Unit Hydrograph Models and Uncertainty Distributions

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ABSTRACT

In this paper, the uncertainty in the effective rainfall distribution (i.e., rainfall less losses) over the catchment (R) is considered to be a dominating influence in causing flood flow modeling errors in discretized models (i.e., subdivision of R into subareas, linked by channel routing). A single area unit hydrograph (UH) model is also used to represent the uncertainty not only in the effective rainfall distribution over R, but also the uncertainty in the catchment hydraulic responses. Both modeling approaches are focused towards the typical case where only one stream gauge and one rain gauge are available for data analysis. It is shown that due to the limited data available, the simple single area UH model includes several uncertainties that the discretized model misrepresents.

INTRODUCTION

A review of the literature which raises questions as to the development, application, and calibration, of hydrologic models is contained in Hromadka and Whitley[1]. In that literature review, it appears that the unknown distribution of effective rainfall (i.e., rainfall less losses) over the catchment, R, is a barrier to the success in the use of hydrologic models for predicting hydrologic responses.

For example, Schilling and Fuchs[2] write "that the spatial resolution of rain data input is of paramount importance to the accuracy of the simulated hydrograph" due to "the high spatial variability of storms" and "the amplification of rainfall sampling errors by the nonlinear transformation" of rainfall into runoff by hydrologic models. In their study, Schilling and Fuchs analyzed an 1,800-acre catchment with three rain gauge densities (all equally spaced) of 81-, 9-, and a single centered gauge. They concluded that variations in runoff volumes and peak flows "is well above 100 percent over the entire range of storms implying that the spatial resolution of rainfall has a dominant influence on the reliability of computed runoff." It is also noted that "errors in the rainfall input are amplified by the rainfall-runoff transformation" so that "a rainfall depth error of 30 percent results in a volume error of 60 percent and a peak flow error of 80 percent." (Other key quotations regarding other studies are contained in Hromadka and Whitley[1].)

In this paper, the uncertainty in the effective rainfall distribution (i.e., rainfall less losses) over the catchment (R) is considered to be a dominating factor in causing hydrologic modeling errors in discretized models (i.e., subdivision of R into subareas, linked by channel routing). A single area unit hydrograph (UH) model is then used which represents the uncertainty not only in the effective rainfall distribution over R, but also the uncertainty in the catchment hydraulic responses. Both modeling approaches are focused towards the situation where only one stream gauge and one rain gauge are available for data analysis and runoff estimates are needed at the stream gauge site.

Because of the uncertainties present in the catchment response, and the uncertainty in effective rainfall over the catchment for each storm, the modeling output is cast as a probability distribution in order to represent the variability in predicted hydrologic estimates given a design storm or hypothetical storm for study purposes. It is shown that the single area UH model can produce such a distribution in output, which represents the natural variance in the correlation of the available rainfall-runoff data; whereas the discretized model typically misrepresents the available data should the discretized model be used for uncertainty analysis.

HYDROLOGIC MODEL DEVELOPMENT

For modeling development purposes, assumptions are made about the catchment, R, and the storm effective rainfall distributions over R.

The catchment, R, is assumed to be sufficiently drained by a free-flowing collector system such that detention or backwater effects are minor throughout R. The catchment is relatively homogeneous such that it can be subdivided into m nearly homogeneous subareas, R_j, $j = 1,2,\ldots,m$. Channel routing along links are assumed to be nearly translation, with a characteristic travel time for each link, for each storm event, i; consequently, a nonlinear response is modeled by using a different travel time for each storm. (Channel flow routing storage effects can be included by assuming a linear routing technique (Hromadka[3]). However, only translation effects will be considered herein in order to simplify the mathematical notation.) Runoff hydrographs are directly summed at confluence points. Finally, a single stream gauge is

located at the downstream end of R. In each subarea, R_j, a unit hydrograph (UH) is defined for each storm event i such that the subarea runoff hydrograph, $Q_j^i(t)$ is given by

$$Q_j^i(t) = \int_{s=0}^{t} e_j^i(t - s)\, \phi_j^i(s)\, ds \tag{1}$$

where $e_j^i(t)$ is the effective rainfall uniformly distributed over subarea R_j. Because the subarea UH's, $\phi_j^i(s)$, may differ between storm events, i, and the channel link travel times also vary on a storm by storm basis, the resulting m-subarea link node model, $Q_m^i(t)$, is a reasonable approximation of any hydrologic model. The global model $Q_m^i(t)$ for R and storm event i is given by

$$Q_m^i(t) = \sum_{j=1}^{m} Q_j^i(t - \tau_j^i) \tag{2}$$

where $Q_j^i(t - \tau_j^i)$ is the runoff hydrograph from R_j offset in time by τ_j^i from the stream gauge; and τ_j^i is the sum of link travel times from R_j to the stream gauge, and is variable between storms, i. For storm event i, the stream gauge runoff hydro-graph is given by $Q_g^i(t)$.

