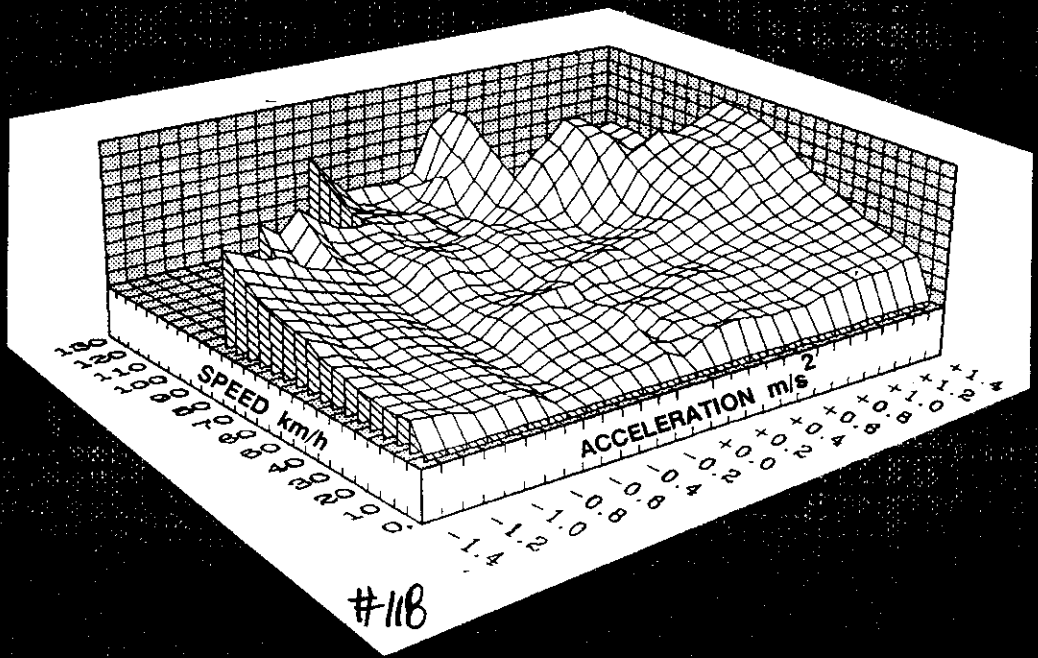


# Computer Techniques in Environmental Studies III

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# A Regionalized Hydrologic Model Distribution

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## ABSTRACT

The single area unit hydrograph model is used to develop a distribution model which accommodates the uncertainty in rainfall over the catchment. By categorizing the available rainfall data into storm classes, the model is then calibrated to the rainfall-runoff data to derive a distribution of unit hydrograph correlations on a storm class basis. The unit hydrograph distributions reflect the unknown variations in the effective rainfall over the catchment, among other factors. The resulting calibrated distribution model is then applied to verification data (not included in the calibration set) to demonstrate the model's utility. For ungauged catchments, a regionalized model distribution is used which implicitly assumes that the distribution of correlation distributions (represented by the variation in the unit hydrograph) is transferable.

## INTRODUCTION

The previous papers (Hromadka<sup>1,2</sup>) develop the background leading to the development, calibration, and use of a single area unit hydrograph UH distribution model,  $[Q_1(t)]$ , where

$$[Q_1(t)] = \int_{s=0}^t e_g(t) [r(s)] ds \quad (1)$$

where  $e_g(t)$  is the effective rainfall distribution measured at the rain gauge site; and  $[r(s)]$  is the distribution of correlations between measured rainfall and runoff data. In the problem setting, only one rain gauge and stream gauge is available for data synthesis purposes. Additionally, the catchment  $R$  is assumed to drain freely to the stream gauge, with negligible backwater effects.

The mathematical underpinnings in the use of Equation (1) is given in detail in the cited papers, and are summarized by the following equation for  $n^i(s)$  for storm event  $i$ ,

$$n^i(s) = \sum_{j=1}^m \sum_{k=1}^{n_j^i} \lambda_{jk}^i \phi_j^i(s - \tau_j^i - \theta_{jk}^i) \quad (2)$$

where  $\lambda_{jk}^i$  are coefficients;  $\phi_j^i(s)$  are subarea UH's for an  $m$ -subarea link node model;  $\tau_j^i$  is the travel time for flow to travel from subarea  $R_j$  to the stream gauge;  $\theta_{jk}^i$  are timesteps;  $n_j^i$  is an index number; and all parameters are evaluated on a storm by storm basis.

