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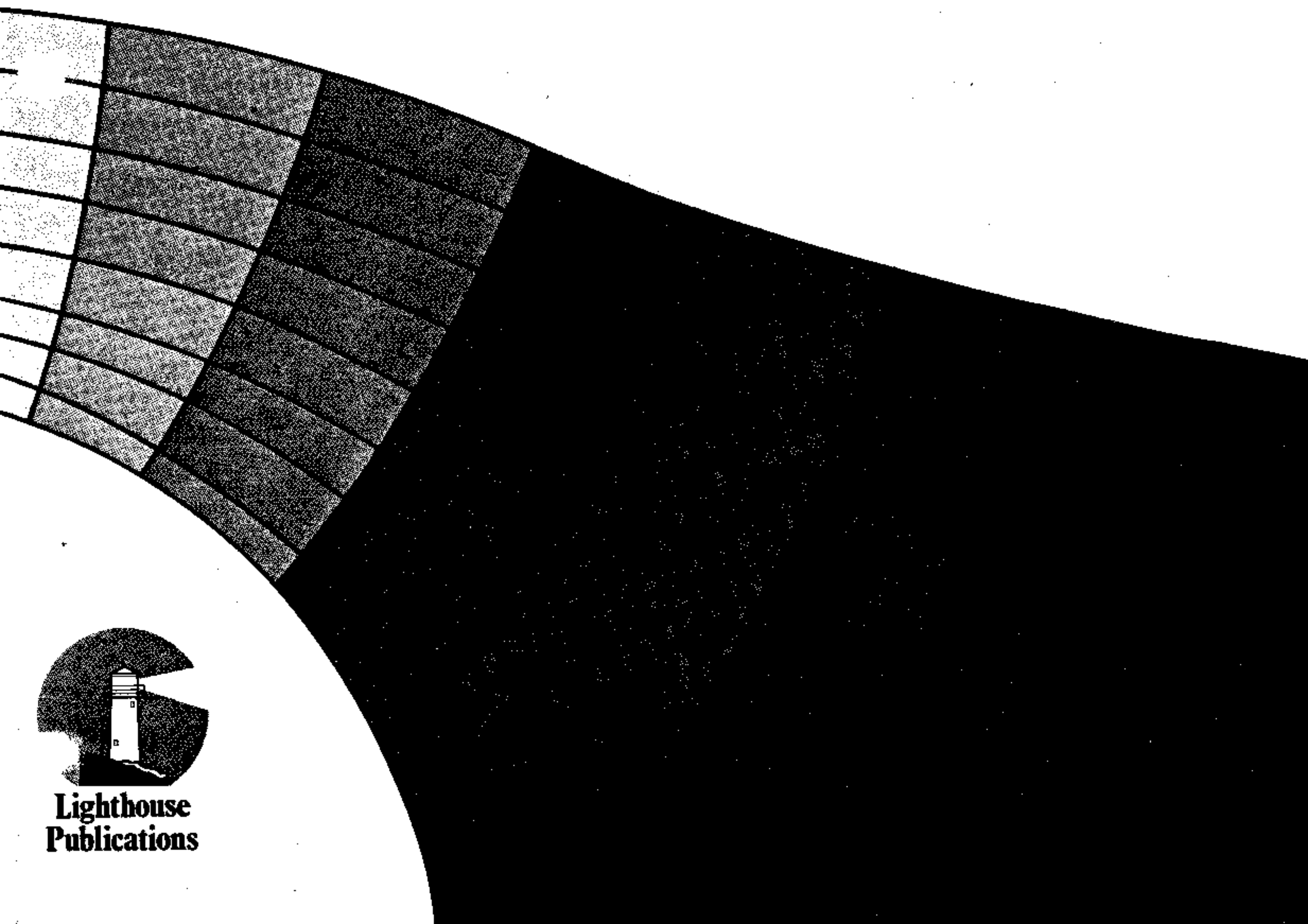

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Including Uncertainty in Hydrology Criterion Variable Predictions

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Abstract

The classic single area unit hydrograph (UH) approach to modelling runoff response from a free draining catchment is shown to represent several important modeling considerations including, (i) subarea runoff response (in a discretized model), (ii) the subarea effective rainfall distribution including variations in magnitude, timing, and storm pattern shape, (iii) channel flow routing translation and storage effects, using the linear routing technique, (iv) subarea runoff hydrograph addition, among other factors. Because the UH method correlates the effective rainfall distribution to the runoff hydrograph distribution, the resulting catchment UH should be considered a correlation distribution in a probabilistic sense. Should the uncertainty in rainfall over the catchment be a major concern in modeling reliability, then the UH output in the predictive setting must be considered to be a random variable.