The available single rain gauge is associated with a runoff measuring system such that the effective rainfall distribution is measured at the rain gauge site, noted as $e_g^i(t)$ for storm i. Each subarea R_j is assumed to be sufficiently small such that the subarea effective rainfall, $e_j^i(t)$, applies uniformly in R_j. The $e_j^i(t)$ are assumed to be linear with respect to $e_g^i(t)$ such that (Hromadka[3])

$$e_j^i(t) = \sum_{k=1}^{n_j^i} \lambda_{jk}^i\, e_g^i(t - \theta_{jk}^i) \tag{3}$$

where λ_{jk}^i are positive coefficients $k - 1,2,\ldots, j^i$; and the θ_{jk}^i are timing offsets. These several constants apply only to R_j and storm i, and enable $e_j^i(t)$ to be written as a finite sum of proportions of the data $e_g^i(t)$, each offset in time by a timestep, θ_{jk}^i.

Combining Eqs. (1), (2), and (3) gives the global m-subarea model of R for storm i,

$$Q_m^i(t) = \sum_{j=1}^{m} \int_{s=0}^{t} e_g^i(t - s) \sum_{k=1}^{n_j^i} \lambda_{jk}^i\, \phi_j^i(s - \theta_{jk}^i - \tau_j^i)\, ds \tag{4}$$

Equation (4) is immediately reduced to the single area UH model (Hromadka[3])

$$Q_m{}^i(t) = Q_1{}^i(t) = \int_{s=0}^{t} W^i \, e_g{}^i(t - s) \, \psi^i(s) \, ds \tag{5}$$

where W^i is the ratio of stream gauge runoff to the rain gauge site's effective rainfall; and $\psi^i(s)$ is the unit hydrograph defined by

$$^{i}(s) = \sum_{j=1}^{m} \sum_{k=1}^{n_j{}^i} \lambda_{jk}{}^i \, \phi_j{}^i(s - \theta_{jk}{}^i - \tau_j{}^i)/W^i \tag{6}$$

In order to evaluate all of the parameters used in Eqs. (5) and (6), a stream gauge is required to be located at each subarea (to evaluate the $\lambda_{jk}{}^i$, $\theta_{jk}{}^i$, $\phi_j{}^i(s)$) and along each channel link (to evaluate $\tau_j{}^i$). A more convenient representation of Eq. (5) is

$$Q_1{}^i(t) = \int_{s=0}^{t} e_g{}^i(t - s) \, \eta^i(s) \, ds \tag{7}$$

where

$$\eta^i(s) = W^i \, \psi^i(s) \tag{8}$$

From the above development, the single area UH model, $Q_1{}^i(t)$, includes all the assumptions leading to the m-subarea link-node model, $Q_m{}^i(t)$. However, with only one stream gauge and rain gauge, the practitioner must use the estimator $\hat{Q}_m{}^i(t)$ which has an associated error due to the need to estimate the hydraulic parameters, $\{\phi_j{}^i(s), \tau_j{}^i\}$ without the benefit of subarea hydroglogic data. There is also the discretization error in artificially defining the effective rainfall distribution over R (i.e., in each subarea R_j). Without subarea rainfall-runoff data, the effective rainfall parameters $\{\lambda_{jk}{}^i, \theta_{jk}{}^i, n_j{}^i\}$ are all incorrectly defined in $\hat{Q}_m{}^i(t)$. The importance of these parameters in hydrologic modeling is reflected by the literature review (Hromadka[3]).

In comparison to $\hat{Q}_m{}^i(t)$, however, the $Q_1{}^i(t)$ integrates the "correct" values for $n_j{}^i(s)$ for storm event i. ($\eta^i(s)$ also includes the effects of channel flow routing storage effects).