Because the rainfall distribution over the catchment  $R$  is unknown, any hydrologic model output must necessarily be a function of at least the random variables  $[\lambda_{jk}^i]$ ,  $[\theta_{jk}^i]$  used in the effective rainfall distribution in  $R_j$  given by (for the linear assumption in storm rainfall)

$$e_j^i(t) = \sum_{k=1}^m \lambda_{jk}^i e_g^i(t - \theta_{jk}^i) \quad (3)$$

Thus in  $R_j$ , the unknown effective rainfall is the random variable

$$[e_j(t)] = \sum_{k=1}^{[n_j^i]} [\lambda_{jk}^i] e_g^i(t - [\theta_{jk}^i]) \quad (4)$$

where brackets are notation for random variables. In Equations (3) and (4),  $e_j^i(t)$  is said to be linear in  $e_g^i(t)$ .

#### Model Application

To apply  $[Q_1(t)]$ , an effective rainfall model is needed to modify the rain gauge data,  $P_g^i(t)$ . Such a model by  $F(P_g^i(t), \{X_i\})$  where the  $\{X_i\}$  are parameters to be selected. To proceed, the  $P_g^i(t)$  record is categorized into storm classes,  $\langle \xi_o \rangle$ , which are considered to result in nearly identical effective rainfall distributions,  $e_g^i(t)$ , at the rain gauge site.

Given a specific class,  $\langle \xi_o \rangle$ , there are several associated data pairs  $\{e_g^i(t), Q_g^i(t)\}$  where the  $Q_g^i(t)$  may differ even though the  $e_g^i(t)$  are nearly identical. This is due to the unknown rainfalls over  $R$ .

The  $Q_1^i(t)$  model is now cast in terms of the UH,  $\psi^i(s)$ , by

$$Q_1^i(t) = \int_{s=0}^t w^i e_g^i(t-s) \psi^i(s) ds \quad (5)$$

where  $\psi^i(s) = \eta^i(s)/W^i$ , and  $W^i$  = ratio of measured stream gauge runoff to effective rainfalls. Because the  $\psi^i(s)$  is unknown for storm  $i$ , as is  $W^i$ , each storm  $i$  has the associated correlation vector  $\eta^i = \langle \text{lag}_i, \psi^i(s), W^i \rangle$ .

For each assumed  $X_i$  used to estimate  $e_{g^i}(t)$ , a different family of  $\{\eta^i\}$  are developed.

If the effective rainfall over  $R$  is linear in  $e_{g^i}(t)$ , and linear routing applies in  $R$ , then each set of parameters used in  $Q_1^i(t)$  gives the same result.

The model now is

$$[Q_1^i(t)] = \int_{s=0}^t F(P_g^i(t), \{X_i\}) [W^0 \psi^0(s)] ds \quad (6)$$

where the correlations  $W^0 \psi^0(s)$  correspond to the fixed  $\{X_i^0\}$ . It is assumed that the  $e_{g^i}(t)$  are linear in  $F(P_g^i(t), \{X_i^0\})$ , and the  $[W^0 \psi^0(s)]$  and  $P_g^i(t)$  are in the appropriate  $\langle \xi_q \rangle$ . If not enough data, the  $\langle \xi_q \rangle$  are grouped together to get the single global distribution,  $[W \psi(s)]$ .

A Regionalized Distribution Model

As a case study, a regionalized distribution model is developed for the Los Angeles and Orange County area in California. A total of 12 urbanized catchments are considered directly, supplemented by additional data prepared by U.S. Army Corps of Engineers (COE) flood control studies (e.g., the ongoing Los Angeles County Drainage Area study, or LACDA).

Table 1 lists the various data quantities used in the regionalized effort. Specific catchment data and storm drain information is available from the Los Angeles Flood Control District (LACFCD) and the Orange County Environmental Management Agency (OCEMA).

In developing correlation distributions for each catchment, only severe storms were used for the derivation of the parameters in Equation (5). This satisfies, as much as possible, in developing statistical correlations on a storm class basis.

