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Use of Subareas in Rainfall-Runoff Models, I: Development of a Multi-Linear Runoff Model Approximation

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ABSTRACT

In all phases of hydrologic modeling (including calibration, design, and watershed evaluation) analyses on all except the smallest watersheds involve watershed subdivision (or discretization). This subdivision is often done without proper discretization guidelines and in most cases little attention has been paid to the effect of this practice on the accuracy of model results. Due to the sparse rainfall-runoff data typically available at a watershed, it is questionable whether the subdivision of the catchment into subareas, when there are no data to calibrate subarea hydrologic model parameters, is a "better" approach to modeling the catchment response. In this paper, we examine how catchment discretization interrelates with the detail that effective rainfall (i.e., rainfall less losses) is known spatially. It is shown that the practice of discretization of the catchment into subareas may introduce conceptual difficulties in the modeling process unless rainfall-runoff data are available in each subarea to validate the subarea hydrologic parameters independent of the other subareas. Further, calibration of the simple model accounts for the probabilistic variation of effective rainfall over the watershed that is not accounted for by the discretized model unless subarea rainfall-runoff data are available and used.

A multi-linear stochastic unit hydrograph model is developed in this paper as an approximation of the rainfall-runoff modeling process. The companion paper evaluates the effects of subdivision by use of the multi-linear model.

KEY WORDS: Rainfall-runoff models, uncertainty, stochastics, subdivision

INTRODUCTION

Many hydrologic models allow for the subdivision of the catchment into subareas, each linked by channel routing submodels (i.e., a link-node model). The effect of subdividing a catchment on modeling accuracy has not been fully investigated. The calibration of a link-node model to available rainfall-runoff data is a related issue, and the method of selecting the model parameters is important to the accuracy of the link-node modeling approach. Also, the uncertainty in the modeling boundary conditions (i.e., the true precipitation distribution over the catchment) is propogated into the fitted parameters of the model itself, and the effect of insufficient knowledge of storm morphology affects model accuracy. These three factors (i.e., watershed subdivision, parameter estimation, and storm morphology effects) are important to the accuracy of hydrologic designs.

In this paper, the unit hydrograph method (UH) is used to develop estimates of runoff modeling error in the frequently occurring cases where the uncertainty in the effective rainfall distribution (i.e., rainfall less losses; rainfall excess) over the catchment dominates all other sources of modeling uncertainty. Indeed, just the uncertainty in the precipitation distribution appears to be a limiting factor in the successful development, calibration, and application of all surface runoff hydrologic models (e.g., Loague and Freeze, 1985; Beard and Chang, 1979; Schilling and Fuches, 1986; Garen and Burges, 1981, Nash and Sutcliffe, 1970; Troutman, 1982).

Based on the literature, the main difficulty in the use, calibration, and development, of rainfall-runoff models appears to be the lack of precise effective rainfall data and the high model sensitivity to (and magnification of) rainfall measurements errors. Nash and Sutcliffe (1970) write that "As there is little point in applying exact laws to approximate boundary conditions, this, and the limited ranges of the variables encountered, suggest the use of simplified empirical relations."

Troutman (1982) also discussed the often cited difficulties with the error in precipitation measurements "due to the spatial variability of precipitation." This source of error can result in "serious errors in runoff prediction and large biases in parameter estimates by calibration of the model."

While surface runoff hydrologic models continue to be developed in technical component complexity, typically including additional algorithms for hydraulic routing effects and continuous soil moisture accounting, the problem setting continues to be poorly posed in a mathematical approximation sense in that the problem boundary conditions (i.e., the storm rainfall over the catchment) remain unknown. Indeed, the usual case in studying catchment runoff response is to have only a single rain gauge and stream gauge available for data analysis purposes; and oftentimes, neither gauge is within the study catchment. As a result, the effective rainfall distribution over the catchment remains unknown; hence, the problem's boundary conditions must be approximated as part of the problem solution.