INTRODUCTION

The current trend in hydrologic surface runoff model development is to discretize the catchment (assumed to be free draining) into several small subareas, each linked by a channel hydraulic flow routing algorithm. The resulting model is then formulated as a link node model which responds hydraulically according to a specified effective rainfall in each subarea. While over 100 such models have been developed in the open literature (Hromadka, 1987), none have been shown to provide consistently "better" results than the classic single area unit hydrograph (UH) methods in the estimation of severe storm runoff of interest in flood control. It is shown in this paper that the classic UH technique provides, (i) a rational modeling structure which properly represents several hydrologic effects which a highly discretized model misrepresents; (ii) a correlation distribution (distribution frequency of UH's) which correlates the effective rainfall to be measured runoff hydrograph; and (iii) a probabilistic model which represents the model output as a random variable, whose variance represents the natural variance between effective rainfall and runoff.

CATCHMENT AND DATA DESCRIPTION

Let R be a free draining catchment with negligible detention effects. R is discretized into m subareas, R_j, each draining to a nodal point which is drained by a channel system. The m-subarea link node model resulting by combining the subarea runoffs for storm i, $Q_j^i(t)$, adding runoff hydrographs at nodal points, and routing through the channel system, is denoted as $Q_m^i(t)$. It is assumed that there is only a single rain gage and stream gage available for data analysis. The rain gage site is monitored for the 'true' effective rainfall distribution, $e_g^i(t)$. The stream gage data represents the entire catchment, R, and is denoted by $Q_g^i(t)$.

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LINEAR EFFECTIVE RAINFALLS FOR SUBAREAS

The effective rainfall distribution (rainfall less losses) in R_j is given by $e_j^i(t)$ for storm i where $e_j^i(t)$ is assumed to be linear in $e_g^i(t)$ by

$$e_j^i(t) = \sum \lambda_{jk}^i \, e_g^i(t - \theta_{jk}^i) \tag{1}$$

where λ_{jk}^i and θ_{jk}^i are coefficients and timing offsets, respectively, for storm i and subarea R_j. In Eq. (1), the variations in the effective rainfall distribution over R due to magnitude and timing are accounted for by the λ_{jk}^i and θ_{jk}^i, respectively. The subareas, R_j, are chosen such that Eq. (1) is a good approximation for each subarea.

SUBAREA RUNOFF

The storm i subarea runoff from R_j is given by $Q_j^i(t)$ where

$$Q_j^i(t) = \int_{s=0}^{t} e_j^i(t-s)\, \phi_j^i(s)\, ds \tag{2}$$

where $\phi_j^i(s)$ is the subarea unit hydrograph (UH) for storm i such that Eq. (2) applies. Combining Eqs. (1) and (2) gives

$$Q_j^i(t) = \int_{s=0}^{t} \sum e_g^i(t - \theta_{jk}^i - s)\, \lambda_{jk}^i\, \phi_j^i(s)\, ds \tag{3}$$

Rearranging variables,

$$Q_j^i(t) = \int_{s=0}^{t} e_g^i(t-s) \sum \lambda_{jk}^i\, \phi_j^i(s - \theta_{jk}^i)\, ds \tag{4}$$

where throughout this paper, arbitrary function F(s - Z) is notation that F(s - Z) = 0 for s < Z.

REFERENCES

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LINEAR ROUTING

Let $I_1(t)$ be the inflow hydrograph to a channel flow routing link (number 1), and $O_1(t)$ the outflow hydrograph. A linear routing model of the unsteady flow routing process is given by

$$O_1(t) = \sum_{k_1=1}^{n_1} a_{k_1} I_1(t - \alpha_{k_1}) \qquad (5)$$

where the a_{k_1} are coefficients which seem to unity; and the α_{k_1} are timing offsets. Again, $I_1(t-\alpha_{k_1}) = 0$ for $t < \alpha_{k_1}$. Given stream gage data for $I_1(t)$ and $O_1(t)$, the best fit values for the a_{k_1} and α_{k_1} can be determined.

