Analysis of a hydrologic model for predicting surface-flow storm runoff

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It is important to consider the uncertainty in a hydrologic model and, in order to prepare such an analysis, a technique to develop model outcome statistics is needed. Due to computational effort limitations, an exhaustion study which considers the total universe of parameter inputs is usually precluded. Two alternatives to an exhaustion study is a Monte Carlo simulation and the more recently advanced Rosenblueth technique. Although the Rosenblueth technique potentially affords a significant saving in computational effort over that usually needed with a Monte Carlo analysis, it was found that in this application the Monte Carlo technique was superior in accuracy, for even the same computational effort.

INTRODUCTION

Advances in hydrologic modeling techniques typically involve the incorporation of higher complexity into the hydrology model by use of hydraulic submodels. With over 150 models reported in the open literature, it is appropriate to review the progress achieved by the complexification of hydrologic models. That is, it is still not clear whether the general level of success afforded by the many types of complex models provide a marked improvement over that achieved by the more commonly used and simpler models such as the unit hydrograph method. Such a review indicates that it is still not clear, in general, whether as modeling complexity increases, modeling accuracy increases.

In the selection of the hydrologic model, the need for both runoff peak flow rates and runoff volumes (for the testing of detention basins) require the selection of a model that produces a runoff hydrograph. The U.S. Army Corps of Engineers (COE) Hydrologic Engineering Center (HEC) Training Document (TD) No. 11, (1980) categorizes all hydrologic models into eight groupings of which three develop a runoff hydrograph; namely, single event (design storm), multiple discrete events, and continuous records (continuous simulation). These models may be used to represent the catchment hydraulics in a design storm model.

In a survey of hydrologic model usage by Federal and State governmental agencies and private engineering firms (U.S. Department of Transportation, Federal Highway Admin., Hydraulic Engineering Circular No. 19, October 1984), it was found that "practically no use is made of watershed models for discrete event and continuous hydrograph simulation." In comparison, however, design storm methods were used from 24 to 34 times more frequently than the complex models by Federal agencies and the private sector, respectively. The frequent use of design storm methods appears to be due to several reasons: (1) design storm methods are considerably simpler to use than discrete event and continuous simulation models; (2) it has not been established in general that the more complex models provide an improvement in computational accuracy over design storm models; and (3) the level of complexity typically embodied in the continuous simulation class of models does not appear to be appropriate for the catchment rainfall-runoff data which is typically available. Consequently, the design storm approach is most often selected for flood control and drainage policies.

The specific components of the design storm/unit hydrograph approach include the design storm, the loss rate function, catchment lag relationships, and unit hydrograph or S-graph development. Inherent in the choice of submodels is the ability to calibrate the model at two levels: (1) calibration of model parameters to represent local or regional catchment rainfall-runoff characteristics, and (2) calibration of the design storm to represent local rainfall intensity-duration-frequency characteristics. Beard and Chang (1979) note that in a hydrologic model, the number of calibration parameters should be as small as possible in order to correlate model parameters with basin characteristics. They also write that a regional study should be prepared to establish the loss rate and unit hydrograph characteristics, "and to compute from balanced storms of selected frequencies (storms having the same rainfall frequency for all durations) and resulting floods."

In order to facilitate these two calibration requirements, the runoff hydrograph model should be as simple as possible. For example, by using a loss rate defined as a fixed percentage of rainfall ($\bar{Y}$) such that the losses do not exceed a maximum value of Fm (phi index), then the design storm pattern shape and location of the peak rainfall are essentially removed as variables in the calibration of the design storm. The parameters for a single area unit hydrograph model are S-graph, lag, and loss rate values of $\bar{Y}$ and Fm. Here, $\bar{Y} = (1 - \text{yield})$, and Fm serves as a phi index loss function.

A criterion for complex and simple models is given by Beard and Chang (1979) as the "difficulty or reliability of model calibration. ... Perhaps the simplest type of model that produces a flood hydrograph is the unit hydrograph model "... and ..." can be derived to some extent from physical drainage features but fairly easily and fairly reliably calibrated through successive approximations by relating the time distribution of average basin rainfall excess to the time distribution of runoff." In comparison, the "most complicated type of model is one that represents each significant element of the hydrologic process by a mathematical algorithm. This is represented by the Stanford Watershed Model and requires extensive data and effort to calibrate."

The literature contains several reports of problems in using complex models, especially in parameter optimization. Additionally, it has not been clearly established whether complex models, such as in the continuous simulation or discrete event classes of models, provided an increase in accuracy over a standard design storm unit hydrograph model.

There are only a few papers and reports in the literature that provided a comparison in hydrologic model performance. From these references, it appears that a simple unit hydrograph model provides as good as or better results than quasi-physically based (or QPB, see the work of Loague and Freeze (1985)) or complex models.

In their paper, Beard and Chang (1979) write that in the case of the unit hydrograph model, "the function of runoff versus rainfall excess is considered to be linear, whereas it usually is not in nature. Also, the variations in shapes of unit hydrographs are not derivable directly from physical factors. However, models of this general nature are usually as representative of physical conditions as can reasonably be validated by available data, and there is little advantage in extending the degree of model sophistication beyond validation capability." It is suggested that "if 50 yr–100 yr of streamflow were available for a specified condition of watershed development, a frequency curve of flows for that condition can be constructed from a properly selected set of flows."

Schilling and Fuchs (1986) write "that the spatial resolution of rain data input is of paramount importance to the accuracy of the simulated hydrograph" due to "the high spatial variability for storms" and "the amplification of rainfall sampling errors by the nonlinear transformation" of rainfall into runoff. Their recommendations are that a model should employ a simplified surface flow model if there are many subbasins; a simple runoff coefficient loss rate; and a diffusion (zero inertia) or storage channel routing technique. Hornberger, et al. (1985) writes that "Even the most physically based models ... cannot reflect the true complexity and heterogeneity of the processes occurring in the field. Catchment hydrology is still very much an empirical science."

Schilling and Fuchs (1986) note that errors in simulation occur for several reasons, including:

"1. The input data, consisting of rainfall and antecedent conditions, vary throughout the watershed and cannot be precisely measured.
2. The physical laws of fluid motion are simplified.
3. Model parameter estimates may be in error."

By reducing the rainfall data set resolution from a grid of 81 gages to a single catchment-centered gage in an 1,800 acre catchment, variation in runoff volumes and peak flows "is well above 100% over the entire range of storms implying that the spatial resolution of rainfall has a dominant influence on the reliability of computed runoff." It is also noted that "errors in the rainfall input are amplified by the rainfall-runoff transformation" so that "a rainfall depth error of 30% results in a volume error of 60% and a peak flow error of 80%."

Schilling and Fuchs (1986) also write that "it is inappropriate to use a sophisticated runoff model to achieve a desired level of modeling accuracy if the spatial resolution of rain input is low" (in their study, the raingauge densities considered for the 1,800-acre catchment are 81–, 9–, and a single centered gage).

In a similar vein, Beard and Change (1979) write that in their study of 14 urban catchments, complex models such as continuous simulation typically have 20 to 40 parameters and functions that must be derived from recorded rainfall-runoff data. "Inasmuch as rainfall data are for scattered point locations and storm rainfall is highly variable in time and space, available data are generally inadequate in this region for reliably calibrating the various interrelated functions of these complex models." Additionally, "changes in the model that would result from urbanization could not be reliably determined." They write that the application "of these complex models to evaluating changes in flood frequencies usually requires simulation of about 50 years of streamflow at each location under each alternative watershed condition."

Garen and Burges (1981) noted the difficulties in rainfall measurement for use in the Stanford Watershed Model, because the K1 parameter (rainfall adjustment factor) and UZSN parameter (upper level storage) had the dominant impact on the model sensitivity. This is especially noteworthy because Dawdy and O'Donnel (1965) concluded that insensitive model coefficients could not be calibrated accurately. Thus, they could not be used to measure physical effects of watershed changes.

Using another complex model, Mein and Brown (1978) write that on "the basis of several tests with the Boughton model it is concluded that for this model at least, relationships derived between any given parameter value and measurable watershed characteristics would be imprecise; i.e., they would have wide confidence limits. One could not be confident therefore in changing a particular parameter value of this model and then claiming that this alteration represented the effect of some proposed land use change. On the other hand, the model performed quite well in predicting flows with these insensitive parameters, showing that individual parameter precision is not a prerequisite to satisfying output performance."

According to Gburek (1971), "... a model system is merely a researcher's idea of how a physical system interacts and behaves, and in the case of watershed research, watershed models are usually extremely simplified mathematical descriptions of a complex physical situation ... until each internal submodel of the overall model can be independently verified, the model remains strictly a hypothesis with respect to its internal locations and transformations ..." (also quoted in McPherson and Schneider, 1974)).

The introduction of a paper by Sorooshian and Gupta (1983) provides a brief review of some of the problems reported by other researchers in attempting to find a "true optimum" parameter set for complex models, including the unsuccessful two man-year effort by Johnston and Pilgrim (1976) to optimize parameters for a version of the Boughton model cited above.

In the extensive study by Loague and Freeze (1985), three event-based rainfall-runoff models (a regression model, a unit hydrograph model, and a kinematic wave quasi-physically based model) were used on three data sets of 269 events from three small upland catchments. In that paper, the term "quasi-physically based" or QPB is used for the kinematic wave model. The three catchments were 25 acres, 2.8 mi^2, and 35 acres in size, and were extensively monitored with rain gage, stream gage, neutron probe, and soil parameter site testing.

For example, the 25 acre site contained 35 neutron probe access sites, 26 soil parameter sites (11 equally spaced), an on-site rain gage, and a stream gage. The QPB model utilized 22 overland flow planes and four channel segments. In comparative tests between the three modeling approaches to measured rainfall-runoff data it was concluded that all models performed poorly and that the QPB performance was only slightly improved by calibration of its most sensitive parameter, hydraulic conductivity. They write that the "conclusion one is forced to draw ... is that the QPB model does not represent reality very well; in other words, there is considerable model error present. We suspect this is the case with some, if not all conceptual models currently in use." Additionally, "the fact that simpler, less data intensive models provided as good or better predictions than a QPB is food for thought."

Based on the literature, the main difficulty in the use, calibration, and development of complex models appears to be the lack of precise rainfall data and the high model sensitivity to (and magnification of) rainfall measurement errors. Nash and Sutcliff (1970) write that "As there is little point in applying exact laws to approximate boundary conditions, this, and the limited ranges of the variables encountered, suggest the use of simplified empirical relations."