In this paper and the companion paper, the relative relationship between the degree of subdivision and the available rainfall-runoff data to evaluate subarea runoff response is considered. By use of linear models in subarea runoff, channel link routing, and the distribution of effective rainfall over the catchment with respect to a given measure of effective rainfall, a distributed parameter link-node rainfall-runoff model may be analyzed as to the effect of each of its individual components. By use of storm classes, the above linear model becomes multi-linear (i.e., nonlinear), and can be used as an approximation of other rainfall-runoff model responses. This multi-linear model is useful in the companion paper to approximately evaluate the effects of subdivision on rainfall-runoff model response.

CATCHMENT AND DATA DESCRIPTION

Let R be a free draining catchment with negligible storage effects. R is discretized into m subareas, R_j, each draining to a nodal point which is drained by a channel system. The m-subarea link node model resulting by combining the subarea runoffs for storm i, adding runoff hydrographs at nodal points, and routing through the channel system, is denoted as $Q_m^i(t)$. It is assumed that there is only a single rain gauge and stream gauge available for data analysis. The rain gauge site is assumed to be also monitored for a "true" point value of effective rainfall, $e_g^i(t)$. That is, we assume that an accurate value of $e_g^i(t)$ is available, even though this data are measured at the rain gauge site which may be located outside of the catchment. The catchment runoff characteristics at the rain gauge site are assumed to be similar throughout R. The stream gauge data represents the entire catchment, R, and is denoted by $Q_g{}^i(t)$ for storm event i.

LINEAR EFFECTIVE RAINFALLS FOR SUBAREAS

The effective rainfall distribution (rainfall less losses) in subarea R_j is given by $e_j{}^i(t)$ for storm i where $e_j{}^i(t)$ is assumed to be linear in $e_g{}^i(t)$ by:

$$e_j{}^i(t) = \sum \lambda_{jk}{}^i \, e_g{}^i(t-\theta_{jk}{}^i), \; j = 1,2,\ldots,m \tag{1}$$

where $\lambda_{jk}{}^i$ and $\theta_{jk}{}^i$ are coefficients and timing offsets, respectively, for storm i and subarea R_j. In Eq.(1), the variations in the effective rainfall distribution over R due to magnitude and timing are accounted for by the $\lambda_{jk}{}^i$ and $\theta_{jk}{}^i$, respectively. Equation 1 provides that the subarea effective rainfalls are similar to that measured (assumed in our development) at the rain gauge site, and that the differences are primarily due to random processes. Figure 1 illustrates the linear effective rainfall corresponding to arbitrary subarea, R_j.

SUBAREA RUNOFF

The storm i subarea runoff from R_j, $q_j{}^i(t)$, is given by the linear convolution integral:

$$q_j{}^i(t) = \int_{s=0}^{t} e_j{}^i(t - s) \, \phi_j{}^i(s) \, ds \tag{2}$$

where $\phi_j{}^i(s)$ is the subarea unit hydrograph (UH) for storm i such that Eq. (2) applies. Combining Eqs. (1) and (2) gives

$$q_j{}^i(t) = \int_{s=0}^{t} \sum e_g{}^i(t - \theta_{jk}{}^i - s) \, \lambda_{jk}{}^i \, \phi_j{}^i(s) \, ds \tag{3}$$

Rearranging variables,

$$q_j{}^i(t) = \int_{s=0}^{t} e_g{}^i(t - s) \sum \lambda_{jk}{}^i \, \phi_j{}^i(s - \theta_{jk}{}^i) \, ds \tag{4}$$

where throughout this paper, the argument of the arbitrary function $F(s - Z)$ is notation that $F(s - Z) = 0$ for $s < Z$.

APPLICATION

To illustrate the linear effective rainfall concept, a simple model will be developed for the severe storm of March 1, 1983 over the 25 square mile Compton catchment in Los Angeles, California. This catchment is fully urbanized and is served by a well-designed storm drain system which would have only minor backwater effects for the subject storm. The catchment has available a single rain gauge and stream gauge. The U.S. Army Corps of Engineers (Los Angeles District Office) or COE previously developed regionalized unit hydrographs for this area and, consequently, synthetic unit hydrographs can be estimated from the catchment characteristics of slope and other physical factors.

