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FLOOD PLAIN ANALYSIS USING A TWO-DIMENSIONAL DIFFUSION HYDRODYNAMIC MODEL

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ABSTRACT Flooding and inundation of large areas can be effectively modeled using the two-dimensional diffusion hydrodynamic model (DHM). The model described here uses the two-dimensional form of the diffusion equations for flow over a plane to represent flow spreading over ground surfaces. The model also deals with channel flows and flows through constrictions (such as bridges or culverts) by means of the one-dimensional diffusion equations. Flow from the one-dimensional channel onto the two-dimensional surface, and return flow from the surface into the channel is handled by a simple interface model which preserves continuity of the flow volumes while forcing an identical water surface at flooding locations between the two-dimensional and one-dimensional models.

An example of the application of the DHM to a flood problem is given to illustrate the ability of the model to accurately define flooding limits in areas where the flow spreads laterally from the river channel. Use of the DHM in general flood control studies is proposed because of the advantages offered by the DHM approach over other commonly used flooding analysis computer models. The DHM is available from the U.S. Geological Survey as non-proprietary computer program.

1. Introduction

Most hydraulic and hydrologic models which are used for analysis of flood effects deal only with peak flows (steady flow models) or use a one-dimensional approach which ignores the lateral spread of water over areas away from the main stream channel. Other drawbacks in the currently available analysis techniques are the inability of most current models to represent unsteady backwater effects in channels and for overland flow, unsteady overflow of channel systems due to constrictions (e.g., culverts, bridges, etc.), unsteady flow of floodwater across watershed boundaries due to two-dimensional (horizontal plane) backwater, and ponding effects.

The current version of the DHM has been successfully applied to a collection of one- and two-dimensional unsteady flow hydraulics problems including dam-break analyses and flood system deficiency studies. Consequently, the DHM promises to provide highly useful, accurate, and simple-to-use computer model which is of immediate help to practicing flood control engineers. The one drawback to the model is that considerable topographic data may be needed, depending on the area being modeled.

2. Background

One approach to studying flood wave propagation is simply to estimate the maximum expected flow rate and route this flow as a steady-state flow through the downstream reaches. This method is excessively conservative in that all effects due to time variations in channel storage and routing are neglected.

A better approach is to rely on one-dimensional (1-D) full dynamic unsteady flow equations (St. Venant eqs.). Some sophisticated 1-D models include terms and parameters to account for complexities in prototype reaches which the basic flow equations cannot adequately handle. The DWOPER Model by Fread [1] incorporates a number of features which allow a 1-D model to be successfully used in many river flooding applications. However, the limits of 1-D models can only be overcome by extending the analysis into the two-dimensional (2-D) realm. Several 2-D models employing full dynamic equations have been developed. Among them is one by Katopodes and Strelkoff [2] particularly aimed at flood flow analysis. Attendant with the increased power and capability of 2-D fully dynamic models, are the greatly increased boundary, initial, geometry and other input data requirements, as well as the need for large amounts of computer memory and computational speed, and increased computational time. Although it is often claimed that the extra computational cost and effort required for a more sophisticated model is negligible compared with the total modeling cost and effort in the 1-D realm, this is not yet the case in the 2-D realm.

The coupled 1-D and 2-D diffusion hydrodynamic model (DHM) described in this paper offers a simple and economic means for the estimation of flooding effects for diverging flood flows.

3. One-Dimensional Model for Unsteady Flow

Generally, the 1-D flow approach is used wherever there is no significant lateral variation in the flow. Land [3,4] examined four such unsteady flow models for prediction of flooding levels and flood wave travel time, and compared the results against observed unsteady flow data. Ponce and Tsivoglou[6] examined the gradual failure of an earth embankment (caused by an overtopping flooding event) and present a detailed model of the total system: sediment transport, unsteady channel hydraulics, and earth embankment failure. Although many dam-break studies involve flood flow regimes which are truly two-dimensional (in the horizontal plane), the 2-D case has not received much attention. In addition to the model of Katopodes and Strelkoff, which relies on the complete 2-D dynamic equations, Xanthopoulos and Koutitas [5] use the diffusion model to approximate a 2-D flow field. The model assumes that the flood plain flow regime is such that the inertia terms are negligible. In a 1-D model, Akan and Yen [6] also use the diffusion approach to model hydrograph confluences at channel junctions. In the latter study, comparisons of model results were made between the diffusion model, a complete dynamic wave model solving the total equation system, and the basic kinematic wave equation model. The comparisons between the diffusion model and the dynamic wave model were good for the study cases, only minor discrepancies existed.