It is noteworthy to consider the HEC Research Note No. 6 (1979) where the Hydrocomp HSP continuous simulation model was applied to the West Branch DuPage River in Illinois. Personnel from Hydrocomp (R. Linsley, N. Crawford, co-authors of the Stanford Watershed Model, principals), HEC, and COE participated in this study which started with a nearly complete hydrologic/meteorologic data base. "It took one person six months to assemble and analyze additional data, and to learn how to use the model. Another six months were spent in calibration and long-record simulation." This time allocation applies to only a 28.5 mi^2 basin. The quality of the final model is indicated by the average absolute monthly volume error of 32.1% and 28.1% for calibration and verification periods, respectively. Peak flow rate average absolute errors were 26% and 36% for calibration and verification periods, respectively. It was concluded that "Discharge frequency under changing urban conditions is a problem that could be handled by simpler, quicker, less costly approaches requiring much less data; e.g., design storms or several historical events used as input to a single-event model, or a continuous model with a less complex soil-moisture accounting algorithm."

The complex model parameter optimization problem has not been resolved. For example, Gupta and Sarooshian (1983) write that "even when calibrated under ideal conditions (simulation studies), it is often impossible to obtain unique estimates for the parameters." Troutman (1982) also discusses the often cited difficulties with the error in precipitation measurements "due to the spatial variability of precipitation." This source of error can result in "serious errors in runoff prediction and large biases in parameter estimates by calibration of the model."

Because it is still not clear (e.g., the Stanford Watershed Model or Hydrocomp HSP has been in operation for over 20 years) whether there is a significant advantage in using a watershed model more complex or physically based than a design storm unit hydrograph approach, the design storm unit hydrograph method is proposed for use in the flood control runoff hydrograph model

HEC (Beard, 1975) provides an in-depth study of the use of design runoff hydrographs for flood control studies. "Hypothetical floods consists of hydrographs of artificial flood flows ... that can be used as a basis for flood control planning, design and operation decisions or evaluations. These floods represent classes of floods of a specified or implied range of severity." Such "floods are ordinarily derived from rainfall or snowmelt or both, with ground conditions that are appropriate to the objectives of the study, but they can be derived from runoff data alone, usually on the basis of runoff volume and peak-flow frequency studies and representative time sequences of runoff."

In complex watershed systems that include catchment subareas, and channel and basin routing components, Beard (1975) writes that "it is usually necessary to simulate the effects of each reservoir on downstream flows for all relevant magnitudes of peaks and volumes of inflows. Here it is particularly important that each hypothetical flood has a peak flow and volumes for all pertinent durations that are commensurate in severity, so that each computed regulated flow will have a probability or frequency that is comparable to that of the corresponding unregulated flow ... In the planing of a flood control project involving storage or in the development of reservoir operation rules, it is not ordinarily known what the critical duration will be, because this depends on the amounts of reservoir space and release in relation to flood magnitude. When alternate types of projects are considered, critical durations will be different, and a design flood should reflect a degree of protection that is comparable for the various types of projects."

Beard (1975) notes that the balanced storm concept is an important argument for not using a historic-storm pattern or sequence of storm patterns (e.g., continuous simulation or discrete event modeling) as "No one historical flood would ordinarily be representative of the same severity of peak flow and runoff volumes for all durations of interest." Indeed, should a continuous simulation study be proposed such that the "project is designed to regulate all floods of record, it is likely that one flood will dictate the type of project and its general features, because the largest flood for peak flows is also usually the largest-volume flood." Hence, a continuous simulation model of say 40 years of data can be thought of as a 40 year duration design storm with its own probability of reoccurrence, which typically reduces for modeling purposes to simply a single or double day storm pattern.

Beard and Chang (1979) write that for design storm construction, "it is generally considered that a satisfactory procedure is to construct an approximately symmetrical pattern of rainfall with uniform area distribution having intensities for all durations corresponding to the same recurrence interval and for that location and size of area" (i.e., depth-area adjustments).

MODEL SELECTION

Of the over 100 models available, a design storm/unit hydrograph model (i.e. "model") is selected for this particular application. Some of the reasons are as follows: (1) the design storm approach—the multiple discrete event and continuous simulation categories of models have not been clearly established to provide better predictions of flood flow frequency estimates for evaluating the impact of urbanization and for design of flood control systems than a calibrated design storm model; (2) the unit hydrograph method—it has not been shown that alternate approaches, (i.e., the kinematic wave modeling technique) provide a significantly better representation of watershed hydrologic response than a model based on unit hydrographs (locally calibrated or regionally calibrated) that represent free-draining catchments; (3) model usage—this class of "model has been used extensively nationwide and has proved generally acceptable and reliable; (4) parameter calibration—the "model" used in this application is based upon a minimal number of parameters, giving higher accuracy in calibration of model parameters to rainfall-runoff data, and the design storm to local flood flow frequency tendencies; (5) calibration effort—the "model" does not require large data or time requirements for calibration; (6) application effort—the "model" does not require excessive computation for application; (7) acceptability—the "model" uses algorithms that are accepted in engineering practice; (8) model flexibility for planning—data handling and computational submodels can be coupled to the "model" (e.g., channel and basin routing) resulting in a highly flexible modeling capability; (9) model certainty evaluation—the certainty of modeling results can be readily evaluated as a distribution of possible outcomes over the probabilistic distribution of parameter values.

UNIT HYDROGRAPH METHOD

The unit hydrograph (UH) method is a synthetic hydrograph approach originally developed by L.K. Sherman (1932). The method assumes that watershed discharge is related to the total volume of runoff, and that the time factors which affect the UH shape are constant. The UH theory was advanced by F. F. Snyder (1938) to transpose storm rainfall-runoff relationships from gauged watersheds to hydrologically similar ungauged watersheds. The additional assumptions used in Snyder's approach are that watershed rainfall-runoff relationships are characterized by watershed area, slope, and shape factors. The method is used to estimate the time distribution of runoff in watersheds where stream gauge information is either unavailable or inadequate to justify statistical analysis.

A slightly modified version of the US Army Corps of Engineers' method for determining the time distribution of runoff is used for this study. This method for determining and utilizing synthetic unit hydrographs is essentially identical to the approach described in the United States Geological Survey, Water Supply Paper No. 772 and in several technical publications of the American Geophysical Union and the American Society of Civil Engineers.

The following is a brief description of the terms used in the discussion of the unit hydrograph model:

Unit Hydrograph is a curve showing the time distribution of rates of runoff which results from a unit one inch of effective rainfall on the tributary watershed upstream of the point of concentration.

Distribution Graph is a unit hydrograph whose ordinates are expressed in terms of percent of ultimate discharge. A distribution graph is generally developed as a block graph with each block representing its associated percent of unit runoff which occurs during the specified unit time.

Effective Rainfall is that portion of rainfall which runs off. It is the total rainfall less infiltration, evaporation, transportation, absorption, and detention.

Summation Hydrograph is a curve showing the time distribution of the rates of runoff that would result from a continuous series of unit effective rainfalls over the tributary watershed upstream of the point of concentration.

Lag is the time from the beginning of a continuous series of unit effective rainfalls over the watershed area to the instant when the rate of runoff equals 50 percent of the ultimate rate of runoff.

Ultimate Discharge is the maximum rate of runoff which results from a specified effective rainfall intensity. Ultimate discharge from a watershed occurs when the rate of runoff on the summation hydrograph is equivalent to the rate of effective rainfall.

S-Graph is a summation hydrograph developed by plotting watershed discharge expressed in percent of ultimate discharge as a function of time expressed in percent of lag.

The model produces a time distribution of runoff $Q(t)$ given by the standard convolution integral representation of:

$$Q(t) = \int_0^t e(s)\phi(t-s)\mathrm{d}s$$

where $Q(t)$ is the catchment flow rate at the watershed point of concentration; $e(s)$ is the effective rainfall intensity; and $\phi(x)$ is the unit hydrograph developed from the S-graph. In the above equation $e(s)$ represents the time distribution of the 24-hour duration design storm pattern modified according to depth-area effects and soil losses.

MODEL COMPONENTS

Design Storm Approach—The design storm approach used in this study is closely related to the methods outlined in the COE HEC TD No. 15 (1982). The method is typical of those used throughout the U.S. and is a hybridization of HEC and SCS procedures. An evaluation of other hydrologic modeling techniques is provided by McCuen (1984) and Hromadka et al. (1986). It is concluded that single event (design storm) models such as is used in this study have not been shown to be less accurate than other more complex and costly models such as continuous simulation (e.g. the Stanford Watershed Model and its successor, the USGS Rainfall-Runoff Model etc.). Additionally, the method is easy to use and provides a straightforward method for watershed planning purposes.

Storm Pattern—the storm pattern used in the model is a synthetic storm with the peak rainfall intensities nested for durations from 5 minutes to 24 hours (i.e., the peak 5 minutes of rainfall is nested within the peak 10 minutes; the peak 10 minutes is nested within the peak 15 minutes; and so forth until the 24 hours storm pattern is completed). This hypothetical storm pattern is referred to in HEC TD-15 (1982) as a "balanced storm", because the consistent depth-frequency relation used for each peak duration storm pattern is comparable to the March 1, 1983 storm that occurred over western Orange County in Southern California during which several recording rain gages measured near 100 year intensities for all durations between 15 minutes and 6 hours. Every watershed is sensitive to a particular duration of rainfall that will produce peak discharge, usually a duration approximately equal to the watershed time-of-concentration or lag. A nested duration design storm insures that each watershed will receive the design frequency depth of rainfall at its critical duration. Durations longer or shorter than the critical duration have little effect on the peak discharge of the watershed, however, longer durations have considerable effect on the total volume of runoff.