For demonstration purposes, a two-subarea model of Compton Creek is used where the upstream subarea, R_1, runoff is modeled to be routed by pure translation (without peak flow attenuation) to the Compton stream gauge where the second subarea, R_2, runoff is directly summed. For the above assumptions, the two-subarea model for storm event i is given by $Q_2^i(t)$ where

$$Q_2^i(t) = q_1^i(t - \tau_1^i) + q_2^i(t) \tag{5}$$

where $q_1^i(t - \tau_1^i)$ is the $q_1^i(t)$ runoff from R_1 for storm i, offset in time by τ_1^i due to translation routing; and $q_2^i(t)$ is the runoff from R_2. From Eqs. (1) and (2), $Q_2^i(t)$ is rewritten as

$$\begin{aligned} Q_2^i(t) &= \int_{s=0}^{t} \sum \lambda_{1k}^i \, e_g^i(t - \theta_{1k}^i) \, \phi_1^i(s - \tau_1^i) \, ds \\ &+ \int_{s=0}^{t} \sum \lambda_{2k}^i \, e_g^i(t - \theta_{2k}^i) \, \phi_2^i(s) \, ds \end{aligned} \tag{6}$$

The subarea UH's, $\phi_1(s)$ and $\phi_2(s)$ are estimated using the COE regionalized data. The appropriate sum of subarea runoffs, $q_1^i(t)$ and $q_2^i(t)$, are then set equal to the stream gauge data for the storm, $Q_g^i(t)$, and the respective parameters λ_{jk}^i and θ_{jk}^i are estimated by minimizing the least-squares error, E, where

$$E = ||\omega_1 Q_g^i(t) - q_1^i(t - \tau_1^i)||_2 + ||\omega_2 Q_g^i(t) - q_2^i(t)||_2 \tag{7}$$

In Eq. (7), ω_1 and ω_2 are proportion factors defined by $\omega_1 = A_1/(A_1 + A_2)$ and $\omega_2 = A_2/(A_1 + A_2)$, where A_1, A_2 are the areas or R_1, R_2, respectively. Additionally, E is minimized with the constraint that all factors λ_{jk}^i are nonnegative. The timing offsets, θ_{jk}^i, used in Eq. (6) for this example are 15-minute offsets for the entire 24-hour storm duration. Thus, there are 96 translates being used to minimize E, for each subarea.

The resulting estimates for $e_j^i(t)$ are shown in Figs. 2a,b for subarea R_1 and R_2, respectively. Shown in the figures are the approximations of the $e_j^i(t)$ in comparison to the measured rain gauge data, $P_g^i(t)$. From the figures it is seen that the estimated $e_j^i(t)$ are quite feasible as being the "true" average effective rainfall distributions over R_1 and R_2. Figure 3 shows the comparison between the modeled $Q_2^i(t)$ results (using the $e_j^i(t)$ estimates in Eq. (7)). However, the main objective of this simple application is only to demonstrate the feasibility and utility of the linear effective rainfall relationship of Eq. (1).

Each subarea's effective rainfall distribution, $e_j^i(t)$, can only be accurately determined by the use of runoff data from each subarea used in the model. Should subarea R_j have a stream gauge to measure $e_j^i(t)$, then $e_j^i(t)$ can be equated to the "available" rain gauge site measured effective rainfall, $e_g^i(t)$, by means of Eq. (1). For example, should subarea R_j experience zero rainfall during storm event i, the λ_{jk} in Eq. (1) would all be zero. Equation (1) provides a means to correlate the subarea R_j runoff for storm i, $q_j^i(t)$, to the available effective rainfall data measured at the rain gauge site, $e_g^i(t)$.

LINEAR ROUTING

Let $I_1(t)$ be the inflow hydrograph to a channel flow routing link (number 1), and $O_1(t)$ the outflow hydrograph. A linear routing model of the unsteady routing process is given by

$$O_1(t) = \sum_{k_1=1}^{n_1} a_{k_1} I_1(t - \alpha_{k_1}) \tag{8}$$

where the a_{k_1} are coefficients which sum to unity; and the α_{k_1} are timing offsets. Again, $I_1(t - \alpha_{k_1}) = 0$ for $t<\alpha_{k_1}$. Given stream gauge data for $I_1(t)$ and $O_1(t)$, the best fit values for the a_{k_1} and α_{k_1} can be determined.