Should the above outflow hydrograph, $O_1(t)$, now be routed through another link (number 2), then $I_2(t) = O_1(t)$ and from the above

$$O_2(t) = \sum_{k_2=1}^{n_2} a_{k_2} I_2(t - \alpha_{k_2})$$

$$= \sum_{k_2=1}^{n_2} a_{k_2} \sum_{k_1=1}^{n_1} a_{k_1} I_1(t - \alpha_{k_1} - \alpha_{k_2}) \qquad (6)$$

For L links, each with their own respective stream gage routing data, the above linear routing technique results in the outflow hydrograph for link number L, $O_L(t)$, being given by

$$O_L(t) = \sum_{k_L=1}^{n_L} a_{k_L} \sum_{k_{L-1}=1}^{n_{L-1}} a_{k_{L-1}} \cdots \sum_{k_2=1}^{n_2} a_{k_2} \sum_{k_1=1}^{n_1} a_{k_1} I_1(t - \alpha_{k_1} - \alpha_{k_2} - \cdots - \alpha_{k_{L-1}} - \alpha_{k_L}) \qquad (7)$$

Using vector notation, the above $O_L(t)$ is written as

$$O_L(t) = \sum_{<k>} a_{<k>} I_1(t - \alpha_{<k>}) \qquad (8)$$

For subarea R_j, the runoff hydrograph for storm i, $Q_j^{\,i}(t)$, flows through L_j links before arriving at the stream gage and contributing to the total measured runoff hydrograph, $Q_g^{\,i}(t)$. All of the constants $a^i_{<k>}$ and $\alpha^i_{<k>}$ are available on a storm by storm basis. Consequently from the linearity of the routing technique, the m-subarea link node model is given by the sum of the m, $Q_j^{\,i}(t)$ contributions,

$$Q_m^{\,i}(t) = \sum_{j=1}^{m} \sum_{<k>_j} a^i_{<k>_j} Q^i_j(t - \alpha^i_{<k>_j}) \qquad (9)$$

where each vector $<k>_j$ is associated to a R_j, and all data is defined for storm i. It is noted that in all cases,

$$\sum_{<k>_j} a^i_{<k>_j} = 1 \qquad (10)$$

LINK-NODE MODEL, $Q_m^{\,i}(t)$

For the above linear approximations for storm i, Eqs. (1), (4), and (9) can be combined to give the final form for $Q_m^{\,i}(t)$,

$$Q_m^{\,i}(t) = \sum_{j=1}^{m} \sum_{<k>_j} a^i_{<k>_j} \int_{s=0}^{t} e_g^{\,i}(t-s) \sum \lambda^i_{jk} \phi_j^{\,i}(s - \theta^i_{jk} - \alpha^i_{<k>_j})\, ds \qquad (11)$$

Because the measured effective rainfall distribution, $e_g^{\,i}(t)$, is independent of the model, Eq. (1) is rewritten in the final form

$$Q_m^{\,i}(t) = \int_{s=0}^{t} e_g^{\,i}(t-s) \sum_{j=1}^{m} \sum_{<k>_j} a^i_{<k>_j} \sum \lambda^i_{jk} \phi_j^{\,i}(s - \theta^i_{jk} - \alpha^i_{<k>_j})\, ds \qquad (12)$$

where all parameters are evaluated on a storm by storm basis, i.

Equation (12) describes a model which represents the total catchment runoff response based on variable subarea UH's, $\phi_j^{\,i}(s)$; variable effective rainfall distributions on a subarea-by-subarea basis with differences in magnitude (λ^i_{jk}), timing (θ^i_{jk}), and pattern shape (linearity assumption); and channel flow routing translation and storage effects (parameters $a^i_{<k>_j}$ and $\alpha^i_{<k>_j}$).

MODEL REDUCTION

The m-subarea model of Eq. (12) is directly reduced to the simple single area UH model (no discretization of R into subareas) given by $Q_1^{\,i}(t)$ where

$$Q_1^{\,i}(t) = \int_{s=0}^{t} e_g^{\,i}(t-s)\, \eta^i(s)\, ds \qquad (13)$$

where $\eta^i(s)$ is the correlation distribution between the data pair $\{Q_g^{\,i}(t), e_g^{\,i}(t)\}$.

From Eq. (13) it is seen that the classic single area UH model represents a highly complex link node modeling structure. For the case of having available a single rain gage and stream gage for data correlation purposes, the derived $\eta^i(s)$ represents the several effects used in the development leading to Eq. (12), integrated according to the sample from the several parameters' respective probability distributions. Because the simple $Q_1^{\,i}(t)$ model structure actually includes most of the effects which are important in flood control hydrologic response, it can be used to develop useful probabilistic distributions of modeling output.