Time Distribution—a (2/3)–(1/3) rainfall distribution is used in the model with the desired return frequency rainfalls nested about hour 16 of the 24 hour storm. HEC TD-15 (1982) uses a (1/2)–(1/2) distribution in which the peak rainfall intensity is placed at the center of the storm. A sensitivity analysis was performed to determine the effect of the two different distributions on peak flow rates. A hypothetical watershed was developed assuming the following parameters:

Watershed Length (L) = 7500 ft.
Watershed Length to the Centroid (LCA) = 3750 ft.
Elevation Drop = 75 ft.
Lag = varied from 0.2 to 2.5 hours
Area (A) = 640 acres
Maximum Soil Loss Rate (Fm) = varied from 0.0 to 0.50 inches/hour
Valley S-graph used
5 Minute Rainfall Depth = 0.51 inches
30 Minutes Rainfall Depth = 1.13 inches
1 Hour Rainfall Depth = 1.53 inches
3 Hour Rainfall Depth = 2.48 inches
6 Hour Rainfall Depth = 3.37 inches
24 Hour Rainfall Depth = 6.19 inches

The results of the sensitivity analysis are shown in Figure 1. It is evident from the plot that there is no significant difference between peak flow rates determined by the use of the two rainfall distributions when using the unit hydrograph and loss rate functions discussed later in this and other sections. It was, therefore, concluded that a reasonable variation in the design storm rainfall distribution (storm pattern) would have a negligible effect on the model results for peak flow rate. However, the distribution of runoff volume varies within the runoff hydrograph

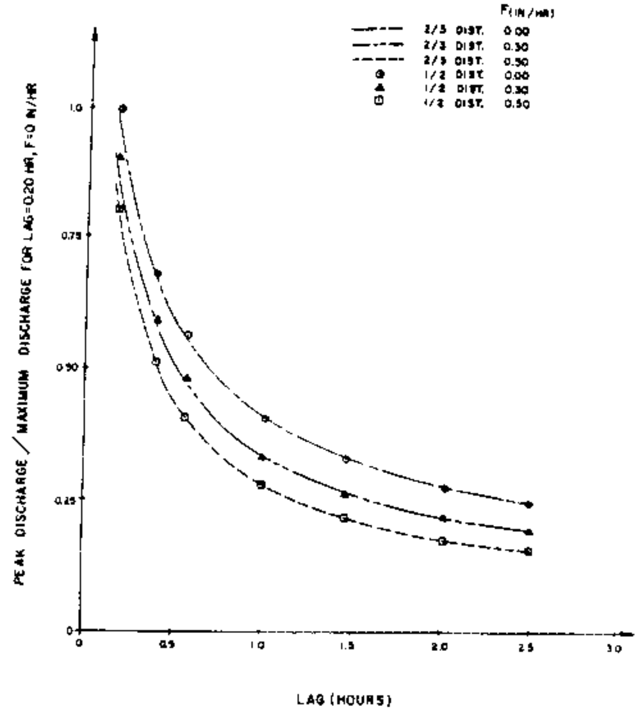

Figure 1 Sensitivity of Time Distribution

depending upon the design storm pattern rainfall distribution.

Precipitation—sources of precipitation data for the study area include the National Weather Service NOAA Atlas 2 (1973) and the State of California DWR (Goodridge, 1982) data. The NOAA Atlas 2 data provides point rainfall depth isohyetals for the 6 and 24 hour durations and provides regression equations for determining other durations. The State of California DWR data provides 10 minute through 24 hour duration data. A comparison of these two data sources indicate that the 6 and 24 hour values are similar but the lesser duration rainfalls (less than 3 hours) differ significantly with the DWR data being approximately 30 to 50 percent higher than the NOAA data.

For this study the DWR data was considered to be reasonable and more up-to-date than the NOAA Atlas 2 data. Precipitation data developed for each station can be used with either the basin average or Theisson Polygon methods, to obtain frequency duration precipitation depths for the study watersheds.

Depth-Area Adjustment—Depth area adjustment factors are used to adjust point rainfall values to correspond to a proper rainfall mass distribution over the watershed. Two different sets of depth area relationships (factors) were considered in this study: the NOAA Atlas 2 (1973) factors, which provides depth area adjustment factors for 30 minute to 24 hour durations; and the March 1943 Sierra Madre storm in Southern California which produced 30 minute to 3 hour duration factors. The 30 minute, 1 and 3 hour depth area curves for each of the two sets are shown for comparison in Figure 2. It can be seen that the Sierra Madre depth area factors provide a

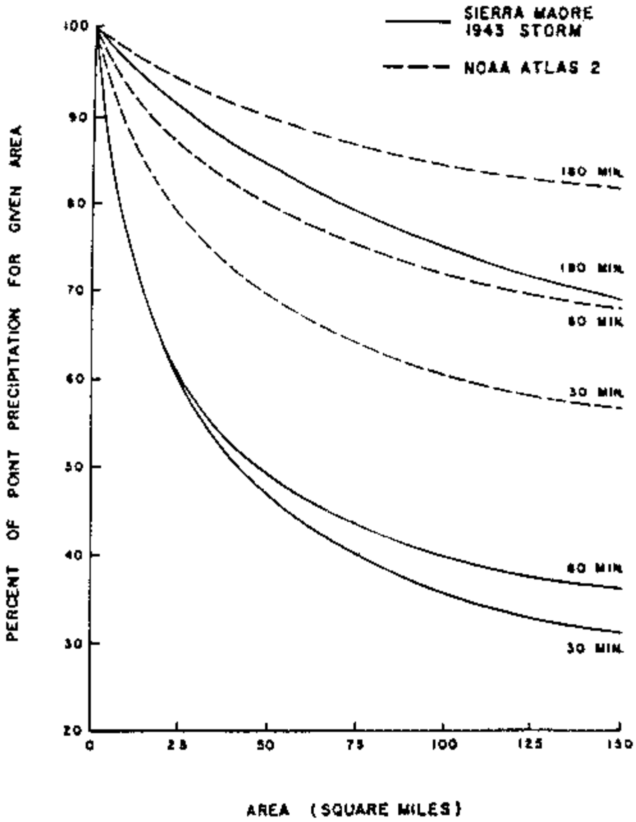

Figure 2 Comparison of Depth Area Curve

significant reduction in design storm rainfalls over that given by the NOAA Atlas 2 values.

Because the Sierra Madre storm actually occurred near the study area, it is considered to be more representative of Southern California storms than the NOAA Atlas 2 curves, which are based on a national average of storms. Therefore, the Sierra Madre relationships for 30 minute to 3 hour durations and the NOAA Atlas 2 relationships for the remaining durations (6 and 24 hours) were used in this study.

Soil Loss Rate—the soil loss rate function used in this study is based on the Soil Conservation Survey's curve number (CN) approach, but modified to have an upper and lower bound. The loss function is defined by the following equation:

$$f(t) = \begin{cases} \bar{Y}I(t) & \text{for} \quad YI(t) < Fm \\ Fm, & \text{otherwise} \end{cases}$$

where

$\bar{Y}$ = the low loss fraction

Fm = the maximum loss rate

$I(t)$ = the design strom rainfall at time (t).

The low loss fraction $\bar{Y}$ acts as a fixed loss percentage, whereas Fm serves as an upper limit to the possible values of $f(t)$. This approach is a hybridization of the SCS CN approach, where the low loss fraction is used to develop runoff hydrograph yields that are consistent with the SCS 24 hour storm yields, and the peak loss rates are consistent with those developed from rainfall-runoff reconstitution studies.

The Fm and $\bar{Y}$ parameters are discussed in greater detail in the following:

Maximum Loss Rate (FM)

The maximum loss rate (Fm) is defined by

$$Fm = FpAp$$

where,

Fp = the maximum loss rate for pervious area Ap for appropriate CN and antecedent moisture condition (AMC);

Ap = the actual pervious area fraction and where the infiltration rate for the impervious area fraction is defined to be zero. Values for Fp can be calibrated from rainfall-runoff reconstitution studies and Ap values can be determined from existing or predicted development patterns.

Low Loss Fraction

The low loss fraction is estimated from the *SCS* loss rate equation by

$$\bar{Y} = 1 - Y$$

where Y is the watershed yield computed by

$$Y = AjYj$$

where Aj = watershed area fraction with corresponding Yj

Yj = watershed yield

Yj for pervious areas is estimated using the *SCS CN* by:

$$Yj = \frac{(P24 - Ia)^2}{P24(P24 + S - Ia)}$$

where

$P24$ = the 24 hour point precipitation

Ia = initial abstraction ($0.2 \times S$)
$S = (1000/CN\text{-}10)$

The $\bar{Y}j$ for impervious areas is defined to be zero.

A distinct advantage afforded by the above function over loss functions such as Green-Amp or Horton is that the effect of the location of the peak rainfall intensities in the design storm pattern on the model peak flow rate (Q) becomes negligible. That is, front-loaded, middle-loaded, and rear-loaded storm patterns all result in nearly equal peak flow estimates. Consequently, the shape (but not-magnitude) or the design storm pattern is essentially eliminated from the list of parameters to be calibrated in the runoff hydrograph "model" (although the time distribution of runoff volumes are affected by location of the peak rainfalls in the storm pattern which is a consideration in dentention basic design).

Antecedent Moisture Condition (AMC)—The AMC conditions I, II, and III represent adjustments for antecedent soil moisture conditions for dry, average, and wet, respectively. The designation of a particular AMC condition of a specific storm is usually determined by the amount of prior rainfall. The effect of AMC is built into the runoff curve number determination by adjusting the *CNs* for AMC I and III (the CNs are given in the tables for AMC II). The AMC adjustment is a probabilistic variable which may be evaluated during calibration. Once the model is calibrated, the AMC variable becomes a constant.

Unit Hydrograph—A unit hydrograph is a flood hydrograph resulting from one inch of effective rainfall occurring uniformly over a watershed in a specified time duration. The unit hydrograph procedure used in this study is developed from an S-graph which is a summation graph of discharge (in percent of lag). Lag is defined as the time in hours from the beginning of effective rainfall to the time for 50 percent of the total volume of runoff to occur. The S-graph and lag time are discussed in more detail below.

S-graph—The S-graph representation of the unit hydrograph as defined above can be used to develop unit hydrographs corresponding to various watershed lag estimates. The S-graph can be developed by rainfall-runoff reconstitution studies for several storms or a single storm for several watersheds, if a regional S-graph is desired by averaging the storm event S-graphs for several storm events of the study watersheds, a representative S-graph may be developed for the watershed. By comparing the representative S-graphs a regional S-graph may be derived to represent a larger area.

Lag Time—The main parameter used to develop a synthetic unit hydrograph from an S-graph is the watershed lag time. A generalized relationship for estimating basin lag values when stream gage data is inappropriate or unavailable for rainfall-runoff reconstitution studies has been developed for the Southern California area, and has been widely used by the Los Angeles District COE.

$$LAG = 24n\left(\frac{L \cdot LCA}{S^{1/2}}\right)^{0.38}$$

$\bar{n}$ = watershed basin factor
L = watershed length (miles)
LCA = length from the watershed centroid to the point of concentration (miles)
S = watershed slope (ft/mile)

This equation was developed from data collected from several Sothern California watersheds ranging in size from 2.3 to 4310 square miles. The uncertainty in using this equation lies in the estimation of the basin actor $\bar{n}$. The COE has developed a table of $\bar{n}$-values estimated for generalized watershed conditions. (See Figure 3). The choice of $\bar{n}$ is very much dependent upon the engineer's judgment and may therefore vary widely between engineers for the same basin. In addition, the above relationship does not seem to give reasonable results for relatively small (less than 5 square miles) watersheds often resulting in lag estimates which are significantly low yielding higher peak discharge estimates.