Should the above outflow hydrograph, $O_1(t)$, now be routed through another link (number 2), then $I_2(t) = O_1(t)$ and from the above

$$\begin{aligned} O_1(t) &= \sum_{k_2=1}^{n_2} a_{k_2} I_2(t - \alpha_{k_2}) \\ &= \sum_{k_2=1}^{n_2} a_{k_2} \sum_{k_1=1}^{n_1} a_{k_1} I_1(t - \alpha_{k_1} - \alpha_{k_2}) \end{aligned} \tag{9}$$

For L links, each with their own respective stream gauge routing data, the above linear routing technique results in the outflow hydrograph for link number L, $O_L(t)$, being given by

$$O_L(t) = \sum_{k_L=1}^{n_L} a_{k_L} \sum_{k_{L-1}=1}^{n_{L-1}} a_{k_{L-1}} \cdots \sum_{k_2=1}^{n_2} a_{k_2} \sum_{k_1=1}^{n_1} a_{k_1} I_1(t-\alpha_{k_1} - \alpha_{k_2} - \cdots - \alpha_{k_{L-1}} - \alpha_{k_L}) \tag{10}$$

Using an index notation, the above $O_L(t)$ is written as

$$O_L(t) = \sum_{<k>} a_{<k>} I_1(t - \alpha_{<k>}) \tag{11}$$

For subarea R_j, the runoff hydrograph for storm i, $q_j^i(t)$, flows through L_j links before arriving at the stream gauge and contributing to the total measured runoff hydrograph, $Q_g^i(t)$. All of the constants $a^i_{<k>}$ and $\alpha^i_{<k>}$ are available on a storm by storm basis. Consequently from the linearity of the routing technique, the m-subarea link node model is given by the sum of the m, $q_j^i(t)$ contributions,

$$Q_m^i(t) = \sum_{j=1}^{m} \sum_{<k>_j} a^i_{<k>_j} q_j^i(t - \alpha^i_{<k>_j}) \tag{12}$$

where each vector $<k>_j$ is associated to a R_j, and all data is defined for storm i. It is noted that in all cases,

$$\sum_{<k>_j} a^i_{<k>_j} = 1 \tag{13}$$

LINK-NODE MODEL, $Q_m^i(t)$

For the above linear approximations for storm i, Eqs. (1), (4), and (12) can be combined to give the final form for the m subarea link-node model, $Q_m^i(t)$.

$$Q_m^i(t) = \sum_{j=1}^{m} \sum_{<k>_j} a^i_{<k>_j} \int_{s=0}^{t} e_g^i(t-s) \sum \lambda_{jk}^i \phi_j^i(s - \theta_{jk}^i - \alpha^i_{<k>_j})\, ds \tag{14}$$

Because the measured effective rainfall distribution, $e_g^i(t)$, is independent of the several indices, Eq. (14) is rewritten in the form

$$Q_m^i(t) = \int_{s=0}^{t} e_g^i(t-s) \sum_{j=1}^{m} \sum_{<k>_j} a^i_{<k>_j} \sum \lambda_{jk}^i \phi_j^i(s-\theta_{jk}^i - \alpha^i_{<k>_j})\, ds \qquad (15)$$

where all parameters are evaluated on a storm by storm basis, i.

Equation (12) described a linear model which represents the total catchment runoff response based on variable subarea UH's, $\phi_j^i(s)$; variable effective rainfall distributions on a subarea-by-subarea basis with differences in magnitude (λ_{jk}^i), timing (θ_{jk}^i), and pattern shape (linearly assumption); and channel flow routing translation and storage effects (parameters $a^i_{<k>_j}$ and $\alpha^i_{<k>_j}$). All parameters employed in Eq. (15) must be evaluated by runoff data where stream gauges are supplied to measure runoff from each subarea, R_j, and stream gauges are located upstream and downstream of each channel reach (link) used in the model. (In practice, such data are seldom available, and one assumes a set of parameter values.)