STORM CLASSIFICATION SYSTEM

To proceed with the analysis, the full domain of effective rainfall distributions measured at the rain gage site are categorized into storm classes, $<\xi_q>$. That is, any two elements of a class $<\xi_q>$ would result in nearly identical effective rainfall distributions at the rain gage site, and hence one would "expect" nearly identical resulting runoff hydrographs from the stream gage. Typically, however, the resulting runoff hydrographs differ and, therefore, the randomness of the effective rainfall distribution over R results in variations in the modeling "best-fit" parameters in correlating the available rainfall-runoff data.

More precisely, any element of a specific storm class $<\xi_0>$ has the effective rainfall distribution, $e_g^{\,0}(t)$. In correlating $\{Q_g^{\,i}(t), e_g^{\,0}(t)\}$, a different $\eta^i(s)$ results due to the variations in the measured $Q_g^{\,i}(t)$ with respect to the single $e_g^{\,0}(t)$.

In the predictive mode, where one is given an assumed (or design) effective rainfall distribution, $e_g^{\,D}(t)$, to apply at the rain gage site, the storm class of which $e_g^{\,D}(t)$ is an element of is identified, $<\xi_D>$, and the predictive output for the input $e_g^{\,D}(t)$ must necessarily be the distribution

$$[Q_1(t)] = \int_{s=0}^{t} e_g^{\,D}(t-s)\, [\eta(s)]_D\, ds \qquad (14)$$

where $[\eta(s)]_D$ is the distribution of $\eta^i(s)$ distributions associated to storm class $[\xi_D]$.

Generally, however, there is insufficient rainfall-runoff data to derive a sufficiently unique set of storm classes, $<\xi_q>$, and hence additional assumptions must be used. For example, one may lower the eligibility standards for each storm class, $<\xi_q>$, implicitly assuming that several distributions $[\eta(s)]_q$ are nearly identical; or one may transfer

$[\eta(s)]_q$ distributions from another rainfall-runoff data set, implicitly assuming that the two- catchment data set correlation distributions are nearly identical. A common occurrence is the case of predicting the runoff response from a design storm effective rainfall distribution, $e_g^D(t)$, which is not an element of any observed storm class. In this case, another storm class distribution of $[\eta(s)]_q$ must be used which implicitly assumes that the two sets of correlation distributions are nearly identical. Consequently for a severe design storm condition, it would be preferable to develop correlation distributions using the severe historic storms which have rainfall-runoff data available for analysis.

EFFECTIVE RAINFALL UNCERTAINTY

The paper by Hromadka, (1987)(1), includes brief statements from several reports which conclude that the variability in the rainfall (and hence the effective rainfall) over the catchment is a dominant factor in the development, calibration, and application, of hydrologic models (e.g., Schilling and Fuchs, 1986; among others)(2). Including this premise in hydrologic studies would indicate that hydrologic model estimates must be functions of random variables, and hence the estimates are random variables themselves.

From Eq. (12), the correlation distribution for storm event i, $\eta^i(s)$, includes all the uncertainty in the effective rainfall distribution over R, as well as the uncertainty in the runoff and flow routing processes. That is, $\eta^i(s)$ must be an element of the random variable $[\eta(s)]$ where

$$\eta^i(s) = \sum_{j=1}^{m} \sum_{\langle k\rangle_j} a^i{}_{\langle k\rangle_j} \sum \lambda_{jk}^i \, \phi_j{}^i(s - \theta_{jk}^i - \alpha^i{}_{\langle k\rangle_j}) \qquad (15)$$

and Eq. (15) applies to a specific storm. For severe storms of flood control interest, one would be dealing with only a subset of the set of all storm classes. In a particular storm class, $\langle\xi_o\rangle$, should it be assumed that the subarea runoff parameters and channel flow routing uncertainties are minor in comparison to the uncertainties in the effective rainfall distribution over R (e.g., Schilling and Fuchs, 1986; among others), then Eq. (15) may be written as

$$[\eta(s)]_o = \sum_{j=1}^{m} \sum_{\langle k\rangle_j} \bar{a}_{\langle k\rangle_j} \sum [\lambda_{jk}] \, \bar{\phi}_j \, (s - [\theta_{jk}] - \bar{\alpha}_{\langle k\rangle_j}) \qquad (16)$$

where the overbars are notation for mean values of the parameters for storm class $\langle\xi_o\rangle$. Although use of Eq. (14) in deriving the $[\eta(s)]$ distributions results in both the uncertainties in both the effective rainfalls and also the submodel algorithms being integrated, Eq. (16) is useful in motivating the use of the distribution concept in design and planning studies for all hydrologic models, based on just the magnitude of the uncertainties in the effective rainfall distribution over R. That is, although one may argue that a particular model is "physically based" and represents the "true" hydraulic response distributed throughout the catchment, the uncertainty in rainfall still remains and is not reduced by increasing hydraulic routing modeling complexity.