In other words, with the available data the $Q_1{}^i(t)$ model represents the "true" $Q_m{}^i(t)$ model with all parameters properly defined; but in practice, use of the m-subarea link-node model results in using the estimator, $\hat{Q}_m{}^i(t)$, given by

$$\hat{Q}_m{}^i(t) = \sum_{j=1}^{m} \int_{s=0}^{t} \hat{e}_j{}^i(t - s) \, \hat{\phi}_j{}^i(s - \hat{\tau}_j{}^i) \, ds \tag{9}$$

where the hats are notation for estimates. Each of the estimates used in (9) are unsupported by subarea rainfall-runoff data. Indeed, the rainfall over each subarea is entirely unknown and is therefore "assigned" the values measured at the rain gauge. The $Q_1{}^i(t)$ model, in comparison, includes the correct variation in rainfall over R (magnitude, timing, and storm pattern shape) in the unit hydrograph, $\psi^i(s)$, as shown in Eq. (6).

HYDROLOGIC MODELING UNCERTAINTY ANALYSIS: DATA REPRESENTATION

The $Q_1{}^i(t)$ model correlates the data pair $\{e_g{}^i(t), Q_g{}^i(t)\}$, for each storm i, for the given modeling assumptions. This correlation is integrated into the time distribution of the parameter, $\eta^i(s)$. Thus for each storm event i, there is an associated $\eta^i(s)$.

To proceed with the uncertainty analysis, it is assumed that there is "sufficient" data at the rain gauge site such as to develop equivalence classes of effective rainfall distributions measured at the rain gauge site. These storm classes are noted as $<\xi_q>$. Any two events in $<\xi_q>$ would be nearly identical (storm duration, antecedent moisture conditions, and other effects) such that the catchment response from R would be thought to be also nearly identical. It is assumed that there is sufficient effective rainfall data to develop a set of classes $<\xi_q>$ such that a reasonable statistical analysis can be made for each class individually.

Let $<\xi_o>$ be a class of storms, (it is recalled that the measured effective rainfall distributions are used in the classification, not the rainfall). Let $e_o{}^i(t)$ be an element of $<\xi_o>$, for $i = 1,2,\ldots,n_o$ where n_o is the number of elements in $<\xi_o>$. To each $e_o{}^i(t)$ there is an associated $Q_o{}^i(t)$ measured at the stream gauge. Correlating each pair $\{e_o{}^i(t), Q_o{}^i(t)\}$ by the single area UH model results in n_o distributions, $\{\eta_o{}^i(s)\}$, $i = 1,2,\ldots,n_o$.

The $\eta_o{}^i(s)$ can be represented by a summation graph, $\bar{S}_o{}^i(s)$, where

$$\bar{S}_o{}^i(s) = \int_{t=0}^{s} \eta_o{}^i(t) \, dt \tag{10}$$

Figure 1 shows a plot of $S_o{}^i(s)$ developed from storms of similar severity from a basin in Los Angeles County, California. By examining the plots, usually a normalization technique is apparent. In Fig. 1, plotting each $\bar{S}_o{}^i(s)$ divided by its ultimate discharge, $U_o{}^i$, (i.e., $U_o{}^i = \bar{S}_o{}^i(s = \infty)$), normalizes the vertical axis from 0- to 100-percent. Defining lag^i to be the time that $\bar{S}_o{}^i(s)$ reaches 50-percent of ultimate discharge ($U_o{}^i$) normalizes the horizontal axis to be time in percent of lag. Figure 2 shows the resulting S-graphs, noted as $S_o{}^i(s)$ for storm class $<\xi_o>$. The several S-graphs can now be identified by a characteristic parameter. A convenient parameter to use is the linear scaling X between the enveloping curves of the S-graph data set. Usually, two of the correlation S-graphs will bound the entire set (see Fig. 3). By identifying an X to each S-graph,

$$S_o^i(X,s) = X\, S_o^A(s) + (1-X)\, S_o^B(s) \qquad (11)$$

where S_o^A and S_o^B are the enveloping S-graphs, and X is the scaling parameter with $0 \leq X \leq 1$.

Based on the above normalizations and parameterizations, each distribution graph, $\bar{S}_o^i(s)$, is identified by the three point vector $\underset{\sim}{\eta}_o^i \equiv \{lag^i, U_o^i, X^i\}$. Consequently, each correlation distribution, $\eta_o^i(s)$, is identified by the vector, $\underset{\sim}{\eta}_o^i$, for $i = 1,2,\ldots,n_o$.

Marginal distributions are developed by plotting frequency-distributions of each point in the vector, $\underset{\sim}{\eta}_o^i$ (see Fig. 4).