RAINFALL-RUNOFF MODEL RESPONSE

The m-subarea model of Eq. (15) is directly simplified to the simple single area UH model (no discretization of R into subareas) given by $Q_1^i(t)$ where

$$Q_1^i(t) = \int_{s=0}^{t} e_g^i(t-s)\, \eta^i(s)\, ds \qquad (16)$$

where $^i(s)$ is the transfer function between the data pair $\{Q_g^i(t), e_g^i(t)\}$, for storm event i.

From Eq. (16) it is seen that the classic single area UH model equates to the highly complex link node modeling structure of Eq. (15), where considerable runoff gauge data is supplied interior of the catchment, R, so that all modeling parameters are accurately calibrated on a storm-by-storm basis. For the case of having available a single rain gauge site (where the effective rainfall is measured, $e_g^i(t)$) and a stream gauge for data correlation purposes, the $^i(s)$ properly represents the several effects used in the development leading to Eq. (15), integrated according to the observed sampling from the several modeling parameters' respective probability distributions. Because the simple $Q_1^i(t)$ model structure actually includes most of the effects which are important in flood control hydrologic response, it can be used to develop useful probabilistic distributions of hydrologic modeling output.

In comparing the two models of Eq. (15) and (16), it is noted that $Q_m^i(t) = Q_1^i(t)$ only when interior runoff data is supplied to accurately evaluate all the modeling parameters used in Eq. (15). For example, should the catchment be discretized into many small subareas with small channel routing links (e.g., such as used in highly subdivided catchments with UH approximations, or as employed in kinematic wave (KW) type models such as MITCAT, or the KW version of HEC-1), then with a stream gauge located at each subarea (or overland flowplane) and at each channel link, all modeling parameters could be accurately evaluated on a storm-by-storm basis, resulting in the formulation of Eq. (15).

Indeed, only by means of subarea stream gauge data can the subarea linear effective rainfall distribution parameters of λ_{jk}^i and θ_{jk}^i be accurately determined for each storm event i. But it is these linear effective rainfall distribution parameters that reflect the important spatial and temporal variability of storm effective rainfall over the catchment which in turn causes the major difficulties in the development, calibration, and use, of hydrologic models (Schilling and Fuchs, 1986; Troutman, 1983; among others).

The current direction of advanced development for hydrologic models is a modeling structure similar to Eq. (15). With subarea and channel-link stream gauge data, the $Q_m^i(t)$ parameters can be accurately determined, and

$$Q_m{}^i(t) = Q_1{}^i(t) \tag{17a}$$

But in the typical case of having only the single rain gauge and stream gauge, all the parameters in Eq. (15) must be approximated, resulting in the estimator, $\hat{Q}_m{}^i(t)$, wherein the subarea linear effective rainfall parameters of Eq. (1) are misrepresented by setting $\theta_{jk}{}^i = 0$ (i.e., zero timing offsets between the measured rainfall at the gauge and the subarea rainfalls), and also by assuming that the magnitudes of rainfall intensities are invariant between subareas and the rain gauge.

From the above discussion, when the formulation of $\hat{Q}_m{}^i(t)$ is appropriate, the estimator model, $Q_m{}^i(t)$, cannot achieve the accuracy of $Q_m i(t)$, (and hence, $Q_1{}^i(t)$).

$$\hat{Q}_m{}^i(t) \neq Q_m{}^i(t) \tag{17b}$$

and from Eq. (17),

$$\hat{Q}_m{}^i(t) \neq Q_1{}^i(t) \tag{17c}$$

From Eqs. (17), the simple single area UH model, $Q_1{}^i(t)$, properly represents the appropriate UH for each subarea (or overland flow plane) for storm i; the appropriate linear routing parameters for each channel link, for storm i; the appropriate timing offsets and proportions of the measured effective rainfalls, for each subarea; and the appropriate summation of runoff hydrograph at each confluence. In contrast, the model estimator, $\hat{Q}_m{}^i(t)$, uses estimates for all of the parameters, and subarea effective rainfall factors, and hence may fail to achieve the accuracy of $Q_1{}^i(t)$ without the addition of interior rainfall-runoff data to accurately validate the parameter values.