DISCRETIZATION ERROR

The need for using the $Q_i(t)$ model in studies where detention effects are minor is made more apparent when examining the effects of discretizing the model into subareas without the benefit of subarea rainfall-runoff data.

In the above typical case, the engineer generally assigns the recorded precipitation from the single available rain gage, $P_g{}^i(t)$, to occur simultaneously over each R_j. That is from Eq. (1), the $\theta_{jk}^i = 0$ and the λ_{jk}^i are set to constants $\hat{\lambda}_j$ which reflect only the variations in loss rate nonhomogeneity. Hence, the 'true' $Q_m{}^i(t)$ model of Eq. (12), (and also Eq. (13)), becomes the estimator $\hat{Q}_m{}^i(t)$ where

$$\hat{Q}_m{}^i(t) = \int_{s=0}^{t} \hat{e}_g{}^i(t-s) \sum_{j=1}^{m} \sum_{\langle k\rangle_j} \hat{a}^i{}_{\langle k\rangle_j} \sum \hat{\lambda}_j \, \hat{\phi}_j{}^i \, (s - \hat{\alpha}^i{}_{\langle k\rangle_j}) \, ds \qquad (17)$$

where hats are notation for estimates. These incorrect assumptions result in 'discretization error'. Indeed, an obvious example of discretization error is the case where a subarea R_j actually receives no rainfall, and yet one assumes that $P_g{}^i(t)$ occurs over R_j in the discretized model. (It is easily shown that the Eq. (13) model accommodates this example case.)

DISCRETIZATION CALIBRATION ERROR

A current trend among practitioners is to develop an m-subarea link-node model estimator $\hat{Q}_m{}^i(t)$ such as Eq. (17), and then "calibrate" the model parameters using the available (single) rain gage and stream gage data pair. Because subarea rainfall-runoff data are unavailable, necessarily it is assumed that the random variables associated to the subarea effective rainfalls are given by

$$\left.\begin{array}{l} [\theta_{jk}] = 0 \\ [\lambda_{jk}] = \hat{\lambda}_j \end{array}\right\} \text{(estimator, } \hat{Q}_m{}^i(t), \text{ assumptions)} \qquad (18)$$

But these assumptions violate the previously stated premise that the uncertainty in the effective rainfall distribtion over R has a major effect in hydrologic modeling accuracy. The impact in using Eq. (18) becomes apparent when calibrating the model to only storms of a single storm class, $\langle\xi_o\rangle$.

Again, for all storms in $\langle\xi_o\rangle$, the effective rainfall distributions are all nearly identical and are essentially given by the single $e_g{}^o(t)$. But due to the variability in rainfall over R_j, the associated runoff hydrographs, $Q_g{}^i(t)$, differ even though $e_g{}^o(t)$ is the single model input.

It is recalled that in Eq. (17), the effective rainfall distribution is now the estimator, $\hat{e}_g{}^o(t)$, which is the true $e_g{}^i(t)$ modified to best correlate $(Q_g{}^i(t), e_g{}^o(t))$. That is, due to the several assumptions leading to Eq. (18) for the discretized model estimator, $\hat{Q}_m{}^i(t)$, the variations due to $[\lambda_{jk}]$ and $[\theta_{jk}]$ are transferred from the $[\eta(s)]$ distribution to the $\hat{e}_g{}^i(t)$ function. For storm class $\langle\xi_o\rangle$, the estimator $\hat{Q}_m{}^i(t)$ can be written approximately from Eqs. (16) and (17) as

$$\hat{Q}_m{}^i(t) = \int_{s=0}^{t} \hat{e}_g{}^i(t-s) \sum_{j=1}^{m} \sum_{\langle k\rangle} \bar{a}_{\langle k\rangle_j} \sum \bar{\lambda}_j \, \bar{\phi}_j (s - \bar{\alpha}_{\langle k\rangle_j}) \, ds \qquad (19)$$

where in Eq. (19), it is assumed that the variations in model output due to using mean values (overbar notation) are minor in comparison to the variations in model output due to $[\lambda_{jk}]$ and $[\theta_{jk}]$. But then Eq. (19) is another single area UH model,

