfall assumed for the entire catchment tributary to the stream gauge, for storm event ω, and y_1 is a constant fraction coefficient. The model estimates of runoff are given by

$$M_1(t,p;\omega)=\int_{s=0}^{t} F_1(t-s,p;\omega)\psi_1(s)ds \tag{13}$$

where $\psi_1(s)$ is the unit hydrograph. The values for y_1 and $\psi_1(s)$ used in Eq. (13) were determined by use of a local flood control agency design criteria. Thus, we are in essence able to test the rainfall-runoff model against actual data. (Normally, the availability of runoff data affords the opportunity to "calibrate" the model by selecting values of y_1 and $\psi_1(s)$ which improve the model's performance. At ungauged catchments we don't have such runoff data; therefore, one is unable to calibrate the model.) Our analysis provides an indication as to how the model may perform at an ungauged catchment.

Using the above 15 storm events, 15 realizations of $E_1(\cdot,p;\omega)$ are obtained for the considered storm class using Eq. (2). Because the model M is a linear operator on the elements of the storm class, one can use either the model output realizations, $M_1(\cdot,p;\omega)$, or the effective rainfall realizations $F_1(\cdot,p;\omega)$, in relating to the approximation error realizations, $E_1(\cdot,p;\omega)$, with equivalent results. That is, for each storm event, ω, in our storm class,

$$E_1(t,p;\omega)=\int_{s=0}^{t} M_1(t-s,p;\omega)h_1(s,p;\omega)ds, \quad or \tag{14}$$

$$E_1(t,p;\omega)=\int_{s=0}^{t} F_1(t-s,p;\omega)g_1(s,p;\omega)ds \tag{15}$$

and from Eq. (13), the transfer functions $g_1(s,p;\omega)$ and $h_1(s,p;\omega)$ are but linear transformations of each other (i.e., a convolution), for all elements in the storm class.

Equation (15) was used in this model analysis, resulting in 15 realizaitons of $g_1(\cdot,p;\omega)$. A hypothetical storm event, ω_D, was defined by simply averaging together the 15 rainfall patterns,

$$P(t,p;\omega_D)=\frac{1}{15}\sum_{j=1}^{15}P(t,p;\omega_j) \tag{16}$$

where each $P(t,p;\omega_j)$ is translated to begin at model time zero. Using ω_D, the model M_1 and the S.I.E.M. result in the sampling of 15 runoff hydrograph realizations shown in Fig. 1. From the figure, the frequency-distribution of a selected criterion variable can be developed by operating on each sampled realization of runoff. An estimate of the frequency-distribution of $[A(\omega_D)]$ resulting from use of M_1 can be readily developed from the figure, where A is the criterion variable of peak flow rate. For model M_1, the expected peak flow rate is 3730 cfs, with a standard deviation of 480 cfs.

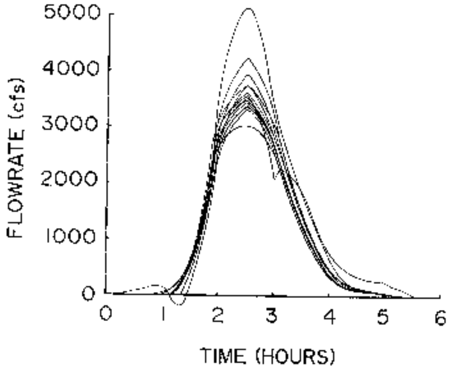

Figure 1

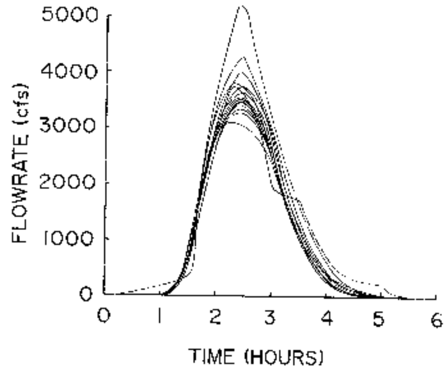

Figure 2

Figure 1. Distribution of Runoff Hydrographs, using model M_1, for hypothetical event ω_D

Figure 2. Distribution of Runoff Hydrographs, using model M_2, for hypothetoical event ω_D

An examination of Fig. 1 indicates that the inclusion of an initial abstraction component, Ia, may be useful. Rainfall-runoff model, M_2, will be based upon the M_1 model and parameters, but with an added Ia component defined as the initial 0.10-inches of rainfall. Thus, the effective rainfall is estimated in M_2 by

$$F_2(t,p;\omega)=\begin{cases} 0;\ \text{for} \int\limits_{s=0}^{t} P(s,p;\omega)ds \le 0.10 \\ \bar{y}_1 P(t,p;\omega);\ \text{otherwise} \end{cases} \tag{17}$$

Using M_2, another set of 15 realizations of $M_2(\cdot,p;\omega)$ are developed. When the S.I.E.M. is applied to the hypothetical storm event, $P(\cdot,p;\omega_D)$, the estimate of runoff is another set of 15 possible outcomes, shown in Fig. 2. For the criterion variable of peak flow rate, the expected peak flow rate is 3685 cfs, with a standard deviation of 460 cfs.