Based on the marginal distributions, the frequency estimate associated to vector, $\underset{\sim}{\eta}_o^i$, is given by $P(\underset{\sim}{\eta}_o^i)$ where

$$P(\underset{\sim}{\eta}_o^i) = P(lag^i, U_o^i, X^i) \qquad (12)$$

Should more identifying characteristics be used to describe the $\eta_o^i(s)$, Eq. (12) is immediately extended. However, there should be sufficient storms in $\langle\xi_o\rangle$ to develop a reliable frequency-distribution for each identifying characteristic. From the above, a distribution $[\eta_o(s)]$ of correlation distributions, $\eta_o^i(s)$, is derived for storm class, $\langle\xi_o\rangle$.

From the above development, storm class $\langle\xi_o\rangle$ is associated with its distribution of vectors, $\underset{\sim}{\eta}_o^i$. Each storm class $\langle\xi_q\rangle$ can be analyzed to determine their respective vector distributions, $\underset{\sim}{\eta}_q^i$. And from Eqs. (4) - (8), the variations between $\underset{\sim}{\eta}_q^i$ can be explained by the variations in the parameters used to describe the spatial and temporal variations of effective rainfall over R, and the variations in the routing approximations.

HYDROLOGIC MODELING UNCERTAINTY ANALYSIS: PREDICTIVE RELATIONSHIPS

An important use of flood flow hydrologic models is in predicting a hydrologic response from R given a design, or hypothetical, storm event.

Given a design storm effective rainfall distribution to be applied at the rain gauge site, $e_g^D(t)$, the hydrologic model is to be used to predict a runoff response from R.

Let $e_g^D(t) \in \langle\xi_o\rangle$.

Then the runoff response is the random variable $[Q_1^D(t)]$ where

$$[Q_1^D(t)] = \int_{s=0}^{t} e_g^D(t-s)\,[\eta_o(s)]\,ds \qquad (13)$$

where brackets indicate a random variable. Note that in Eq. (13), the correlation distributions, $\eta_o^i(s)$, are now shown as a random variable, as there is no information in $e_g^D(t)$ which determines a particular distribution from the total set. $[Q_1^D(t)]$ is the collection of runoff hydrographs which are possible outcomes associated to the design storm effective rainfall, $e_g^D(t)$, applied at the rain gauge site. $[\eta_o(s)]$ is the collection of correlation distributions associated to storm class $\langle\xi_o\rangle$ where $e_g^D(t)$ is considered to be sufficiently similar to the elements in $\langle\xi_o\rangle$. Because Eq. (13) is a prediction, any of the elements in $[\eta_o(s)]$, and hence $[Q_1^D(t)]$, are candidates as realization of the stochastic process. The model structure is seen to be a causal linear filter.

The variation in any hydrologic quantity is reflected by use of Eq. (13). For example, the variation in flow rate estimates at storm time t_o is given by

$$[Q_1^D(t_o)] = \int_{s=0}^{t} e_g^D(t_o - s)\,[\eta_o(s)]\,ds \qquad (14)$$

Letting t_p be the time of the peak flow rate (where t_p is a function of the random variable, $[\eta_o(s)]$), the uncertainty in peak flow rate estimates, q_p, is given by

$$[q_p] = [Q_1^D(t_p)] = \int_{s=0}^{t_p} e_g^D(t_p - s)\,[\eta_o(s)]\,ds \qquad (15)$$

Figure 5 shows the distribution of q_p for a catchment in Los Angeles, California. The frequency distribution of q_p in Fig. 5 is determined by evaluating Eq. (15) using the marginal distributions shown in Figs. 3 and 4, according to the mutually dependent probability of occurrence given by Eq. (12). By scanning the entire distribution of $[\eta_o(s)]$ available in storm class $\langle\xi_o\rangle$, the q_p frequency distribution is constructed.

Generally, the available data is insufficient to develop highly specific classes of storms, $\langle\xi_q\rangle$, nor develop precise statistical estimates within storm classes, should a storm class categorization be made. Consequently to proceed with the uncertainty analysis, another assumption must be invoked. One approach is to transfer the distribution information of the correlation distributions, $[\eta_q(s)]$, from another catchment considered hydrologically similar with respect to the correlation of rainfall-runoff data, (i.e., regionalization). Another approach is to assume the $[\eta_q(s)]$ to be distributed identically for "similar" storm classes $\langle\xi_q\rangle$, and combine the several storm rainfall-runoff correlations, $\eta^i(s)$, into the same distribution. The first technique may be utilized for any catchment, but the second technique obviously requires local rainfall-runoff data.