Peak Loss Rate,  $F_p$

In this application, the loss function structure used in Equation (6) is a simple phi-index which is calibrated on a storm basis with the unit hydrograph. From Table 1, several peak rainfall loss rates,  $F_p$ , are tabulated which include two loss rates for double-peak storms. The range of  $F_p$  estimates lie between 0.30 and 0.65 inch/hour with the highest value occurring in Verdugo Wash which has substantial open space in foothill areas. Except for Verdugo Wash,  $0.20 \leq F_p \leq 0.60$  which is a variation in values of the order noted for Alhambra Wash along. Figure 1 shows a frequency-distribution of  $F_p$  for the several watersheds combined. It is evident from the figure that 92-percent of

$F_p$  values are between 0.20 and 0.45 inch/hour, with 80-percent of the values falling between 0.20 and 0.40 inch/hour. Consequently, a regional mean value of  $F_p$  equal to 0.30 inch/hour contains 80-percent of the  $F_p$  values, for all watersheds, for all storms, within 0.10 inch/hour.

### S-Graph

Each of the watersheds listed in Table 1 has S-graphs developed for each of the storms where peak loss rate values were developed. For example, Fig. 2 shows the several S-graphs developed for Alhambra Wash. By averaging the several S-graph ordinates (developed from rainfall-runoff data), an average S-graph is obtained. By combining the several watershed average S-graphs (Fig. 3) into a single plot, and weighting the ordinates by the associated number of storm events, an average of averaged S-graphs is obtained. This regionalized S-graph (Urban S-graph in Fig. 3) is an estimate of the expected S-graph for the region. It is noted that the variation in S-graphs for a single watershed for different storms (see Fig. 2) is of the order of magnitude of variation seen between the several catchment averaged S-graphs.

In order to quantify the effects of variations in the S-graph due to the variations in storms and in watersheds (i.e., for ungauged watersheds not included in the calibration data set), a scaling is used where the variable "X" signifies the average value of an arbitrary S-graph as a linear combination of the steepest and flattest S-graphs obtained. That is, all the S-graphs (all storms, all catchments) lie between the February 1978 storm Alhambra S-graph ( $X = 1$ ) and the San Jose S-graphs ( $X = 0$ ). To approximate a particular S-graph of the sample set,

$$S(X) = X S_1 + (1-X) S_2 \quad (7)$$

where  $S(X)$  is the S-graph as a function of  $X$ , and  $S_1$  and  $S_2$  are the Alhambra (February 1978 storm) and San Jose S-graphs, respectively. Figure 4 shows the frequency distribution of  $X$  where each watershed is weighted equally in the total distribution (i.e., each watershed is represented by an equal number of  $X$  entries).

### Catchment Lag

In Table 1, the Urban S-graph, which represents a regionalized expected S-graph for urbanized watersheds in valley type topography, has an associated  $X$  value of 0.85. Because the Urban S-graph is a near duplicate of the SCS S-graph, it was assumed that catchment lag (COE definition) is related to the catchment time of concentration,  $T_c$ , as is typically assumed in the SCS approach.

Catchment  $T_c$  values are estimated by subdividing the watershed into subareas with the initial subarea less than 10 acres and a flow-length of less than 1000 feet. Using Kirpich formula, an initial subarea  $T_c$  is estimated, and a  $Q$  is calculated. By subsequent routing downstream of the peak flowrate ( $Q$ ) through the various conveyances (using normal depth flow velocities) and adding estimated successive

subarea contributions, a catchment  $T_c$  is estimated as the sum of travel times analogous to a mixed velocity method.

Lag values are developed directly from available COE rainfall-runoff calibration data, or by using a calibrated lag formula (COE):

$$\text{lag}(\text{hours}) = 24 n \left( \frac{L L_{ca}}{s^{0.5}} \right)^{0.38} \quad (8)$$

where  $L$  is the watershed length in miles;  $L_{ca}$  is the length of the centroid along the watercourse in miles;  $s$  is the slope in ft/mile; and  $n$  is the basin factor.

Because Eaton Wash, Arcadia Wash and Alhambra Wash are all contiguous, have similar shape, slopes, development patterns, and drainage systems, the basin factor of  $n = 0.015$  calibrated from Equation (8) for Alhambra Wash was also used for the other three neighboring watersheds. Then the lag was estimated using Equation (8).

Compton Creek has two gauges, and the  $n = 0.015$  developed for Compton2 was also used for the Compton1 gauge. The Dominguez catchment, which is contiguous to Compton Creek, is also assumed to have a lag calculated using  $n = 0.015$ .