$$\hat{Q}_m{}^i(t) = \int_{s=0}^{t} \hat{e}_g{}^i(t-s) \, \hat{\eta}(s) \, ds \qquad (20)$$

where $\hat{\eta}(s)$ is an estimated distribution which is essentially 'fixed' for all storms in a specified storm class $\langle\xi_o\rangle$. The $\hat{\eta}(s)$ is fixed due to nearly the same input being applied to each subarea for each storm in $\langle\xi_o\rangle$. In calibrating $\hat{Q}_m{}^i(t)$, therefore, the work effort is focused towards finding the best fit effective rainfall distribution, $\hat{e}_g{}^i(t)$, which correlates the several pairs $(Q_j{}^i(t), \hat{\eta}(s))$. That is, the 'true' single $e_g{}^o(t)$ is forced to be modified to be $\hat{e}_g^{\,i}(t)$ in order to correlate the $\{Q_g^{\,i}(t), \hat{n}(s)\}$, for each storm, i. This contrasts with finding the best fit $n^i(s)$ which correlates the pairs, $\{Q_g^{\,i}(t), e_g^{\,o}(t)\}$. It is recalled that from Eqs. (16), (17), and (20), $\hat{n}(s)$ is "fixed" due to the assumptions of Eq. (18), and due to using a single storm class, $<\xi_o>$.

Because the effective rainfall submodel used in $\hat{Q}_m^{\,i}(t)$ has a prescribed structure, it cannot match the best fit $\hat{e}_g^{\,i}(t)$ for all storms and, consequently, modeling error is introduced into the calibration parameters of the loss rate submodel in order to (1) modify the true $e_g^{\,o}(t)$ due to the effects of $[\lambda_{jk}]$ and $[\theta_{jk}]$; (2) the derivation of loss rate parameters which are not "physically based".

Another error which results due to use of Eq. (18) is that the estimator modeling distribution $[\hat{Q}_m(t)]$ for storm class $<\xi_o>$ will be imprecise due to the variation in derived loss rate parameters not achieving the true variation in $\hat{e}_g^{\,i}(t)$.

The above results indicate that for the given assumptions, the calibration of a highly discretized catchment model will generally lead to a model that is no more reliable in the predictive mode than the simple single area UH model. These results appear to be validated by the open literature (Hromadka, 1987).

EXPECTED VALUE ESTIMATES

In practice, the single area UH model is used to correlate several record data pairs $\{Q_g^{\,i}(t), e_g^{\,i}(t)\}$ of the same or similar storm class $<\xi_o>$ to derive the associate correlation distributions, $\{n^i(s)\}$. Although the $\{n^i(s)\}$ are often integrated and normalized, and the several normalizing parameters averaged together, the net effect of all this is finding the expected value of the distribution of correlation distributions, denoted by $E[n(s)]$. Then, the model used for predictive purposes (for storms of the same class, $<\xi_o>$ used to develop $[n(s)]$) is the expected distribution $E[Q_1(t)]$ given by

$$E[Q_1(t)] = \int_{s=0}^{t} e_g(t)\, E[n(s)]\, ds \qquad (21)$$

where $e_g(t)$ is a design input in $<\xi_o>$. From Eqs. (12) and (13), $E[Q_1(t)] = E[Q_m(t)]$ which is the 'true' expected distribution for the given assumptions leading to Eq. (12).

In comparison, after calibrating the estimator, $\hat{Q}_m^{\,i}(t)$, to the available data, the averaging of parameters results in the model (for storms in $<\xi_o>$)

$$E[\hat{Q}_m(t)] = \int_{s=0}^{t} E[\hat{e}_g(t)]\, \hat{n}(s)\, ds \qquad (22)$$

where $E[\hat{e}_g(t)]$ is the "best fit" to the expected value of the true effective rainfalls (needed to correlate the $\{Q_g^{\,i}(t), \hat{n}(s)\}$) using a specified rigid link-node model structure.

Comparing Eqs. (21) and (22), it is seen that for storm class $<\xi_o>$, Eq. (21) is the 'true' expected value.

VERIFICATION TESTS OF MODELS

From Eqs. (21) and (22), the standard use of verification tests on the models of $E[Q_1(t)]$ and $E[\hat{Q}_m(t)]$ simply test the distribution of $[Q_g(t)]$ about the mean estimates of $E[Q_1(t)]$ and $E[\hat{Q}_m(t)]$ for storm class $<\xi_o>$. The discrepancies reported in the literature for verification tests indicates that the natural variance between the $e_g^{\,i}(t)$ and $Q_g^{\,i}(t)$ is usually quite large.