STORM CLASSIFICATION SYSTEM

To proceed with the analysis, the full domain of effective rainfall distributions measured at the rain gauge site are categorized into storm classes, $<\xi_x>$. Because the storm classifications are based upon effective rainfalls, the measured precipitations, $P_g{}^i(t)$, may vary considerably yet produce similar effective rainfall distributions. That is, any two elements of a class $<\xi_x>$ would result in nearly identical effective rainfall distributions at the rain gauge site, and hence one would "expect" nearly identical runoff hydrographs recorded at the stream gauge. Typically, however, the resulting runoff hydrographs differ and, therefore, the randomness of the effective rainfall distribution over the catchment, R, results in variations in the modeling "best-fit" parameters (i.e., in the $Q_1{}^i(t)$, the $\eta^i(s)$ variations) in correlating the available rainfall-runoff data.

This, in effect, results in a multi-linear rainfall-runoff model that has a different probable runoff response depending on the severity of the storm's effective rainfall (i.e., storm class). Such a storm class system may involve effective rainfalls classified as severe, major, moderate, and mild.

More precisely, any element of a specific storm class $<\xi_o>$ has the effective rainfall distribution, $e_g{}^o(t)$. However, there are several runoffs associated to the single $e_g{}^o(t)$, and are noted by $Q_g{}^{oi}(t)$. In correlating $\{Q_g{}^o(t),\ e_g{}^o(t)\}$, a different $\eta^i(s)$ results due to the variations in the measured $Q_g{}^o(t)$ with respect to the single known input at the rain gauge site, $e_g{}^o(t)$.

In the predictive mode, where one is given an assumed (or design) effective rainfall distribution, $e_g{}^D(t)$, to apply at the rain gauge site, the storm class of which $e_g{}^D(t)$ is an element of is identified, $<\xi_D>$, and the predictive output for the input, $e_g{}^D(t)$, must necessarily be the random process or distribution, developed from the stochastic integral equation

$$[Q_1{}^D(t)] = \int_{s=0}^{t} e_g{}^D(t - s)\ [\eta(s)]_D\ ds \tag{18}$$

where $[\eta(s)]_D$ is the distribution of $\eta^i(s)$ outcomes associated to storm class $[\xi_D]$. That is, $[\eta(s)]_D$ is the set of $\eta(s)$ realizations derived from rainfall-runoff data, using the model of Eq. (16), for storm class D.

Generally, however, there is insufficient rainfall-runoff data to derive a statistically significant set of storm classes, $<\xi_x>$, and hence additional assumptions must be used. For example, one may lower the eligibility standards for each storm class, $<\xi_x>$, implicitly assuming that several distributions $[\eta(s)]_x$ are nearly identical; or one may transfer $[\eta(s)]_x$ distributions from another rainfall-runoff data set, implicitly assuming that the two catchment data set transfer function distributions are nearly identical. A common occurrence is the case of predicting the runoff response from the design storm effective rainfall distribution, $e_g{}^D(t)$, which is not an element of any observed storm class. In this case, another storm class distribution must be used, which implicitly assumes that the two sets of transfer function distributions are nearly identical. Consequently for a severe design storm condition, it would be preferable to develop correlation distributions using the severe historic storms which have rainfall-runoff data available for the appropriate condition of the catchment.

CONCLUSIONS

Developed in this paper is the conclusion that the single area UH modeling structure represents a highly complex link-node model where all parameters are validated by data. The single area model UH integrates several effects occurring during storm event i; namely, (1) variation in the individual subarea UH across storm events, (2) the distribution of the individual runoff hydrograph channel routing effects, and (3) the variations in the effective rainfall magnitude, timing, and pattern shape over the catchment. When correlating stream gauge runoff to effective rainfall, the single area UH determined by calibration will include the above described effects.

In contrast, using a highly discretized model during calibration will result in a "rigid" UH which transfers the unknown variations in the above cited effects to the model's effective rainfall distribution, resulting in a less reliable calibration of the loss function parameters.