4. Mathematical Development for Two-Dimensional Model

The two-dimensional diffusion hydrodynamic model (DHM) is based on a diffusion scheme in which gravity, friction, and pressure forces are assumed to dominate the flow equations. Xanthopolous and Koutitas [5] employed such an approach in the prediction of dam-break flood plains in Greece. Good results were also obtained in their studies when they applied the 2-D model to flows that were essentially 1-D in nature.

In the following sequence of equations, which follows the development of Hromadka and Yen [7,8], an integrated finite difference model is developed which solves (1) the two-dimensional flood wave propagation over surfaces, (2) the one-dimensional flood wave for channel flow, and (3) the interface between the two models to accomodate flooding effects. The set of fully dynamic 2-D unsteady flow equations consists of the following:

an equation of continuity

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = \frac{\partial H}{\partial t} = 0 \tag{1}$$

and two equations of motion

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x} \frac{q_x^2}{h} + \frac{\partial}{\partial y} \frac{q_x q_y}{n} + gh \quad S_{fx} + \frac{\partial H}{\partial x} = 0 \tag{2}$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial}{\partial y} \frac{q_y^2}{h} + \frac{\partial}{\partial x} \frac{q_x q_y}{h} + gh \quad S_{fy} + \frac{\partial H}{\partial y} = 0 \tag{3}$$

in which q_x and q_y are flow rates per unit width in the x and y-directions; S_{fx} and S_{fy} represent friction slopes in x and y; H, h, g stand for, respectively, water-surface elevation, flow depth, and gravitational acceleration; and x,y,t are spatial and temporal coordinates.

The above equation set is based on assumptions of constant fluid density with no sources or sinks in the flow field, hydrostatic pressure distributions, and relatively uniform bottom slopes.

The local and convective acceleration terms can be grouped together and rewritten as

$$m_z + \quad S_{fz} + \frac{\partial H}{\partial z} = 0, \; z = x,y \tag{4}$$

where m_z represents the sum of the first three terms in Eqs. (2) and (3) divided by gh. Assuming the friction slope to be approximated by steady flow conditions, Manning's formula can be used to estimate q_z

$$q_z = \frac{1}{n} \quad h^{5/3} \; S_{fz} \;^{1/2}, \; z = x,y \tag{5}$$

Equation 5 can be rewritten as

$$q_z = -K_z \frac{\partial H}{\partial z} - K_z \, m_z, \; z = x,y \tag{6}$$

where

$$K_z = \frac{1}{n} h^{5/3} \quad \frac{\partial H}{\partial s} + m_s \quad ^{1/2}, \; z = x,y \tag{7}$$

The symbol s indicates the flow direction which makes an angle $\theta = \tan^{-1}(q_y/q_x)$ with the +x-direction.

Values of m are assumed to be negligible, resulting in the simple diffusion model:

$$q_z = -K_z \frac{\partial H}{\partial z}, \; z = x,y \tag{8}$$

The 2-D flood flow model is formulated by substituting Eq. (8) into Eq. (1) to give

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} = \frac{\partial H}{\partial t} \tag{9}$$

6. Numerical Model Formulation (Grid Elements)

For uniform grid elements, the numerical modeling approach used is the integrated finite difference version of the nodal domain integration (NDI) method. For grid elements, the *NDI nodal equation is based on the* usual nodal system (see Fig. 1). Flow rates along the boundary Γ are estimated using a linear trial function between nodal points.

For a square grid of width, δ,

$$q_{\Gamma_E} = - K_{x\,\Gamma_E} \; H_E - H_c \; /\delta \tag{10}$$

where

$$K_{x_{\Gamma_E}} = \begin{cases} \left. \frac{1}{n} \; \bar{h}^{5/3} \left(\frac{H_E - H_c}{\delta \cos \theta} \right)^{1/2} \right|_{\Gamma_E} ; \; \bar{h} > 0 \\ 0 \; ; \; \bar{h} \le 0 \text{ or } |H_E - H_c| < 10^{-3} \end{cases} \tag{11}$$

In Eq. (11), $\bar{h}$ and $\bar{n}$ are both averages of the values at c and E, i.e. $\bar{h} = (h + h_E)/2$ and $\bar{n} = (n_c + n_E)/2$. Additionally, the denominator of K_x is checked, and K_x is set to zero if $|H_E - H_c|$ is less than a tolerance such as 10^{-3} m.