CONCLUSIONS

Given a catchment and rainfall-runoff data, a flood flow model can only serve as a means to statistically correlate the two forms of data. In an single area UH model, the unit hydrograph serves as the link which correlates the effective rainfall data to runoff data. The single area UH model actually represents a complex link-node model, had subarea hydrologic data been available to evaluate subarea runoff, and had stream gauge data been available to evaluate all link-node hydraulic parameters. Without subarea data, however, the link-node model representation becomes an estimator which is in error due to the approximation of hydraulic parameters and the misrepresentation of the effective rainfall distribution over the entire catchment, (i.e., discretization error).

REFERENCES

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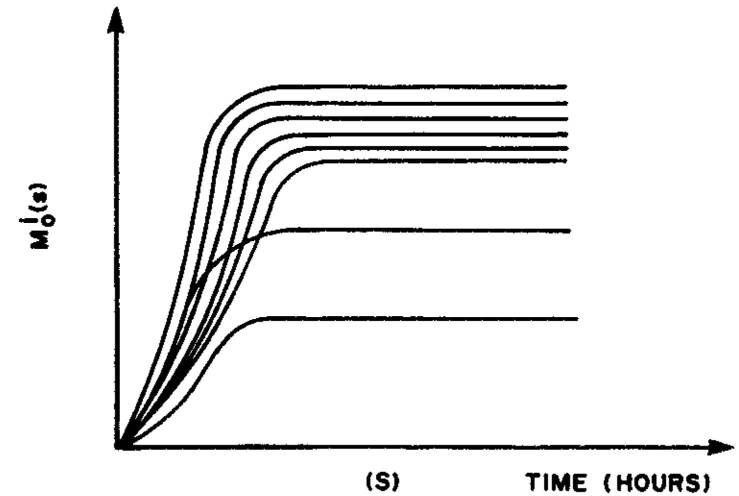

Fig. 1. Example Summation Graphs, $\bar{S}_0^i(s)$, for Storms in Class $<\xi_0>$.

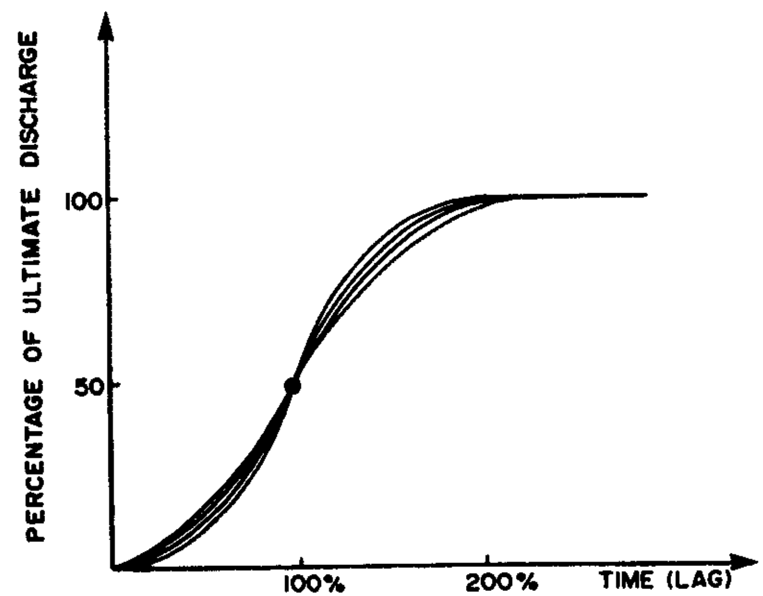

Fig. 2. S-Graphs, $S_0^i(s)$, for Storm Class $<\xi_0>$.