The correlation of the effective rainfall to the runoff hydrograph from the catchment R will result in a different UH (for the single area model) for each storm event. However, the resulting collection of UH's reflect the dominating uncertainty in the variation in the magnitude, timing, and shape of the effective rainfall distribution over R. When the data base consists of only a single rain gauge and stream gauge these three uncertainties cannot be reduced by including additional complexities into the hydrologic model (e.g., subareas linked by hydraulic routing submodels, additional soil-moisture accounting algorithms, etc.). Only additional measured rainfall-runoff data within the catchment R will reduce the uncertainty. Without this additional data, the uncertainty in the effective rainfall over R will remain and should be included in flood control design and planning studies by the development of confidence levels in the modeling results.

The well-known single area unit hydrograph approach may be used on a storm class basis to develop a multi-linear version. This multi-linear model can be used to approximate the response of other rainfall-runoff models. The companion paper will employ the multi-linear UH model in analyzing the effects of subdivision in rainfall-runoff models.

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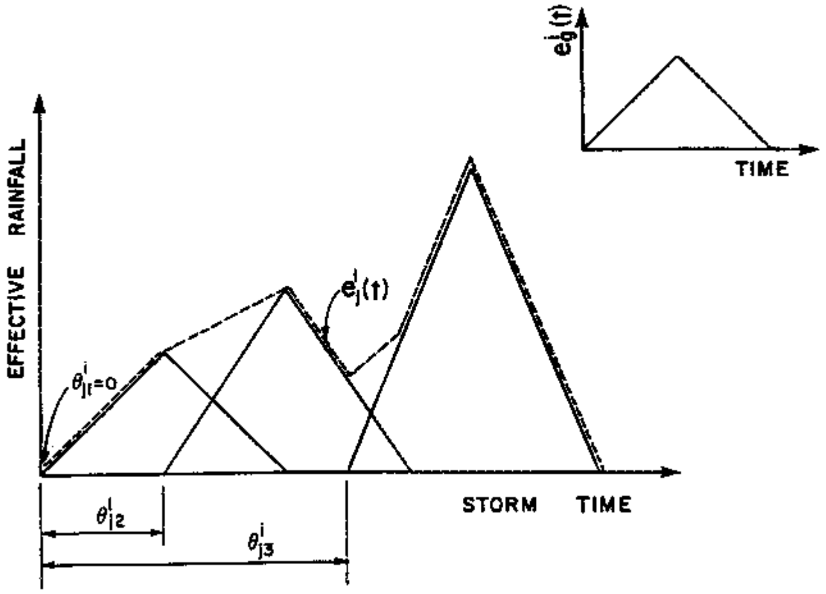

Fig. 1. Subarea Effective Rainfall as a Linear Combination of Rain Gauge Measured Effective Rainfall.

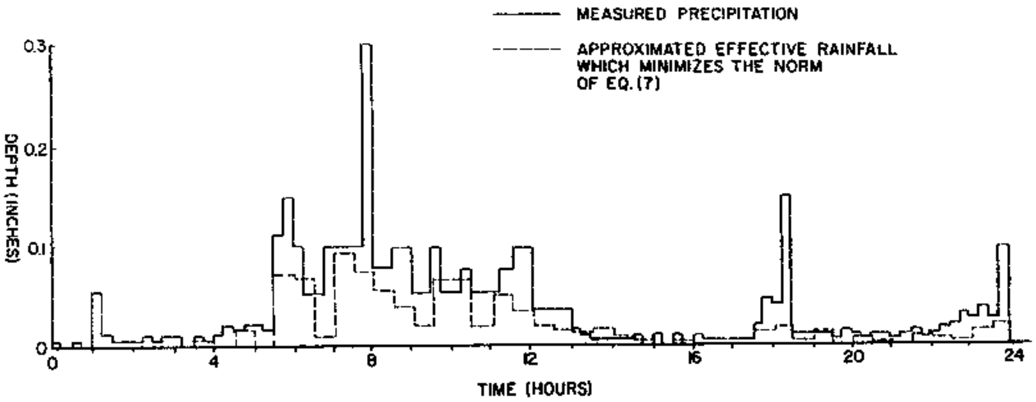

Fig. 2a. Subarea R_1 Effective Rainfalls, $e_1{}^i(t)$, for March 1 Storm, which Minimize the Approximation Error of Eq. (7). Note that $e_1{}^i(t)$ is not derived from the precipitation, but is derived by correlating measured runoff to the assumed unit hydrograph for subarea R_1.