A third rainfall-runoff model, M_3, is based upon the unit hydrograph employed in models M_1 and M_2, but uses the effective rainfall estimate of

$$F_3(t,p;\omega)=\begin{cases} P(t,p;\omega)-f(t,p);\ \text{when positive} \\ 0\ ;\ \text{otherwise} \end{cases} \tag{18}$$

where $f(t,p)$ is an exponential loss rate defined by

$$f(t,p)=f_o+(f_\infty-f_o)e^{-kt} \tag{19}$$

In M_3, parameters f_o, f_∞, k are defined by f_o=0.8 inch/hour, f_∞=0.3 inch/hour, and k is defined such that $f(t,p)$ is within 5-percent of f_∞ at storm of 25-minutes. These parameter values are based upon the same local flood control agency criteria used for developing the synthetic $\psi_1(s)$.

Upon application of model M_3 to the rainfall-runoff data, another set of total error realizations, $E_3(\cdot,p;\omega)$, is developed. For the hypothetical storm event, ω_D, and the criterion variable of peak flow rate, the expected peak flow rate is 4,205 cfs with a standard deviation of 520 cfs.

Rainfall-runoff models, M_4, M_5, and M_6, are each defined to be a three-subarea unit hydrograph model, where subareas are selected to approximately conform to the catchment area wherein each rain gauge is assumed to apply. A unit hydrograph is estimated for each subarea by the same local flood control agency design criteria used in the previous models. The selected effective rainfall estimators for M_4, M_5, and M_6, conform to an application of the loss functions used in the previous models M_1, M_2, M_3, respectively. The resulting expected peak flow rates, m_x, and standard deviations, α_x, for model M_x, are as follows: $M_4 : m_4$=3855 cfs, σ_4=480 cfs; $M_5 : m_5$=3710 cfs, σ_5=490 cfs; $M_6 : m_6$=4260 cfs, σ_6=540 cfs.

Rainfall-runoff models M_7, M_8, and M_9, are based upon a highly subdivided version of models M_4, M_5, and M_6. A total of 68 subareas are used (each approximately 140 acres in size) in each of the three models. The drainage system is modeled as a combination of pipeflow and open channel flow hydraulic links, using a kinematic wave routing technique. The only difference between models M_7, M_8, and M_9, are the subarea effective rainfalls, which conform to the estimators used in models M_1, M_2, and M_3, respectively. Subarea unit hydrographs are estimated using the previously mentioned criteria. The results in predicting the runoff criterion variable of peak flow rate, at the stream gauge location, for the hypothetical storm ω_D, are as follows:

$M_7 : m_7$=3910cfs, σ_7=450cfs; $M_8 : m_8$=3820cfs, σ_8=460cfs;

$M_9 : m_9$=4455cfs, σ_9=530cfs.

Table 1 summarizes the several rainfall-runoff model estimates for this example problem. It is seen that the expected peak flow rate estimates range from 3685 cfs to 4455 cfs, and the standard deviations range from 450 cfs to 540 cfs. It is also seen that, for this problem, use of the practice of subdivision into subareas and channel routing tends to effect peak flow rate estimates in a consistent manner, resulting in higher estimates, of the expected peak flow rate.

5 Rainfall-runoff model calibration

Each of the above rainfall-runoff models can be calibrated. Because of the data availability, the loss function parameters can be uniformly modified. Additionally, runoff timing effects of a model can be adjusted by uniformly modifying all channel routing link hydraulic parameters (i.e., Manning's friction-factors), or modifying the single area unit hydrograph model lag times. Such a calibration effort was performed for each of the nine rainfall-runoff models by using the following objective: choose model parameters, by trial-and-error, as to minimize the difference between the model estimates and the measured data for the peak 30-minutes of runoff, for each storm event. Obviously, other objective functions could be used. The above objective was selected as a simple test for achieving reasonable estimates of peak flood flow characteristics which would be important to the flood control system being studied.