McCuen et al.<sup>3</sup> provides additional measured lag values and mixed velocity  $T_c$  estimates which, when lag is modified according to the COE definition, can be plotted with the local data such as shown in Fig. 5. A least-square best fit results in:

$$\text{Lag} = 0.80 T_c \quad (9)$$

#### The Regionalized Distribution Model, $[Q_1(t)]$

Each of the model parameters are assumed to have the probability distribution functions (pdf) shown in a frequency distribution form in Figs. 1, 4, and 6 for  $F_D$ ,  $S(X)$ , and  $\text{lag} = 0.8 T_c$ , respectively. For example, if the model distribution is applied at a gauged site, say Alhambra Wash, then the variability in the S-graph is not given by Fig. 4 for  $S(X)$ , where  $0.60 \leq X \leq 1$ , but for  $0.75 \leq X \leq 1$ . Similarly, the estimate for lag is much more certain for Alhambra Wash than shown in Fig. 6. Consequently, the model distribution  $[Q_1(t)]$  for a gauged site will show a significantly smaller range in possible outcomes than if the total range of parameter values of Fig. 1 and 4 are assumed (as is done when using a regionalized model distribution for the ungauged sites, or sites where an inadequate length of data exist for a constant level of watershed development).

To evaluate the model distribution,  $[Q_1(t)]$ , a simulation that exhausts all combinations of parameter values shown in the pdf distribution is prepared. Because the lag/ $T_c$  plot is a function of  $T_c$ , several  $T_c$  values must be assumed and lag values varied freely according to Fig. 6. An important hydrologic output is the peak flow rate,  $Q$ . The distribution of  $[Q]/E[Q]$  is shown in Fig. 7 for the case of  $T_c$  equal to 1 hour and a watershed area of 1 square mile (hence, depth-area adjustments are not involved). In the figure,  $[Q]$  is the distribution of possible model peak flow rate estimates, and  $E[Q_m]$  is the peak flow rate obtained from the model using the expected parameters of lag equal to  $0.8 T_c$ ,  $F_p$  equal to 0.30 inch/hour, and  $X$  equal to 0.85 (Urban S-graph). The  $[Q]/E[Q]$  plots were all very close to Fig. 7 as a function of  $T_c$ ; therefore, Fig. 7 is taken to represent the overall  $[Q]/E[Q]$  distribution for watershed areas less than 1 square mile.

## CONCLUSIONS

The single area unit hydrograph model is used to develop a distribution model which accommodates the uncertainty in rainfall over the catchment. By categorizing the available rainfall data into storm classes, the model is then calibrated to the rainfall-runoff data to derive a distribution of unit hydrograph correlations on a storm class basis. The unit hydrograph distributions reflect the unknown variations in the effective rainfall over the catchment, among other factors. The resulting calibrated distribution model is then applied to verification data (not included in the calibration set) to demonstrate the model's utility. The hydrologic model distribution approach can be regionalized for use on ungauged catchments.

## REFERENCES

1. Hromadka II, T.V., 1988, Rainfall-Runoff Models: A Review of 1988, Invited Paper, Proceedings: ENVIROSOFT '88 Conference, Computational Mechanics Publications, Porto Carras, Greece.
2. Hromadka II, T.V., 1988, Back to the Unit Hydrograph Method, Proceedings: ENVIROSOFT '88 Conference, Porto Carras, Greece, Computational Mechanics Press.
3. McCuen, R., et al, 1984, Estimating Urban Time of Concentration, Journal of Hydraulic Engineering, Vol. 110, No. 7.