For these reasons, a correlation of watershed lag to the rational method time-of-concentration (Tc) was

$\bar{n}$ = 0.015
1. Drainage area has fairly uniform, gentle slopes
2. Most watercourses either improved or along paved streets
3. Groundcover consists of some grasses - large % of area impervious
4. Main water course improved channel or conduit

$\bar{n}$ = 0.020
1. Drainage area has some graded and non-uniform, gentle slopes
2. Over half of the area watercourses are improved or paved streets
3. Groundcover consists of equal amount of grasses and impervious area
4. Main watercourse is partly improved channel or conduit and partly greenbelt (see $\bar{n}$ = 0.025)

$\bar{n}$ = 0.025
1. Drainage area is generally rolling with gentle side slopes
2. Some drainage improvements in the area - streets and canals
3. Groundcover consists mostly of scattered brush and grass and small % impervious
4. Main watercourse is straight channels which are turfed or with stony beds and weeds on earth bank (greenbelt type)

$\bar{n}$ = 0.030 (Foothill Area)
1. Drainage area is generally rolling with rounded ridges and moderate side slopes
2. No drainage improvements exist in the area
3. Groundcover includes scattered brush and grasses
4. Watercourses meander in fairly straight, unimproved channels with some boulders and lodged debris

$\bar{n}$ = 0.040 (Foothill Area)
1. Drainage area is composed of steep upper canyons with moderate slopes in lower canyons
2. No drainage improvements exist in the area
3. Groundcover is mixed brush and trees with grasses in lower canyons
4. Watercourses have moderate bends and are moderately impeded by boulders and debris with meandering courses

$\bar{n}$ = 0.050 (Mountain Areas)
1. Drainage area is quite rugged with sharp ridges and steep canyons
2. No drainage improvements exist in the area
3. Groundcover, excluding small areas of rock outcrops, includes many trees and considerable underbrush
4. Watercourses meander around sharp bends, over large boulders and considerable debris obstruction

$\bar{n}$ = 0.200
1. Drainage area has comparatively uniform slopes
2. No drainage improvements exist in the area
3. Groundcover consists of cultivated crops or substantial growths of grass and fairly dense small shrubs, cacti, or similar vegetation
4. Surface characteristics are such that channelization does not occur

BASIN $\bar{n}$
PARAMETER DESCRIPTIONS

Figure 3

developed for this study. The relationship developed,

$$LAG = 0.8 \times Tc$$

reduces the uncertainty and arbitrary selection errors associated with estimating lag from generalized basin $\bar{n}$ charts.

MODEL SENSITIVITY

A separate study was conducted to test the sensitivity of the design storm approach to three of the model's key parameters: watershed lag (basin factor, $\bar{n}$), maximum loss rate (Fm), and low loss fraction ($\bar{Y}$).

The following watershed characteristics are assumed:

Parameters			
Area (sqmi)	1 s	5 s	40 s
Length of longest water course (ft)	7,500	20,000	50,000
Length from centroid to the point of concentration (ft)	3,750	10,000	25,000
Elevation drop (ft)	75	200	500
Basin factor	varied from 0.015 to 0.055		
Maximum loss rate (in/hr)	varied from 0.100 to 0.550		
Low loss fraction	varied from 0.000 to 1.000		

The above watershed parameters were input into a computer model of the design storm approach. Figure 4 shows the sensitivity of the peak flow rate varying the basin factor from 0.015 to 0.055 while keeping the maximum loss rate and low loss fraction constant at

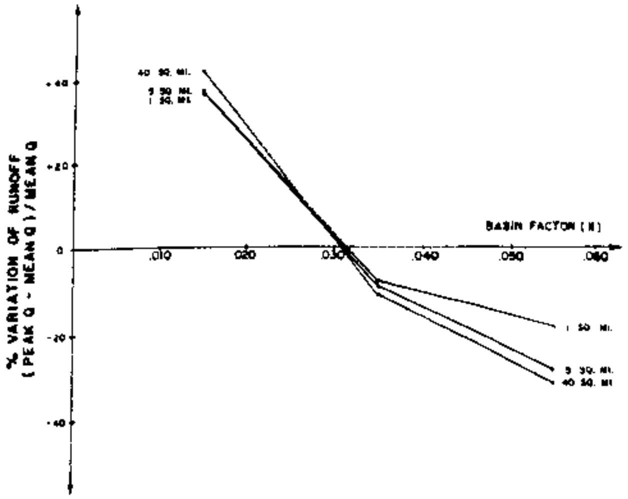

Figure 4 Sensitivity of $\bar{n}$

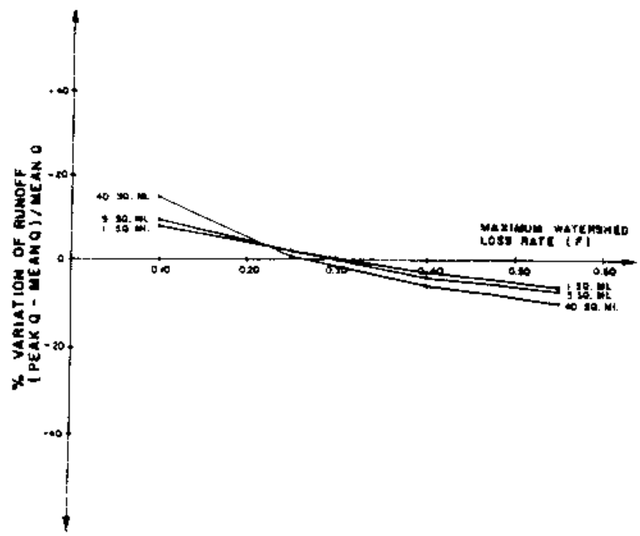

Figure 5 Sensitivity of Fm

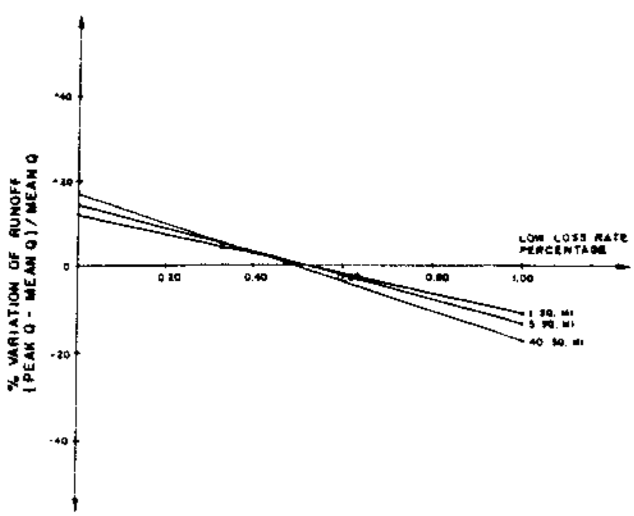

Figure 6 Sensitivity of $\bar{Y}$

0.10 in/hr and 1.0, respectively. From the figure, a reduction in the basin factor from 0.030 to 0.020 (33% reduction) results in a 20 percent increase in peak discharge.

Figure 5 shows the sensitivity of the maximum loss rate by keeping the basin factor and low loss fraction constant at 0.015 and 0.01, respectively. The results of the analysis show that increasing the maximum loss rate from 0.30 to 0.50 in/hr results in an 8 percent decrease in peak discharge, whereas decreasing the maximum loss rate from 0.30 to 0.10 in/hr results in a 10 percent increase in peak discharge.

Figure 6 shows the sensitivity in peak discharge to the low loss fraction. The basin factor and maximum loss rate are held constant at 0.015 and 0.55 in/hr, respectively. Increasing the low loss fraction from 0.50 to 0.80 results in an 8 percent decrease in peak discharge while decreasing the low loss fraction from 0.50 to 0.20 results in a 9 percent increase.

From the three figures, it is concluded that the design storm model used in this study is only slightly sensitive to the two loss parameters, but is significantly sensitive to the basin factor. For this reason, a procedure to better estimate watershed lag is developed by relating lag to the watershed time-of-concentration.

MODEL CALIBRATION

Model parameters are calibrated most often by using recorded precipitation and runoff data to solve the "inverse" problem. That is, given the system input (precipitation) and the system output (runoff), the inverse problem (or calibration) is to define the characteristics of a system that produces the transformation from input to output. In the model the functions that define the transformation are preselected; calibration requires only selection of the parameters of these transformation functions.

If the model was a perfect model of watershed hydrology, and if the precipitation and runoff could be measured accurately, the parameters of the precipitation-runoff transformation functions for a particular storm event could be determined directly by inverse solution of the transformation equations. These conditions are not generally satisfied in reality, and the inverse solution of the equations is difficult. Thus the parameters cannot be determined directly. Instead, the parameters are found by selection of those values that yield the "best" fit of some recorded runoff event with the available recorded precipitation and the model. This parameter selection is generally accomplished by a systematic trial-and-error procedure: first, parameter values are selected; next, the model is run with these values; and then, the resulting runoff hydrograph is compared with the recorded hydrograph. If the "fit" is less than satisfactory, different parameter values can be selected, and the process can be repeated.

Considerable rainfall-runoff calibration data have been prepared by the Los Angeles Office of the Corps of Engineers (COE) for use in local flood control design and planning studies. Some of this information has been prepared during the course of routine flood control studies in Orange County and Los Angeles County, but the major focus of the data has been compiled by the COE in their

Table 1. Watershed characteristics

Watershed Name	Area (mi²)	Length (mi)	Length of Centroid (mi)	Slope (ft/mi)	Percent Impervious (%)	Tc (hrs)	Storm Date	Peak F_p (in/hr)	Lag (hrs)	Basin factor
Watershed Geometry							Calibration Results			
Alhambra Wash[1]	13.67	8.62	4.17	82.4	45	0.89	Feb. 78 Mar. 78 Feb. 80	0.59, 0.24 0.35, 0.29 0.24	0.62	0.015
Compton2[1]	24.66	12.69	6.63	13.8	55	2.22	Feb. 78 Mar. 78 Feb. 80	0.36 0.29 0.44	0.94	0.015
Verdugo Wash[1]	26.8	10.98	5.49	316.9	20	—	Feb. 78	0.65	0.64	0.016
Lemekiln[1]	10.3	7.77	3.41	295.7	25	—	Feb. 78 Feb. 80	0.27 0.27	0.73	0.026
San Jose[2]	83.4	23.00	8.5	60.0	18		Feb. 78 Feb. 80	0.20 0.39	1.66	0.020
Sepulveda[2]	152.0	19.0	9.0	143.0	24	—	Feb. 78 Mar. 78 Feb. 80	0.22, 0.21 0.32 0.42	1.12	0.017
Eaton Wash[1]	11.02[4] (5%)	8.14	3.41	90.9	40	1.05	—	—	—	0.015[7]
Rubio Wash[1]	12.20[5] (3%)	9.47	5.11	125.7	40	0.68	—	—	—	0.015[7]
Arcadia Wash[1]	7.70[6] (14%)	5.87	3.03	156.7	45	0.60	—	—	—	0.015[8]
Compton1[1]	15.08	9.47	3.79	14.3	55	1.92	—	—	—	0.015[8]
Dominguez[1]	37.30	11.36	4.92	7.9	60	2.08	—	—	—	0.0159
Santa Ana Delhi[3]	17.6	8.71	4.17	16.0	40	1.73	—	—	—	0.053[9] 0.040[1]
Westminster[3]	6.7	5.65	1.39	13	40	—	—	—	0.079[9]	0.040[1]
El Modena-Irvine[3]	11.9	6.34	2.69	52	40	0.78	—	—	—	0.028[9]
Gargen Grove-Wintersberg[1]	20.8	11.74	4.73	10.6	64	1.98	—	—	—	—
San Diego Creek[1]	36.8	14.2	8.52	95.0	20	1.39	—	—	—	—