CERTAINTY IN FLOOD CONTROL DESIGN

Recalling the premise that the variations in the effective rainfall distribution over the catchment, R, has a major impact on modeling accuracy, it may be questioned whether using the expected value of a model output is the proper use of a probabilistic distribution.

For example, suppose that a rain gage station with an extremely long record length shows that a severe storm condition occurs fairly frequently (say, about every 100 years), and each occurrence results in a nearly identical effective rainfall distribution at the rain gage site. Hence, a storm class of design interest is well defined, $<\xi_D>$, where each element has a nearly identical input, $e_g^{\,D}(t)$, for any catchment hydrologic model. Yet the catchment stream gage shows a variation in the runoff hydrographs, $Q_g^{\,i}(t)$, for each event of $e_g^{\,D}(t)$. From this information, a model distribution is derived from Eqs. (12) and (13) to give

$$[Q_D(t)] = \int_{s=0}^{t} e_g^{\,D}(t-s)[n^D(s)]\, ds \qquad (23)$$

Equation (23) is the distribution of hydrologic modeling estimates (see Figure 1), and is the best estimate available. Given another design storm event, with the same $e_g^{\,D}(t)$ resulting, the best a model can do in estimating the resulting runoff hydrograph is reflected in Eq. (23), and Figure 1.

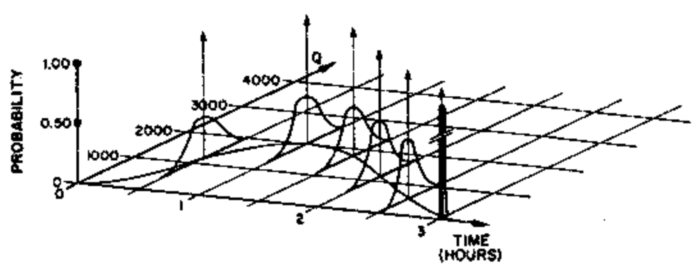

Figure 1. The Hydrologic Model Distribution (Eq. 23)) for a Predicted Response, $[Q_D(t)]$, from Input, $e_g^D(t)$. Heavy line is the Expected Distribution, $E[Q_D(t)]$

Should the expected model $E[Q_D(t)]$ be used for design study purposes, this expected runoff hydrograph typically would not be the most severe design condition for flood control facilities. Instead, the true distribution $[Q_D(t)]$ should be used to evaluate the flood control system performance, and a level of confidence selected as to the success in predictive design. That is, using the $E[Q_D(t)]$ model for design purposes often results in a design product that has only a 50-percent confidence level of protecting for the specified design event, $e_g^{\,D}(t)$, given the available rainfall-runoff data. Perhaps a higher level of confidence, such as 85-percent or 95-percent, may be more appropriate in the interest of public safety, and to reduce the exposure to flood damage liability.

USING THE HYDROLOGIC MODEL DISTRIBUTION $[Q_i(t)]$

From the development leading to the model of Eq. (12), use of the standard single area UH model of Eq. (13) has a powerful representation of the catchment response including: random variations in the effective rainfall distribution pattern shape, magnitude, and timing, on an arbitrarily discretized subarea basis; variations in the subarea runoff response and channel flow routing effects on a storm by storm basis; storage effects in channel routing; among others. Calibration of the $Q_i(t)$ model to rainfall-runoff data on a storm class basis results in a distribution of correlation distributions, $[n(s)]$, which reflects the natural variance between the record data. The resulting model distribution, $[Q_i(t)]$, reflects the natural variance in predicting runoff quantities for storms of the same class used to derive $[n(s)]$.

The link node model estimator, $\hat{Q}_m^{\,i}(t)$, however, cannot achieve the true distribution of $[Q_i(t)]$. Only if rainfall-runoff data were available in each subarea (in order to determine the λ_{jk}^i and θ_{jk}^i on a storm by storm basis) would the model parameters (e.g., the loss rate model parameters be properly calibrated and the variance due to the rainfall effects (i.e., $[\lambda_{jk}]$ and $[\theta_{jk}]$ in Eq. (12)) be properly reflected. Consequently, $[Q_i(t)]$ should be used. The distribution $[\hat{Q}_m(t)]$, developed by varying the loss rate parameters (as the routing parameters are nearly invariant for storms of the same class), cannot achieve the true variance between rainfall-runoff due to the loss rate algorithm structure. If $\hat{Q}_m^{\,i}(t)$ were supplied subarea rainfall-runoff data, and stream gage data to evaluate all routing parameters, then $\hat{Q}_m^{\,i}(t) = Q_m^{\,i}(t) = Q_i^{\,i}(t)$. That is, given enough runoff data to evaluate all model parameters on a subarea and link basis, the link node model will achieve the distribution variance between model output and the given rainfall data as achieved by the classic single area UH model.