The calibrated models are then again applied to estimating the peak flow rate for the hypothetical storm event, ω_D. The results are itemized in Table 2. Table 2 indicates a range in expected peak flow rates, using the calibrated rainfall-runoff models, of 3895 cfs - 4500 cfs, with a range in standard deviations of 410 cfs - 530 cfs.

6 Confidence (prediction) interval estimates

The results contained in Tables 1 and 2 can be directly used to obtain one-sided confidence (prediction) interval estimates for the runoff criterion variable of peak flow rate. Table 3 provides the final tabulation for our example problem. In Table 3, normality is assumed for convenience only. The computational results used to develop Tables 1 and

Table 1. Peak flow rate estimates for example problem (without calibration of models)

Model Number	Loss Function Elements	Number of Subareas	Number of Routing Links	Number of Peak Flow Rate (cfs)	Expected Deviation (cfs)
1	$yP(\cdot,p;\omega)$	1	-	3730	480
2	$yP(\cdot,p;\omega);Ia$	1	-	3685	460
3	$f_0+(f_\infty-f_0)e^{-kt}$	1	-	4205	520
4	see #1	3	-	3855	480
5	see #2	3	5	3710	490
6	see #3	3	5	4260	540
7	see #1	68	27	3910	450
8	see #2	68	27	3820	460
9	see #3	68	27	4455	530

Table 2. Peak flow rate estimates for example problem (with calibration of models)

Model Number	Loss Function Elements	Number of Subareas	Number of Routing Links	Number of Peak Flow Rate (cfs)	Expected Deviation (cfs)
1	$yP(\cdot,p;\omega)$	1	-	3895	480
2	$yP(\cdot,p;\omega);Ia$	1	-	3910	410
3	$f_0+(f_\infty-f_0)e^{-kt}$	1	-	4400	490
4	see #1	3	-	3960	475
5	see #2	3	5	3950	430
6	see #3	3	5	4450	510
7	see #1	68	27	3960	454
8	see #2	68	27	3900	460
9	see #3	68	27	4500	530

2, can be used to prepare a frequency-histogram of the runoff criterion variable values, and estimates of confidence intervals obtained directly from the computed data. In the considered example problem, the one and two-standard deviation estimates from the histogram were found to be within a few percent of the corresponding values given in Table 3.

Table 3 summarizes the demonstrated performance of nine different rainfall-runoff models for the two case studies of (i) where a standardized (or regionalized) rainfall-runoff model is used at an ungauged catchment; and (ii) where the model is calibrated to local runoff data when such data are available. Obviously, there are a wide spectrum of rainfall-runoff modeling approaches that are not considered in the table, but any of these other models can be included in our comparison by use of the above S.I.E.M. procedures.

7 Conclusions

The stochastic integral equation method (S.I.E.M.) is used to evaluate the relative performance, with respect to prediction error, of a set of both calibrated and uncalibrated rainfall-runoff models. The S.I.E.M. is also used to estimate confidence (prediction) interval values of a runoff criterion variable, given a prescribed rainfall-runoff model. Although only the peak flow rate is considered in this paper, the extension of the various concepts and procedures to other criterion variables follows directly.

Several topics need to be addressed in future research. Among them include, and examination of similarity measures, an examination of probability distributions used to estimate confidence intervals, regionalization of transfer function realizations for use at ungauged locations, and several other topics.

Table 3. One-sided confidence (prediction) interval estimates for peak flow (example problem)

Model Number	Calibrated	m (50%)	σ	$m+\sigma$ (84%)	$m+2\sigma$ (98%)
1	no	3730	(480)	4210	4690
2	no	3685	(460)	4145	4605
3	no	4205	(520)	4725	5245
4	no	3855	(480)	4335	4815
5	no	3710	(490)	4200	4690
6	no	4260	(540)	4800	5340
7	no	3910	(450)	4360	4810
8	no	3820	(460)	4280	4740
9	no	4455	(530)	4985	5515
1	yes	3895	(450)	4345	4795
2	yes	3910	(410)	4320	4730
3	yes	4400	(490)	4890	5380
4	yes	3960	(475)	4435	4910
5	yes	3950	(430)	4380	4810
6	yes	4450	(510)	4960	5470
7	yes	3960	(450)	4410	5064
8	yes	3900	(460)	4360	4820
9	yes	4500	(530)	5030	5560

Notes: 1. m= expected peak flow rate, 2. (50%) = 50-percent confidence (prediction) interval
3. (σ)= standard deviation of peak flow rate distribution

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