Notes:
1: Watershed Geometry based on review of quadrangle maps and LACFCD storm drain maps.
2: Watershed Geometry based on COE LACDA Study.
3: Watershed Geometry based on COE Reconstitution Study for Santa Ana Delhi and Westminster Channels (June, 1983
4: Area reduced 57% due to several debris basins and Eaton Wash Dam reservoir, and groundwater recharge ponds.
5: Area reduced 3% due to debris basin.
6: Area reduced 14% due to several debris basins.
7: 0.013 basin factor reported by COE (subarea characteristics, June, 1984).
8: 0.015 basin factor assumed due to similar watershed values of 0.015.
9: Average basin factor computed from reconstitution studies.
10: COE recommended basin factor for flood flows.

on-going study for the massive Santa Ana River project (Los Angeles County Drainage Area, or LACDA). The watershed information available includes rainfall-runoff calibration results for three or more significant storms from each watershed. The calibration provided optimized estimates for the watershed loss rates, S-graph, and lag at the peak rainfall intensities. A total of 12 watersheds were considered in detail for this study. Seven of the watersheds are located in Los Angeles County and the remaining 5 are located in Orange County (see Figure 7). Several other watersheds were also considered in light of previous COE studies that resulted in additional estimates of loss rates, S-graphs, and lag values. Table 1 provides a summary for the watershed data from the LACDA study as well as the additional catchments considered hydrologically similar to the COE study catchments.

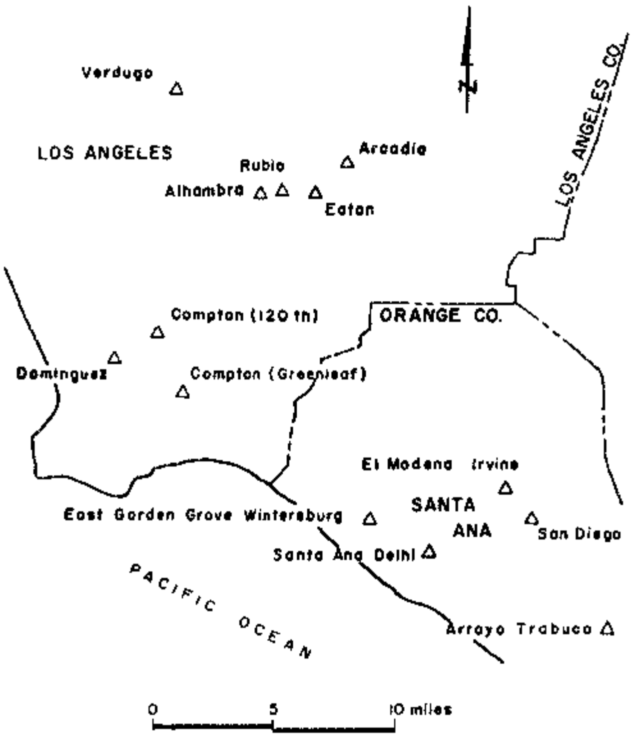

Figure 7 Location of Drainage Basins Used in Model Parameter Calibration Effort.

The key catchments utilized for the calibration of the design storm are Alhambra Wash, Arcadia Wash, Eaton Wash, Rubio Wash, Compton Creek (2 locations), and Dominguez Channel. Although the other watersheds were considered in the parameter calibration, the above seven key watersheds were considered hydrologically similar to the region where the model is intended for use (the valley area of Orange County).

Four of the seven watersheds have been fully urbanized for 20 to 30 years, with efficient interior storm drain systems draining into major concrete channels. Additionally, all of the major storm events have occured during the gage record of relatively constant urbanization; hence, the gage record is assumed to be essentially homogeneous.

PARAMETER OPTIMIZATION

Peak Loss Rate, F_p. From Table 1, several peak rainfall loss rates that include, when appropriate, two loss rates for double-peak storms, are tabulated. The range of values of all F_p estimates lie between 0.30 and 0.65 inch/hour with the highest value occurring in Verdugo Wash, which has substantial open space in the foothill areas. Except for Verdugo Wash, F_p is in the range from 0.2 to 0.60, which is a variation in values of the order noted for Alhambra Wash alone. Figure 8 shows a histogram of F_p values for the several watersheds. It is evident from the figure that 88 percent of F_p values are between 0.20 and 0.45 inch/hour, with 77 percent of the values falling between 0.20 and 0.40 inch/hour. Consequently, a regional mean value of F_p equal to 0.30 inch/hour is recommended for the model; this value contains nearly 80 percent of the F_p values, for all watersheds and for all storms, within 0.10 inch/hour.

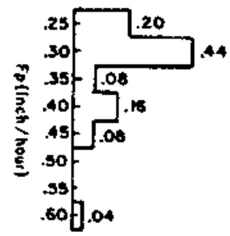

Figure 8 Distribution-Frequency of Pervious Area Loss Function, F;iP;t.

S-graph Each of the watersheds listed in Table 1 has S-graphs developed for each of the storms where peak loss rate values were developed. For example, Fig. 9 shows the S-graphs developed for Alhambra Wash. By averaging the S-graph ordinates (developed from rainfall-runoff data), an average watershed S-graph was obtained. By combining the several watershed average S-graphs into a single plot (Fig. 10), a regional S-graph is obtained. The variation in S-graphs for a single watershed for different storms (see Fig. 9) is of the same order of magnitude of variation seen between the several catchment averaged S-graphs.

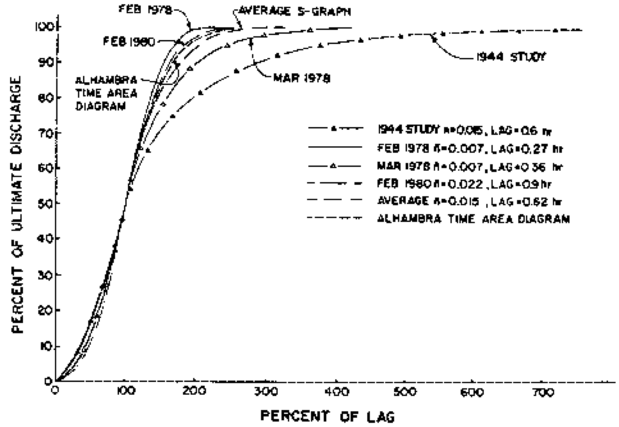

Figure 9 Alhambra Wash Best-Fit S-Graphs

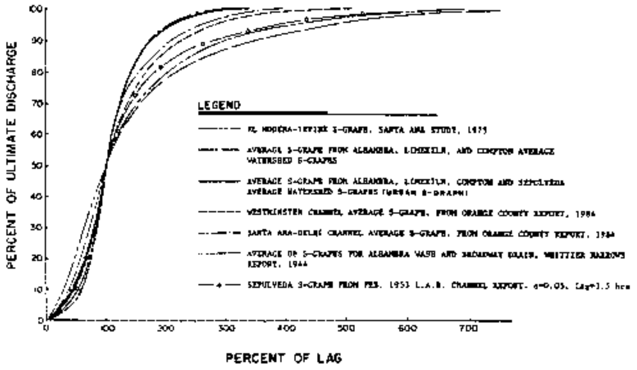

Figure 10   Catchment Averaged S-Graphs

In order to quantify the effects of variations in the S-graph due to variations in storms and in watersheds (i.e., for ungauged watersheds not included in the calibration data set), the scaling of Fig. 11 was used where the variable "X" signifies the average value of an arbitrary S-graph as a linear combination of the steepest and flattest S-graphs obtained. That is, all the S-graphs (all storms, all catchments) lie between the Feb. 1978 storm Alhambra S-graph (X = 1) and the San Jose S-graph (X = 0). To approximate a particular S-graph of the sample set.

$$S(X) = XS_1 + (1 - X)S_2$$

where $S(X)$ is the S-graph as a function of X, and S_1 and S_2 are the Alhambra (Feb. 1978 storm) and San Jose S-graphs, respectively. Figure 12 shows the population distribution of X where each watershed is weighted equally in the total distribution (i.e., each watershed is represented by an equal number of X entries). Table 2 lists the X values obtained from the Fig. 11 scalings of each catchment S-graph. In Table 2, an "upper" and "lower" X-value that corresponds to the X coordinate at 80 percent and 20 percent of ultimate discharge values, respectively, is listed. An average of the upper and lower X values is used in the population distribution of Fig. 12.

Table 2.   Catchment S-graph X-values

Watershed	Storm	X(upper)	X(lower)	X(avg)
Alhambra	Feb. 78	1.00	1.00	1.00
	Feb. 80	0.95	0.60	0.78
	Mar. 78	0.70	0.80	0.75
Limekiln	Feb. 78	0.50	0.80	0.65
	Feb. 80	0.80	1.00	0.90(2)
Sepulveda	Avg.	0.90	0.80	0.85(3)
Compton	Avg.	0.90	1.00	0.95(3)
Westminster	Avg.	0.60	0.60	0.60(3)
Santa Ana Delhi	Avg.	0.80	1.00	0.90(3)
Urban	Avg.	0.90	0.80	0.85

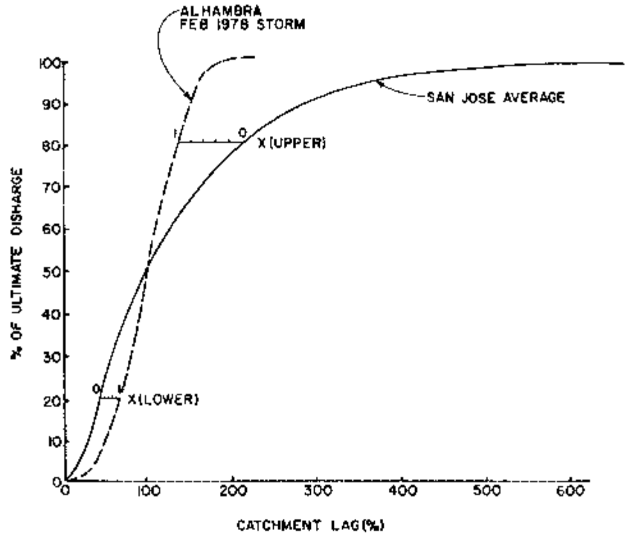

Figure 11   S-Graph Scaling for S(X)