APPLICATION: DETENTION BASIN VOLUME SIZING

The above developments are now applied to a simple application. A catchment of 1,800-acres is studied to size a detention basin. The design objective is to protect for a historic design storm. Based on the available stream gage and rain gage data, a class of severe storms, $<\zeta_o>$, is developed and the $Q_i^{\,i}(t)$ model is calibrated for each element of $<\zeta_o>$. The resulting $[n(s)]$ distribution is shown in mass curve form, $[M(s)]$, where

$$M^i(s) = \int_{x=0}^{t} n^i(x)\,dx \qquad (24)$$

A frequency distribution for $[M(s)]$ is shown in Fig. 2.

Using $[M(s)]$, the $[n(s)]$ is found by differentiation and the model distribution, $[Q_D(t)]$, is given by Eq. (23) and shown in Fig. 1. Routing the $[Q_D(t)]$ through the detention basin resulted in the volume requirement distribution shown in Fig. 3. Shown in the figure is the expected volume requirement using $E[Q_D(t)]$, and also the 50-percent and 85-percent confidence estimates. Note that in this case, the "expected" volume requirement derived by using $E[Q(t)]$ (such as done in usual practice) is slightly less than the 50-percent confidence estimate.

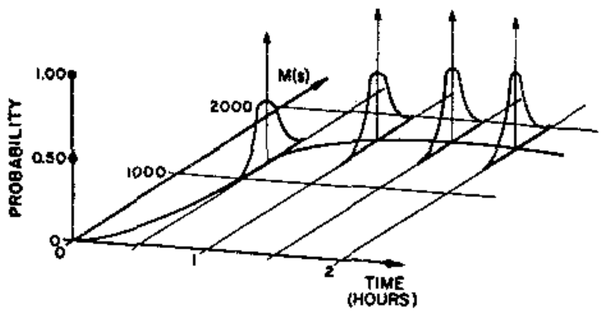

Figure 2. Frequency Distribution for $[M(s)]$. Heavy Line is the Expected Distribution, $E[M(s)]$

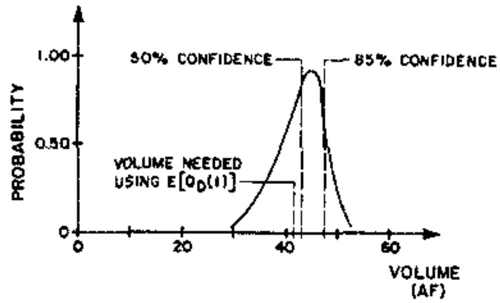

Figure 3. Detention Basin Volume Requirements

CONCLUSIONS

The classic single area unit hydrograph approach to modeling runoff response from a free draining catchment is shown to represent several important modeling considerations including, (i) subarea runoff response (in a discretized model), (ii) the subarea effective rainfall distribution including variations in magnitude, timing, and storm pattern shape, (iii) channel flow routing translation and storage effects, using the linear routing technique, (iv) subarea runoff hydrograph addition, among other factors. Because the UH method correlates the effective rainfall distribution to the runoff hydrograph distribution, the resulting catchment UH should be considered a correlation distribution in a probabilistic sense. Should the uncertainty in rainfall over the catchment be a major concern in modeling reliability, then the UH output in the predictive setting must be considered to be a random variable. In this paper, the UH method is shown to have a rational modeling structure for free-draining catchments. The correlations represented by the class of UH's derived from similarly categorized storms, properly reproduces the natural variance between the effective rainfall and runoff hydrograph. By using the full set of observed UH's (from the same storm category), a design product can be developed which accommodates modeling uncertainty due to the uncertainty in rainfall and other factors. The resulting UH model is then interpreted to be a probabilistic distribution, in which a flood control design needs to be tested by probabilistic simulation, varying the UH according to its frequency distribution. As a case study, a distribution of runoff hydrographs is used to estimate multi-outlet retarding basin design volume requirements.