TABLE 1. WATERSHED CHARACTERISTICS

Watershed Name	Watershed Geometry					Tc (Hrs)	Calibration Results			
	Area (mi <sup>2</sup> )	Length (mi)	Length of Centroid (mi)	Slope (ft/mi)	Percent Impervious (%)		Storm Date	Peak F <sub>p</sub> (inch/hr)	Lag (hrs)	Basin factor
Alhambra Wash <sup>1</sup>	13.67	8.62	4.17	82.4	45	0.88	Feb.78 Mar.78 Feb.80	0.59,0.24 0.35,0.29 0.24	0.62	0.015
Compton <sup>2</sup>	24.66	12.69	6.63	13.8	55	2.22	Feb.78 Mar.78 Feb.80	0.36 0.29 0.44	0.94	0.015
Verdugo Wash <sup>1</sup>	26.8	10.98	5.49	316.9	20	--	Feb.78	0.65	0.64	0.016
Linnasin <sup>1</sup>	10.3	7.77	3.41	295.7	25	--	Feb.78 Feb.80	0.27 0.27	0.73	0.026
San Jose <sup>2</sup>	83.4	23.00	8.5	60.0	18		Feb.78 Feb.80	0.20 0.38	1.66	0.020
Sepulveda <sup>2</sup>	152.0	19.0	9.0	143.0	24	--	Feb.78 Mar.78 Feb.80	0.22,0.21 0.32 0.42	1.12	0.017
Eaton Wash <sup>4</sup>	11.02 <sup>4</sup> (57%)	8.14	3.41	90.9	40	1.05	---	---	---	0.015 <sup>7</sup>
Rubio Wash <sup>4</sup>	12.20 <sup>5</sup> (3%)	9.47	5.11	125.7	40	0.68	---	---	---	0.015 <sup>7</sup>
Arcadia Wash <sup>1</sup>	7.70 <sup>6</sup> (14%)	5.87	3.03	156.7	45	0.60	---	---	---	0.015 <sup>8</sup>
Compton <sup>1</sup>	15.08	9.47	3.79	14.3	55	1.92	---	---	---	0.015 <sup>8</sup>
Dominguez <sup>1</sup>	37.30	11.36	4.92	7.9	60	2.08	---	---	---	0.015 <sup>8</sup>
Santa Ana Delhi <sup>3</sup>	17.6	8.71	4.17	16.0	40	1.73	---	---	---	0.053 <sup>9</sup> 0.040 <sup>10</sup>
Westminster <sup>3</sup>	6.7	5.65	1.39	13	40		---	---	---	0.075 <sup>9</sup> 0.040 <sup>10</sup>
El Modena-Irvine <sup>3</sup>	11.9	6.34	2.69	52	40	0.78	---	---	---	0.028 <sup>9</sup>
Garden Grove-Wintersberg <sup>4</sup>	20.8	11.74	4.73	10.6	64	1.98	---	---	---	---
San Diego Creek <sup>1</sup>	36.8	14.2	8.52	95.0	20	1.39	---	---	---	---

- Notes 1: watershed Geometry based on review of quadrangle maps and LACFCD storm drain maps.
- 2: Watershed Geometry based on COE LACDA Study.
- 3: Watershed Geometry based on COE Reconstitution Study for Santa Ana Delhi and Westminster Channels (June, 1983).
- 4: Area reduced 57% due to several debris basins and Eaton Wash Dam reservoir, and groundwater recharge ponds.
- 5: Area reduced 3% due to debris basin.
- 6: Area reduced 14% due to several debris basins.
- 7: 0.013 basin factor reported by COE (subarea characteristics, June, 1984).
- 8: 0.015 basin factor assumed due to similar watershed values of 0.015.
- 9: Average basin factor computed from reconstitution studies
- 10: COE recommended basin factor for flood flows.

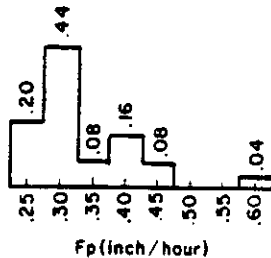


Figure 1. Frequency-distribution for F<sub>p</sub>.