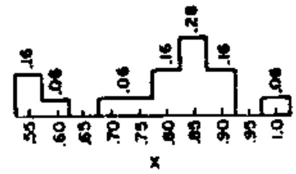

Figure 12   Distribution-Frequency for S-Graph 'X' Parameter

Catchment Lag From Table 2, the Urban S-graph, which represents a regionalized S-graph for urbanized watersheds in valley type topography, has an associated X value of 0.85. When the Urban S-graph is compared to the standard SCS S-graph, a striking similarity is seen (Fig. 13). Because the new Urban S-graph is a near duplicate of the SCS S-graph, it was assumed that catchment lag (COE definition) is related to the catchment

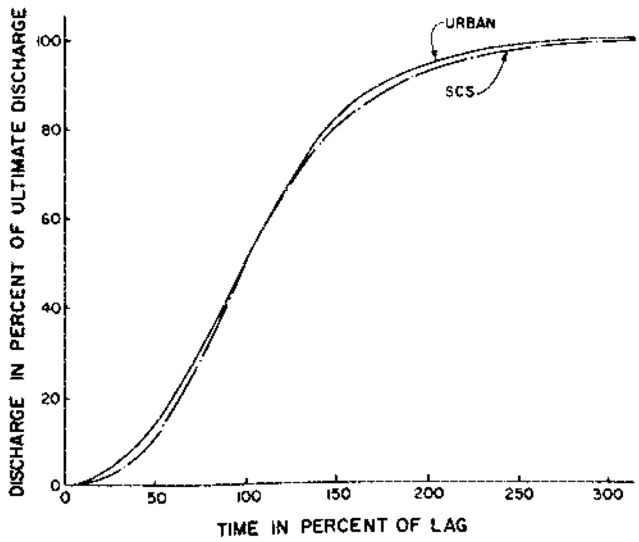

Figure 13   Comparison of the Standard SCS and the Regionalized S-Graphs

time of concentration, Tc, as is typically assumed in the SCS approach.

Catchment Tc values are estimated by subdividing the watershed into subareas with the initial subarea less than 10 acres and a flow length of less than 1000 feet. Using a Kirpich formula, an initial subarea Tc is estimated and a Q is calculated. By subsequent routing downstream of the peak flow rate (Q) through the various conveyances (using normal depth flow velocities) and adding estimated successive subarea contributions, a catchment Tc is estimated as the sum of travel times analogous to a mixed velocity method.

Lag values are developed directly from available COE storm event calibration data, or by using a "basin factor" calibrated from neighboring catchments (see Fig. 7). The COE standard lag formula is:

$$\text{lag(hours)} = 24\bar{n}\left(\frac{LL_{ca}}{s^{0.5}}\right)^{0.38}$$

where L is the watershed length in miles; L_{ca} is the length to the centroid along the watercourse in miles; s is the slope in ft/mile; and $\bar{n}$ is the basin factor.

Because Eaton Wash, Rubio Wash, Arcadia Wash and Alhambra Wash are all contiguous (see Fig. 7), have similar shapes, slopes, development patterns, and drainage systems, the basin factor of $\bar{n} = 0.015$ calibrated for Alhambra Wash was also used for the other three neighboring watersheds. Then the lag was estimated using the COE formula.

Compton Creek has two gages, and the $\bar{n} = 0.015$ calibrated for Compton 2 was also used for the Compton 1 gage. The Dominguez catchment, which is contiguous to Compton Creek, is also assumed to have a lag calculated by using $\bar{n} = 0.015$.

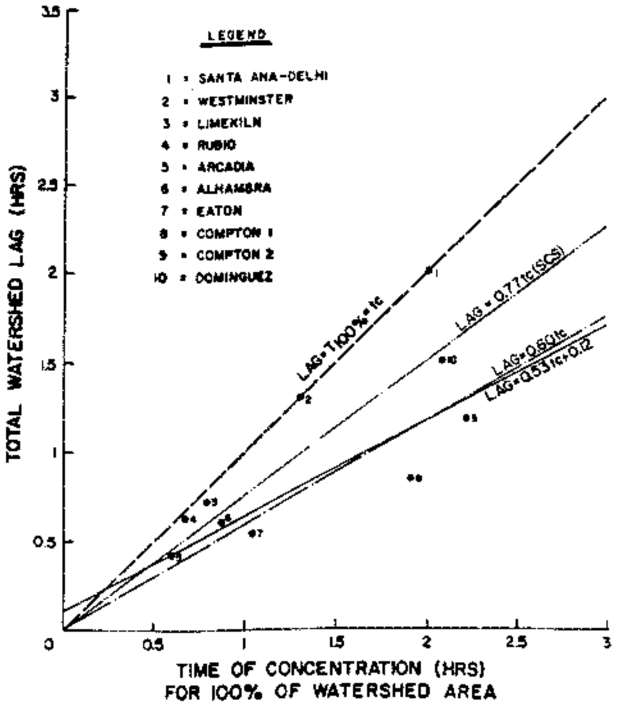

Figure 14 Relationship Between Catchment Lag and Tc Values for Local Region

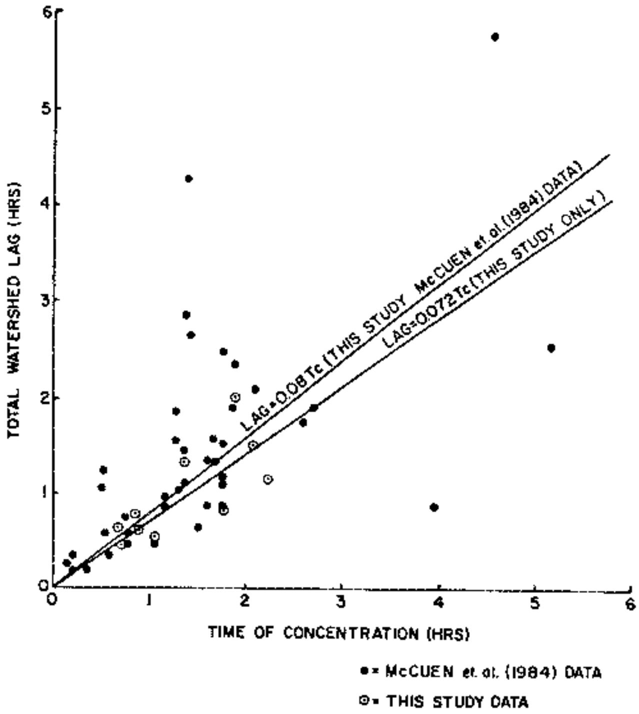

Figure 15 Relationship Between Measured Catchment Lag and Computed Tc

Figure 16 Distribution-Frequency of Lag 0.8Tc

The Santa Ana-Delhi and Westminster catchment systems of Orange County have lag values developed from prior COE calibration studies. Figure 14 provides a summary of the local lag versus Tc data. A least-squares best fit results in

$$\text{lag} = 0.72Tc$$

McCuen et al. (1984) provide additional measured lag values and mixed velocity Tc estimates which, when lag is modified according to the COE definition, can be plotted with the local data such as shown in Fig. 15. A least-squares best fit results in:

$$\text{lag} = 0.80Tc$$

In comparison, McCuen (1982) gives standard SCS relationships between lag, Tc, and time-to-peak (Tp) which, when modified to the COE lag definition, results in:

$$\text{lag} = 0.77Tc$$

Assuming a lag of $0.80Tc$, the distribution of (lag/Tc) values with respect to Lag $= 0.80Tc$ is shown in Fig. 16.

STATISTICAL ANALYSIS

Many hydrology models involve complex systems of submodels whose performance fluctuates because of

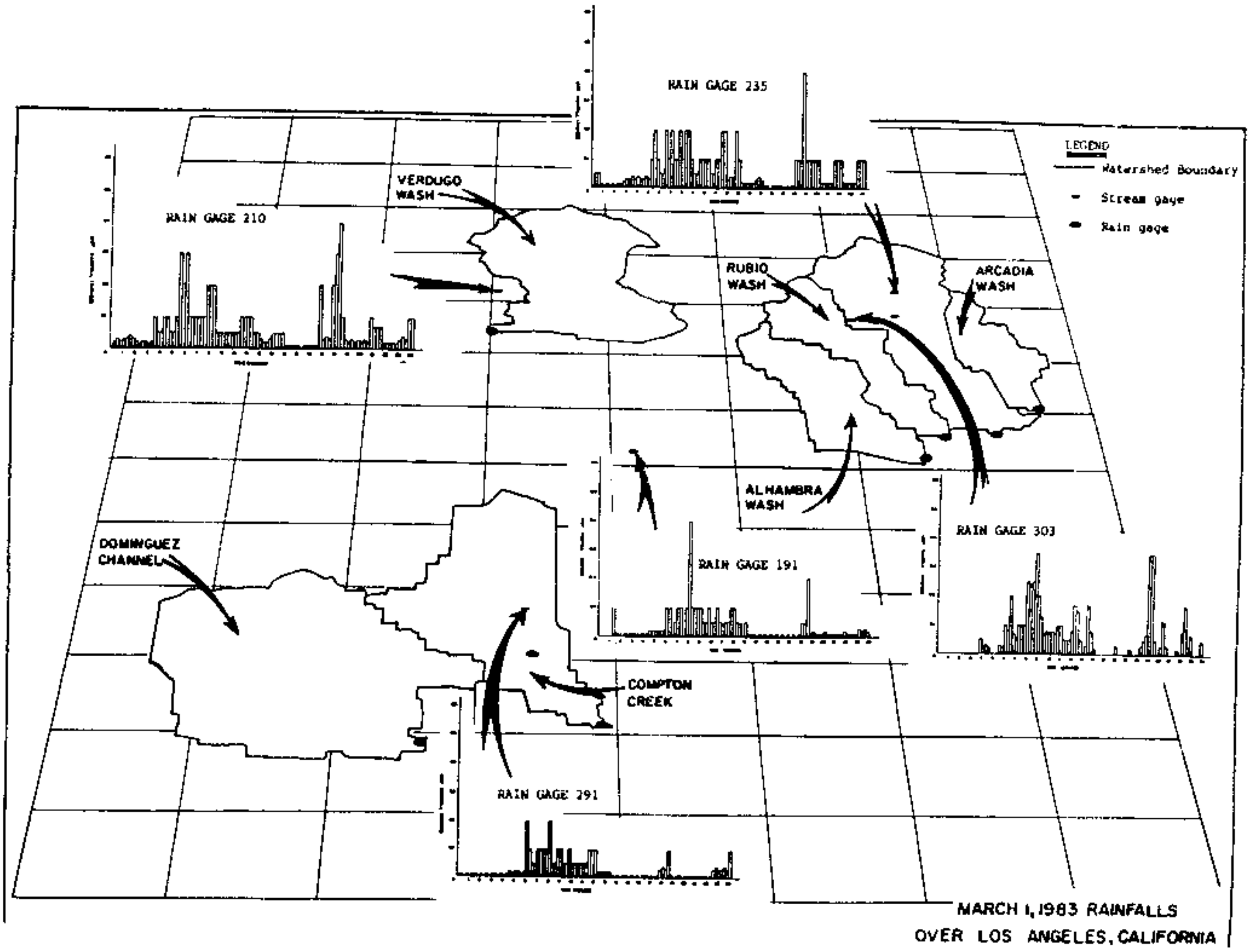

Figure 17 March 1, 1983 Recorded Storm Rainfall Distribution

variations in their parameters. From a knowledge of the model parameter distribution and an understanding of the model structure, the model output distribution due to parameter uncertainty can be evaluated.