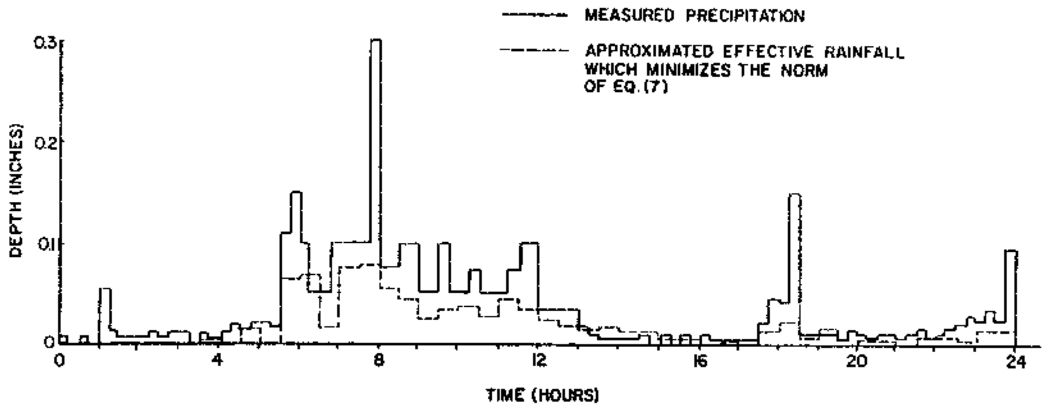

Fig. 2b. Subarea R_2 Effective Rainfalls, $e_2{}^i(t)$. Note that $e_2{}^i(t)$ differs from $e_1{}^i(t)$ of Fig. 2a.

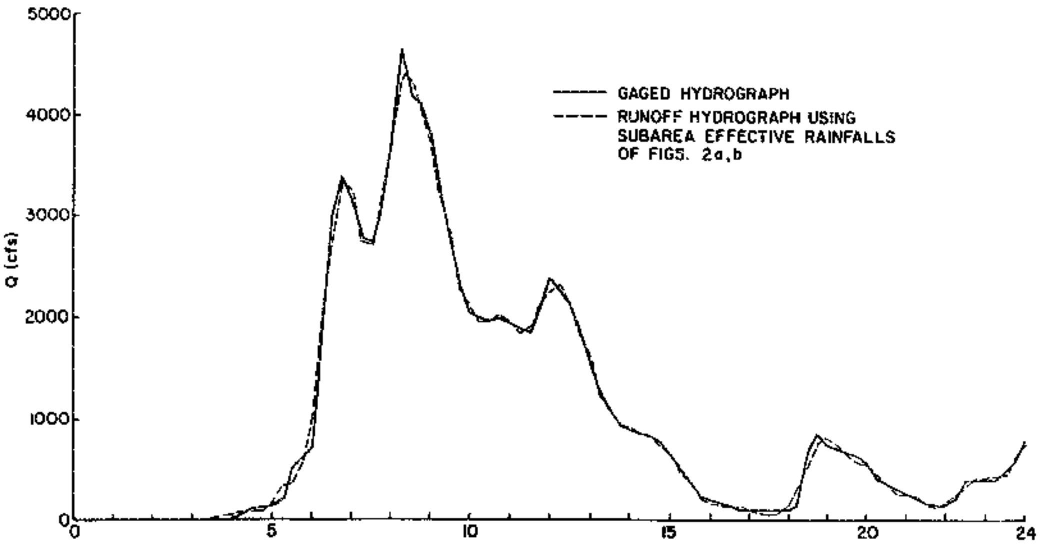

Fig. 3. Comparison of Measured Runoff Hydrograph, $Q_g{}^i(t)$, and Modeled Runoff, $Q_2{}^i(t)$, using Subarea Effective Rainfalls shown in Figs. 2a,b and Derived from Eq. (7).