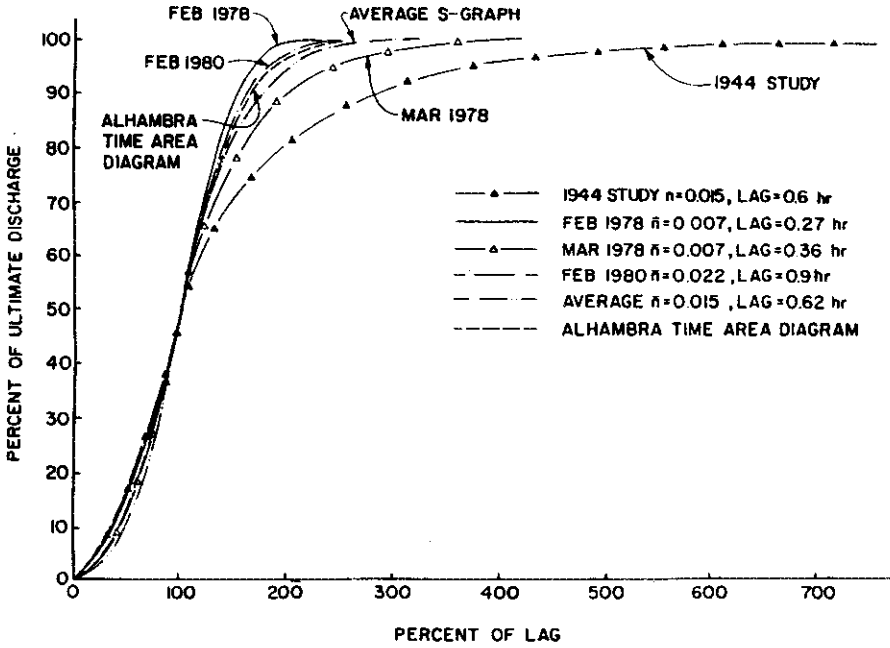


Figure 2. Alhambra Wash S-Graphs.

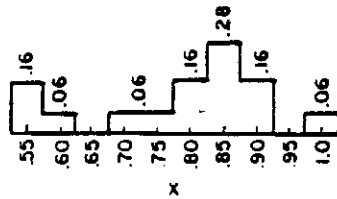


Figure 4. Frequency-distribution for S-Graphs Parameter, X.

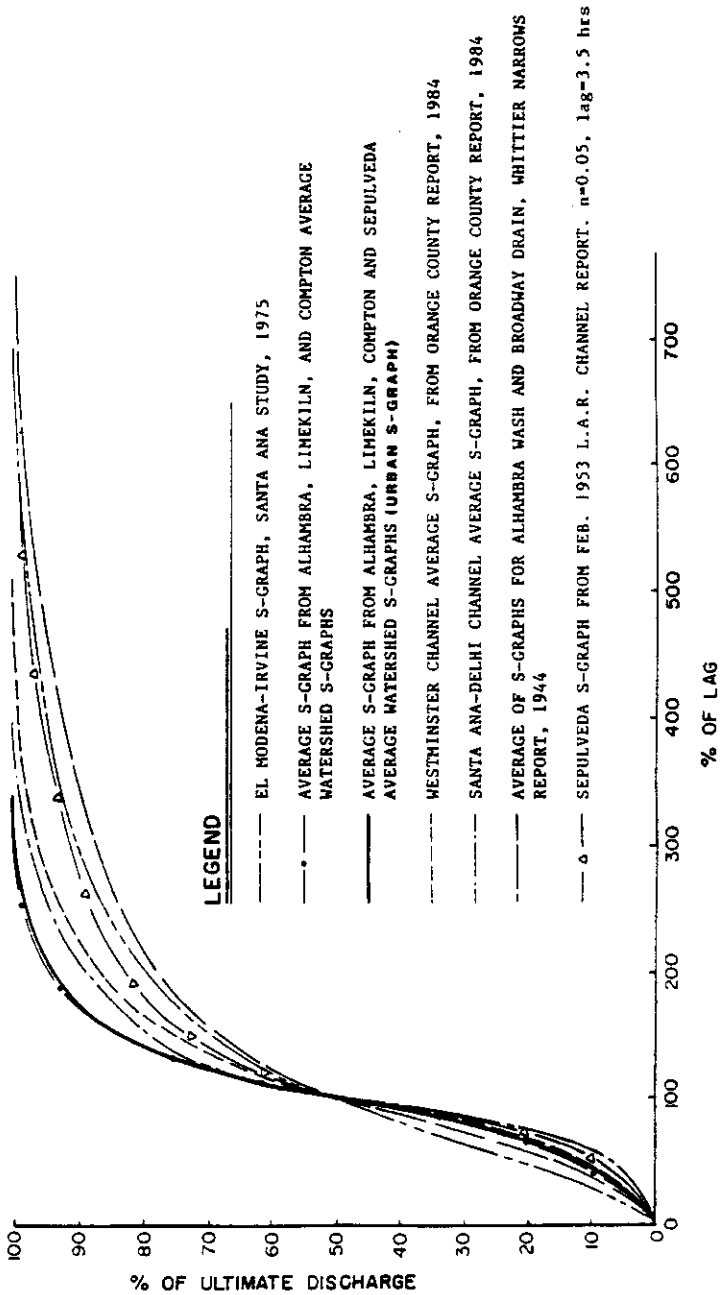


Figure 3. Average S-Graphs.