The model advances in time by an explicit approach

$$\underset{\sim}{H}^{i+1} = \underset{\sim}{K}^{i} \; \underset{\sim}{H}^{i} \tag{12}$$

where the assumed input flood flows are added to the specified input nodes at each time step. After each time step, the conduction parameters of Eq. (11) are reevaluated, and the solution of Eq. (12) reinitiated. Using grid sizes with uniform lengths of about 1000 m, time steps of about 4 sec gave satisfactory results.

7. One-Dimensional Model

The one-dimensional formulation is developed by eliminating a *directional component* in Eq. (9). This model provides a good approximation of one-dimensional unsteady flow routing including backwater effects and subcritical/supercritical flow regimes. A study by Hromadka et al. [9] indicated that good results can be obtained from a one-dimensional version of DHM for modeling unsteady flow effects.

8. Interface Model (Flooding Source/Sink Term)

To model flood flows leaving and returning to a one-dimensional channel, an interface model is needed to couple the 1-D (channel) DHM and 2-D (topography) DHM. Figure 2 illustrates the mass conservation scheme used to represent the source/sink term of flows flooding or draining the topographic model to the channel model.

9. Channel Structures

A major problem in flood plain modeling is the existence of channel obstructions and constrictions such as bridges, culverts, etc. These features are effectively modeled in the DHM by specifying a stage-discharge relationship at these points within the channel system or at an appropriate point on the flood plain topography.

10. Application of DHM to a Flood Plain Problem

The DHM was used to make a detailed study of flooding during a 100-yr flood event on the Santa Ana River in Orange County, California, in the city of Garden Grove. The schmatic of the finite elements used is shown in Fig. 3. The local terrain slopes southwesterly at a mild gradient. The area is fully developed with mixed residential and commercial structures. The Garden Grove Freeway forms a barrier on the southerly side of the area, with the exception of an outlet at an undercrossing at Garden Grove Boulevard. The flood flows leaving the Santa Ana River are large and can spread easily in a lateral direction; thus the analysis must include the effects of both unsteady flow and two-dimensional flow.

The detailed model is based on topographic data from an aerial survey. In the model, effective areas are used which represent that area where rapid water volume changes occur. Additionally, effective flowpaths are used which represent the length of each grid boundary where flows can cross. In all cases, the buildings are assumed to "survive" the flood. This represents a conservative condition in that with less volume available, higher flood depths will be predicted.

The upstream boundary condition was a specified inflow hydrograph at Nodes 1,2,3, and 4. A second boundary condtion was the inflow through the freeway underpass as a function of time. The DHM includes this flow characteristic by using diffusion routing according to the width of the underpass. That is, the topographic model accommodates this restricted flow by using the appropriate hydraulic flow-width and Manning's friction factor.

Results from the modeling of this situation by the DHM are given in Figs. 4 and 5. The DHM predicted maximum flood depths of less than 1.5 m (5 ft) for this area. In a previous flood plain study utilizing a standard one-dimensional model (HEC-2) the maximum flood plain depth was indicated to be about 3 m (10 ft). Figure 6 compares the one-dimensional (HEC-2) flood plain and the DHM flood plain. The differences in predictions by the two approaches are significant.

11. Conclusions

With the DHM, two-dimensional unsteady flow characteristics can be evaluated over the study area rather than only at particular cross sections when using the traditional one-dimensional methods (e.g., HEC-2) typically utilized in engineering studies of flood plains.

Because the DHM provides a two-dimensional hydrodynamic response, it eliminates the differences in predicted flood depths due to the choice of cross-sections used in standard one-dimensional models. That is, model users usually attempt to select cross-section perpendicular to the direction of flow, but in areas in which flows can spread laterally the cross section selection becomes somewhat arbitrary for the 1-D model. Additionally, the DHM accommodates both two-dimensional backwater and unsteady flow effects, which are typically neglected in most flood plain studies with steady-flow one-dimensional models. The DHM is straight-forward to use and does not requre a level of expertise beyond that needed for application of one-dimensional flooding models.

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