Several statistical models are available for studying the relationship between the model output distribution and parameter uncertainty. In a statistical model, the parameters of a hydrology model are considered as random variables sampled from respective value distributions usually developed from regionalized data analysis. For parameters with discrete probability density functions (pdf) an exhaustion model can be used to calculate the mean and standard deviation of the model output. However in the general case, the number of trials needed to exhaust the parameter vector field is so large, that this technique cannot be used. If continuous or discrete probability density functions of each parameter are known, then the Monte Carlo simulation method can be used to develop estimates of the mean and variance of model output. The moments generation method is another technique which can be applied to a simple performance-parameters relationship model with known mean and standard deviation for each parameter. If the coefficient of variation (which is defined as the ratio between standard deviation and mean) for each parameter is small, then a simplified two-point estimate method (Rosenblueth, 1975) may be considered.

EXHAUSTION MODEL

Simple discrete probability density functions can be derived using histograms for each random variable. For demonstration purposes, consider a hydrology model output Q based upon three parameters; i.e., $Q = F(X_1, X_2, X_3)$, where the X_i are represented by three different discrete probability density functions (Fig. 17).

The mean of the subject model output, Q, is

$$\bar{Q} = \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} P_{1i} P_{2j} P_{3k} F(X_{1i}, X_{2j}, X_{3k})$$

and the standard deviation is

$$S = \left(\sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} P_{1i} P_{2j} P_{3k} [F(X_{1i}, X_{2j}, X_{3k}) - \bar{Q}]^2 \right)^{1/2}$$

where P_{1i} is the probability weighting assigned to X_1 at outcome i. The above two equations can be extended directly for any finite number of parameters and number of parameter-histogram values.

MONTE CARLO SIMULATION METHOD

If probability density functions for each of the parameters can be obtained or estimated, the Monte Carlo simulation method can be used. Because the Monte Carlo simulation method involves randomly selected input vectors, the computed model output statistics (e.g., mean, variance) are themselves random variables and the estimates are also subject to statistical fluctuations. Thus any estimate will be a random variable and will have an associated error band. The larger the number of trials in the simulation, it is hoped the more precise will be the estimates for the statistics.

MOMENTS GENERATION METHOD

Generally, the density functions are not available for most of the parameters in a hydrology model. However, oftentimes the mean and standard deviation of each parameter can be estimated from the limited information. By using the previous example, the mean of the model output can be estimated from the moments generation method as

$$E[Q] = E[F(\bar{X}_1, \bar{X}_2, \bar{X}_3)] + 1/2 \sum_{i=1}^{3} \frac{\partial^2 F(\bar{X}_1, \bar{X}_2, X_3)}{\partial X_i^2} \mathrm{VAR}(X_i)$$

and the variance of the system performance can be estimated as

$$\mathrm{Var}[Q] = \sum_{i=1}^{3} \left(\frac{\partial F(\bar{X}_1, \bar{X}_2, \bar{X}_3)}{\partial X_i} \right)^2 \mathrm{VAR}(X_i) \qquad (16)$$

in which X_1, X_2, X_3 are the estimated means and $\mathrm{Var}(X_i)$ is the estimated variance for each parameter, respectively.

TWO-POINT ESTIMATE METHOD

If the first and second partial derivatives in the above equations are not available, then the two-point estimate method (Rosenblueth, 1975) can be considered. The estimated mean ($\bar{Q}$) and standard deviation(s) for the model output (Q) from the two-point estimate method are

$$\begin{aligned}
\bar{Q} = {} & 1/8F(X_1+s_1, X_2+s_2, X_3+s_3) \\
& + 1/8F(X_1+s_1, X_2+s_2, X_3-s_3) \\
& + 1/8F(X_1+s_1, X_2-s_2, X_3+s_3) \\
& + 1/8F(X_1-s_1, X_2+s_2, X_3+s_3) \\
& + 1/8F(X_1-s_1, X_2-s_2, X_3+s_3) \\
& + 1/8F(X_1-s_1, X_2+s_2, X_3-s_3) \\
& + 1/8F(X_1+s_1, X_2-s_2, X_3-s_3) \\
& + 1/8F(X_1-s_1, X_2-s_2, X_3-s_3) \qquad (17)
\end{aligned}$$

and

$$\begin{aligned}
s = {} & \{1/8[F(X_1+s_1, X_2+s_2, X_3-s_3) - Q_2]^2 \\
& + 1/8[F(X_1+s_1, X_2+s_2, X_3-s_3) - Q]^2 \qquad (18) \\
& \cdot \\
& \cdot \\
& \cdot \\
& + 1/8[F(X_1-s_1, X_2-s_2, X_3-s_3) - Q]^2\}^{1/2}
\end{aligned}$$

in which s_1, s_2, s_3 are the standard deviations of X_1, X_2, X_3, respectively.

The exhaustion model is suitable for models involving few parameters with sparse discrete pdf's. When the discrete pdf's become dense, or the number of parameters becomes large, resulting in an infeasible number of model outcome runs for the exhaustion technique, then the Monte Carlo or Rosenblueth techniques become more attractive. In most engineering problems, only the means and variances of the parameters need to be estimated; in which case, the moment generation method is preferred when the first and second partial derivatives can be evaluated or approximated. If the model output-parameters relationship is linear, then either the moment generation method or the two-point estimate method is suitable. For a non-linear system with parameters that have small coefficients of variation, the results from the moment generation method and the two-point estimate method are usually satisfactory. The exhaustion model and the Monte Carlo simulation method are suitable for non-linear systems and/or for parameters having large coefficients of variation.

Since different criteria and information are needed for each statistical model, the selection of a statistical model should be conducted with care, and attention must be given to development of a realistic description of the underlying physical situation to serve as input to a statistical model.

COMPARISON OF MODEL UNCERTAINTY EVALUATION TECHNIQUES: PEAK FLOW ESTIMATES

For simplicity, the hydrologic model is assumed to be a function of only the three representative parameters of soil loss (phi index), watershed time of concentration (Lag) and the unit hydrograph (S-graph form) variable, (X). Figures 8, 12 and 16 show the assumed histograms for these three random variables. In this application, the model output considered is the runoff hydrograph peak flow rate, or Qpeak. Using the several parameter values from each histogram, an exhaustion study was performed. For the second analysis using the Monte Carlo technique, a uniform [0, 1] distribution is used to represent each histogram shown in Figures 8, 12 & 16. Then a Monte Carlo simulation is used to develop random model outcomes by use of random input vectors containing randomly selected parameter values. Finally, the mean and standard deviation values from each histogram were used in the Rosenblueth method as the third model. Table 3 shows the results developed from a runoff hydrograph model for 1 square mile. Because only 3 parameters are used in this model setting, the Rosenblueth technique requires only 2^3 or 8 trials of the model, potentially affording a considerable cost saving over an exhaustion analysis.

As shown in Table 3, the Rosenblueth model gives the highest estimated mean value of the Qpeak and also the largest standard deviation of the estimated Qpeak. This probably reflects the high coefficients of variation of watershed time of concentration (and similarly, lag) and soil loss function (ph index) in the above study.

For further comparison of the Monte Carlo and Rosenblueth models, the two simplified pdf's of watershed lag time and soil loss function were modified to have small coefficients of variation. Table 3 also includes the

Table 3

Model	Iterations	Mean	Standard Deviation
Exhaustion	378	683 (687)	190 (101)
Monte Carlo[1]	10	649 (675)	166 (102)
	25	688 (676)	199 (101)
	100	670 (682)	171 (94)
	500	683 (685)	181 (94)
	1000	681 (684)	185 (94)
	5000	683 (685)	187 (93)
Rosenblueth	8	728 (712)	404 (98)

Notes
1. Single sample results. All values for the mean and standard deviation for Q are estimates (see text).
2. Values in parentheses indicate results by using smaller coefficent of variation for the time of concentration and soil loss function.

results of the three statistical approaches for this second study. In this case, the Rosenblueth model showed acceptable results compared to the exhaustion model and the Monte Carlo method.

It is noteworthy that in the actual field case study where the coefficients of variation of two model parameters are "large", the Rosenblueth technique performed poorly, even when compared to a Monte Carlo analysis with a small sample set. Of course, the Monte Carlo technique estimates are random variables themselves, but in this study it was found that for over 90% of the time, the Monte Carlo technique resulted in better estimates of the mean and standard deviation than the Rosenblueth method for the same number of trials (i.e., sample size of 8). And when the model parameters are modified to have a "small' coefficient of variation, the Rosenblueth technique produced acceptable estimates but so did the Monte Carlo method for the same effort.

Because it is not clear beforehand whether a model uncertainty analysis based upon the Rosenblueth technique will result in adequate estimates of the mean and standard deviation, and because the Monte Carlo performed as good as or better than the Rosenblueth technique (as applied to this hydrologic model), the Monte Carlo technique may be preferable for use in other studies as well where information about the parameters' pdfs are known.