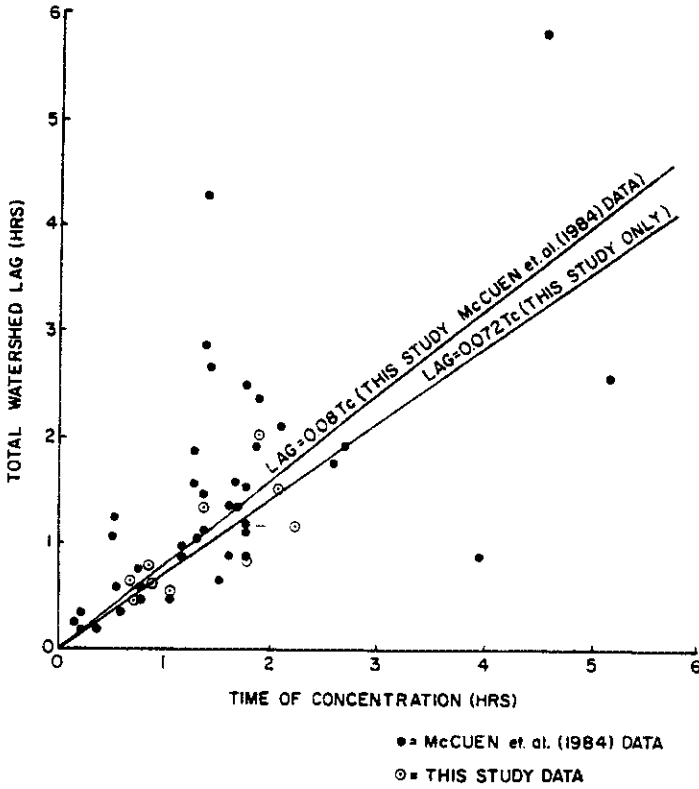


Figure 5. Relationship Between Measured Catchment Lag Computed Tc.

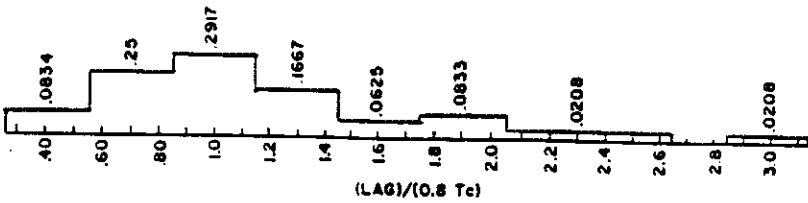


Figure 6. Frequency-distribution for (Lag)/(0.8 Tc).

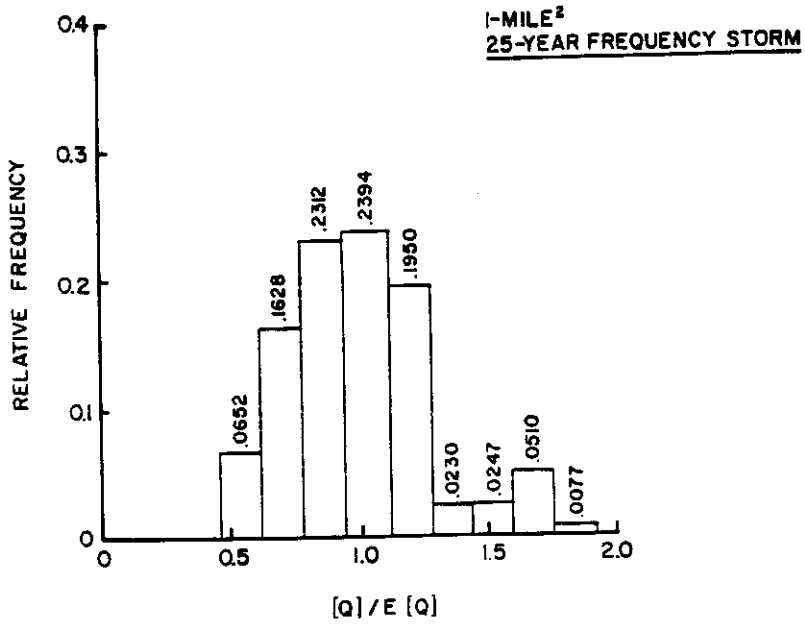


Figure 7.  $[Q]/E[Q]$  Distribution for  $T_c = 1$ -hour,  
Area = 1 mile square.