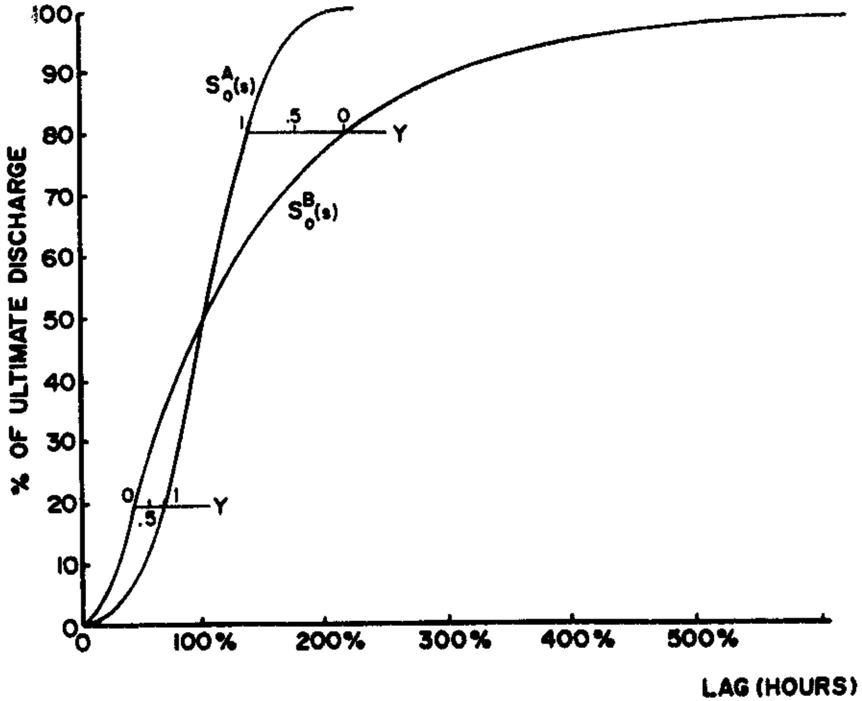

Fig. 3. Definition of S-Graph Parameter X using $S_0^A(s)$ and $S_0^B(s)$.

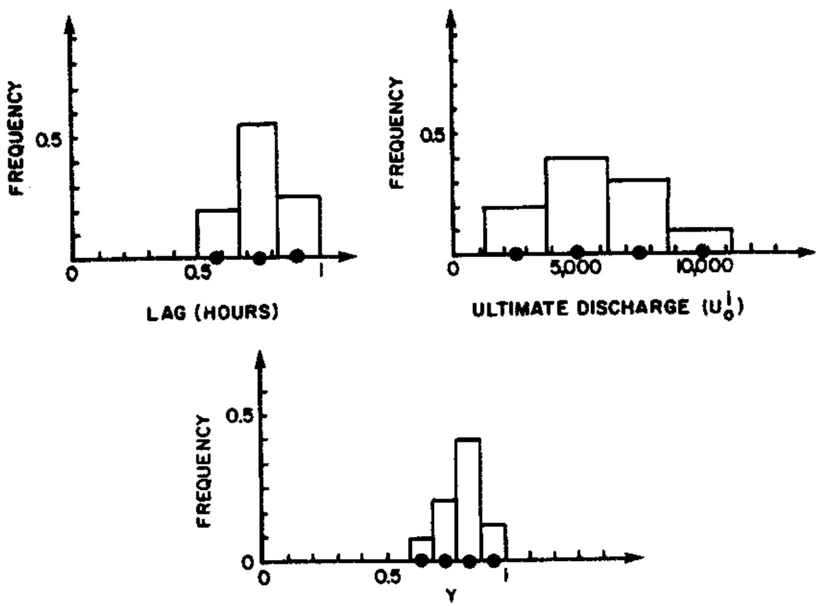

Fig. 4. Marginal Distributions for Vector, $\underset{\sim}{\eta}_0^i$, components.

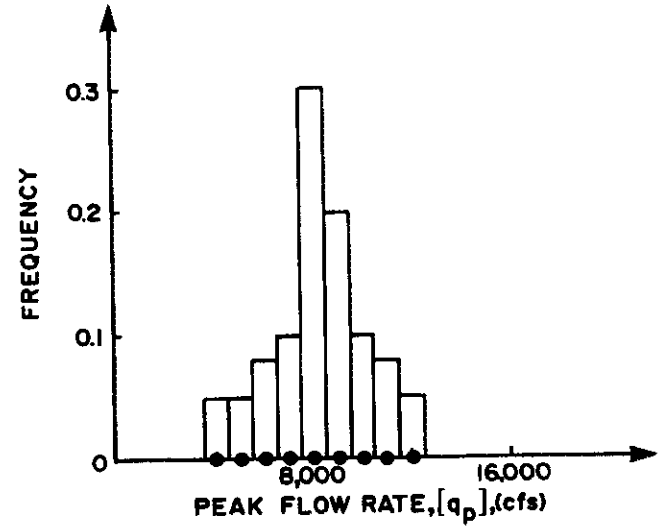

Fig. 5. Uncertainty Distribution for the Estimate of a Peak Flow Rate, q_p.