MODEL VERIFICATION

The hydrologic model was applied to a severe storm condition which occurred on March 1, 1983 in southern California. This storm resulted in various rainfall intensities ranging between 10-year and 200-year return frequencies, causing severe distributions in the Los Angeles area. As can be seen, the variation in rainfall is significant even though the storm was of a rare return frequency. Stream gauge data was also available at several catchments. Consequently, it is feasible to prepare a rainfall-runoff analysis using the previously described hydrologic model. It is noted that this storm was not

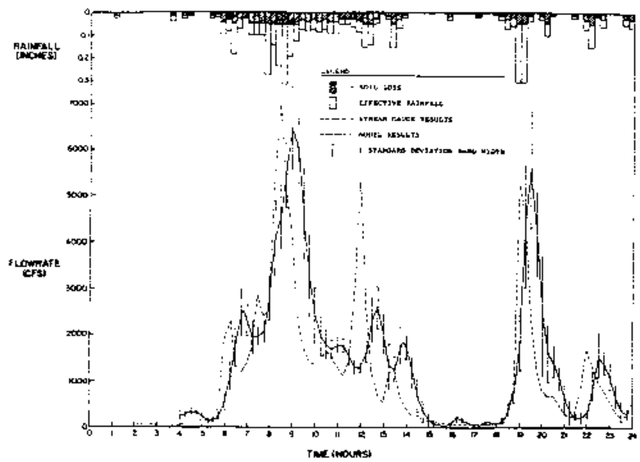

Figure 18 Rosenblueth Analysis of the March 1, 1983 Storm for the Alhambra Wash Watershed

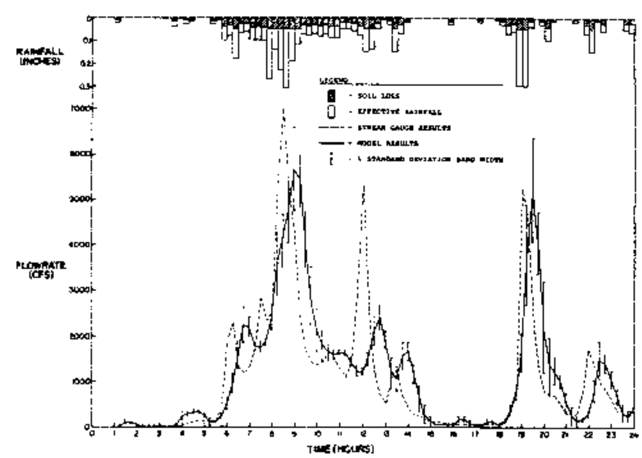

Figure 19 Monte Carlo Analysis of the March 1, 1983 Storm for the Alhambra Wash Watershed

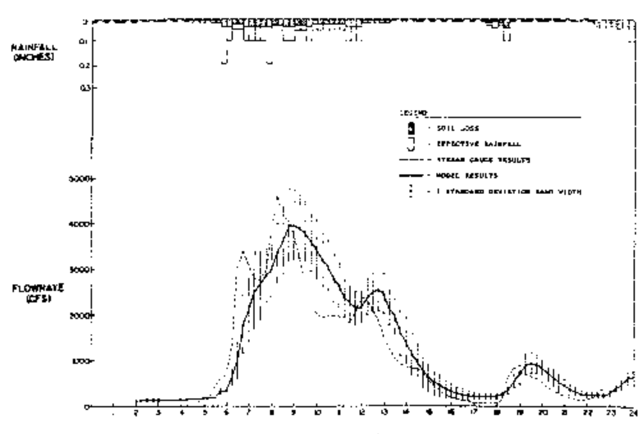

Figure 20 Rosenblueth Analysis of the March 1, 1983 Storm for the Compton Creek Watershed

Table 4. Comparison of Rosenblueth and Monte Carlo Techniques in Modeling March 1, 1983 Storm for Two Catchments

Watershed	Runoff Hydrograph 5-minute Interval No	Rosenblueth $\bar{Q}$	Rosenblueth s	Monte Carlo $n=8$ $\bar{Q}$	$n=8$ s	$n=50$ $\bar{Q}$	$n=50$ s	$n=100$ $\bar{Q}$	$n=100$ s	$n=500$ $\bar{Q}$	$n=500$ s
Alhambra Wash	82	2511	491	2303	363	2155	362	2101	339	2086	336
	109	6486	858	5758	864	5444	798	5329	713	5305	739
	152	2618	530	2404	338	2285	315	2227	302	2211	293
	234	5602	1263	5218	1154	4740	1170	4600	1088	4535	1058
	271	1585	443	1521	343	1387	320	1342	307	1327	295
Compton Creek	81	1828	647	3583	681	3279	740	3171	680	3145	656
	05	3984	784	4232	650	4036	513	3942	465	3919	436
	(25)1	2570	451	2717	250	2637	195	2621	189	2616	177
	233	946	257	1507	184	1428	191	1417	182	1410	181
	291	762	139	1018	33	1004	60	1002	57	999	47

Notes
n = sample size
Q = mean value
s = standard deviation

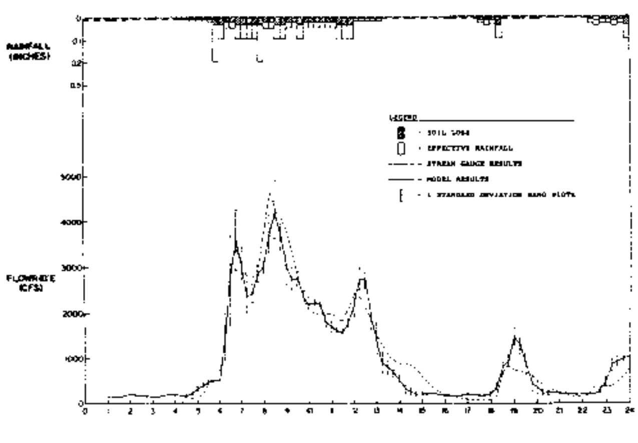

Figure 21 Monte Carlo Analysis of the March 1, 1983 Storm for the Compton Creek Watershed

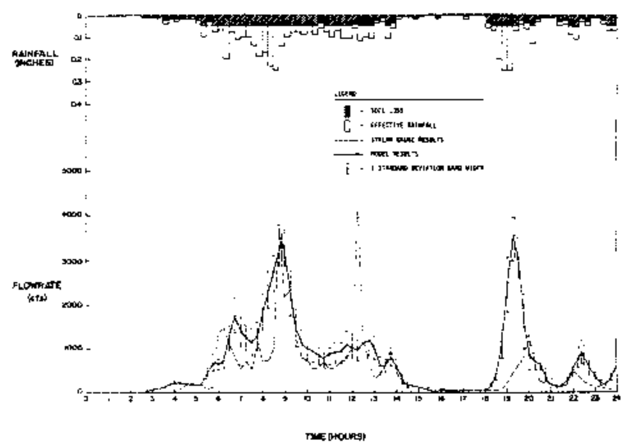

Figure 22 Rosenblueth Analysis of the March 1, 1983 Storm for the Arcadia Wash Watershed

included in the data set used to determine the parameter calibration of the hydrologic model. It is also noted that in this study effort, the focus is upon the modeling of a particular storm event rather than the development of a probabilistic design storm runoff hydrograph for flood control purposes. Hence, the nested design storm pattern used for flood control purposes is replaced, in this application, by the actual measured storm pattern recorded at the available rain gauges.

The subject model is based upon 5-minute unit intervals for both rainfall and runoff. Assuming each 5-minute unit interval of runoff to be a random variable, an uncertainty analysis can be prepared for the entire runoff hydrograph as a collection of 5-minute unit interval random variables. Such an analysis was prepared for two of the study watersheds using the parameter value histograms of Figs. 8, 12, and 16, for both the Rosenblueth and Monte Carlo techniques. A comparison of results in the estimates of the several peak flow rate statistics are contained in Table 4. Figures 18 through 21 show the modeling outcomes developed from the two uncertainty analysis techniques considered as compared to the stream gauge records. Figures 22 through 25 show the modeling ooutcomes developed for the remaining four watersheds using the Rosenblueth technique only.

As with the previous application, the monte Carlo

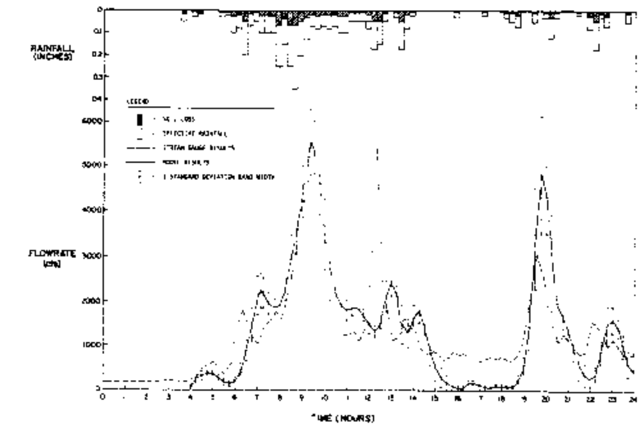

Figure 23 Rosenblueth Analysis of the March 1, 1983 Storm for the Eaton Wash Watershed

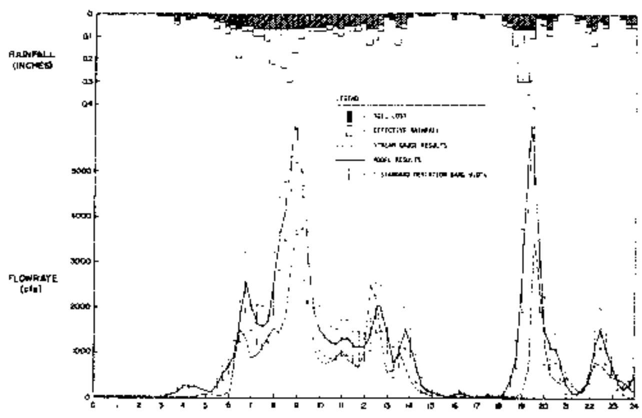

Figure 24 Rosenblueth Analysis of the March 1, 1983 Storm for the Rubio Wash Watershed

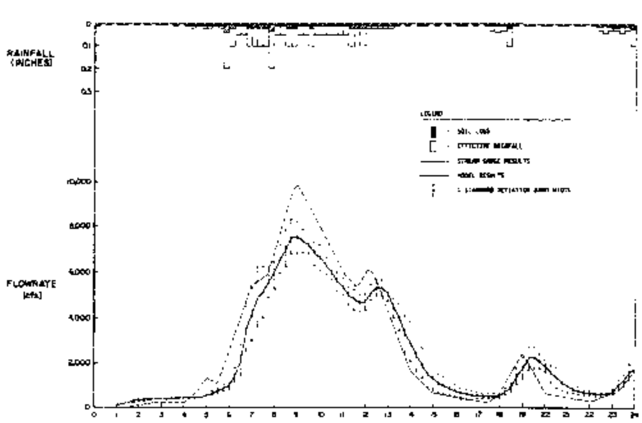

Figure 25 Rosenblueth Analysis of the March 1, 1983 Storm for the Dominguez Channel Watershed

technique provides in general, a "better" estimate for the mean and standard deviation than provided by the Rosenblueth technique. Even with the same computational effort (8 trials), the Monte Carlo technique outperformed (for this study) the Rosenblueth technique.

CONCLUSIONS

It is important to consider the uncertainty in a hydrologic model and, in order to prepare such an analysis, a technique to develop model outcome statistics is needed. Due to computational effort limitations, an exhaustion study which considers the total universe of parameter inputs is usually precluded. Two alternatives to an exhaustion study is a Monte Carlo simulation and the more recently advanced Rosenblueth technique. Although the Rosenblueth technique potentially affords a significant saving in computational effort over that usually needed with a Monte Carlo analysis, it was found that in this application the Monte Carlo technique was superior in accuracy, for even the same computational effort.

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