Checking Flood Frequency Curves Using Rainfall Data

By T. V. Hromadka II[1] and R. J. Whitley[2]

Analysis

A typical procedure for deriving flood frequency information in the United States is to analyze the annual flood series, without utilizing the available rainfall data, and follow the guidelines of Water Resources Bulletin 17B (1981). In this way, an estimate of the flood frequency curve is developed for decision making or policy statements for flood control. There are numerous sources of uncertainty in such an estimation. For example, there is the uncertainty as to whether the log Pearson III distribution is the "correct" distribution from which to extrapolate infrequent floods, as well as the uncertainty in the distribution parameters chosen which is caused by the uncertainty in estimates of the mean, standard deviation and skew of the log Pearson III distribution (Stedinger 1983). So it is important to have a procedure which furnishes a check on the flood frequency analysis results.

This note suggests a simple statistical procedure using rainfall data. Generally, rainfall data are much more plentiful than runoff data and are not subject to questionable adjustments used to account for changing watershed conditions, for loss of flow data, for transmission losses, or for variations in runoff volume data. Also, rainfall data are usually available at several rain gauge stations within a meteorologically homogeneous region and these data can be subjected to a regional statistical analysis. In fact, local government agencies often collect and synthesize both rainfall and stream gauge data and prepare regional rainfall data summaries which give various estimates of rainfall depth-duration values versus return frequency.

For a watershed with a computed flood frequency curve, one estimate of "goodness of fit" of this curve is to compare the probabilities computed from this curve with the probabilities of the rainfall which produced the individual peak flow rates used in the flow rate annual series. These probabilities from the curve can be obtained either: (1) By an analysis of the peak rainfall depth-duration frequency of the storm event which produced the annual peak flow rate; or (2) by assuming that the return frequency of peak rainfall depth corresponding to a storm duration approximately equal to the critical duration of the catchment (which is usually assumed to be the time of concentration) is equal to the maximum recorded depth return frequency for that year. This analysis in terms of the time of concentration is most appropriate for smaller watersheds.

As an example of method (2), suppose that the catchment is assumed to have a critical duration of about one hour. Then the peak one hour depth of rainfall corresponding to the record annual peak flow rate is determined for each data point in the flood annual series. These rainfall depths are then ranked according to the return frequency as estimated from the regionalized rainfall data, which is another source of error. The resulting set of T-year return frequency estimates are then compared to the expected return frequencies by means of the well-known chi-square test. In this way an estimate is obtained of how likely, or unlikely, the rainfall occurrences are and therefore how likely it is that the flood frequency is correct. Although the record peak rainfalls typically may not coincide with the runoff peak flow rates, their use still provides a useful estimate of likelihood for decision making purposes.

[1]Dir. of Water Resour. Engrg., Williamson and Schmid, Irvine, CA 92714.
[2]Prof. of Mathematics, Univ. of California, Irvine, CA 92717.

Note. Discussion open until September 1, 1989. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 27, 1986. This paper is part of the *Journal of Hydraulic Engineering*, Vol. 115, No. 4, April, 1989. ©ASCE, ISSN 0733-9429/89/0004-0544/$1.00 + $.15 per page. Paper No. 23375.

Examples

To illustrate the procedure, rainfall data corresponding to the flood peak annual series are determined and compared for two regions in southern California: (a) The valley area of the county of San Bernardino; and (b) the coastal area of the neighboring Orange county.

We apply method (2) discussed above to one hour storms for area (a), obtaining number of occurrences (A1), also to two hour storms for area (a) to obtain the number of occurrences (A2), and finally for one hour storms for area (b) obtaining the number of occurrences (B). Return periods for these precipitations were obtained from the regional data given in (Department of Water Resources 1981); these regional data were generated from regionalized parameters under the assumption that the underlying distribution is log Pearson III. Therefore the chi-squared comparison given below comprises a test of the consistency of this assumption with the observed data. Table 1 summarizes the results.

We now want to apply a chi-square test to these data, for which see, for example, Kendall and Stuart (1979) or Breiman (1973).

The first step is to group the data (which have already been grouped to some extent) in an attempt to satisfy the rule-of-thumb of having at least five occurrences in each group and as many groups as possible (Breiman 1973); group together the observations for return periods of greater than or equal to six years in one group for (A1), (A2), and 6–10 years for (B).

The next step is to compute the theoretical expected number of occurrences in each group. The definition of a magnitude M_T with a return period T is that magnitude such that the probability of seeing an event of size $X \geq M_T$, is $1/T$:

TABLE 1. Number of Occurrences of Rainfall with Given Return Periods (A1 = San Bernardino County—One Hour Storms; A2 = San Bernardino County—Two Hour Storms; and B = Orange County—One Hour Storms)

Return period (1)	A1 (2)	A2 (3)	B (4)
1 year	22	17	21
2–5 years	9	15	19
6–10 years	3	1	6
11–20 years	1	2	2
≥21 years	0	0	2

TABLE 2. Probability of T-year Flood (Eq. 2)

$T = 1$ (1)	2 (2)	3 (3)	4 (4)	5 (5)	6 (6)	7 (7)	8 (8)	9 (9)	10 (10)	≥11 (11)
0.500	0.167	0.083	0.050	0.033	0.024	0.018	0.014	0.011	0.009	0.091

$$Prob(X \geq M_T) = \frac{1}{T} \quad (1)$$

(Since the underlying distribution is assumed to be absolutely continuous, the event $\{X \geq M_T\}$ has the same probability as the event $\{X > M_T\}$, and it is of no importance whether or not the endpoint M_T is included in Eq. 1.) The usual interpretation of a "T-year event" is that it is one of size X where X is in the range $M_T \leq X < M_{T+1}$. In this case,

$$Prob(M_T \leq X < M_{T+1}) = \left\{1 - \frac{1}{T+1}\right\} - \left\{1 - \frac{1}{T}\right\} = \frac{1}{T(T+1)} \quad (2)$$

Note that to apply this interpretation of "T-year event" in the computation of chi-square it is necessary to also apply it when making up the table of observed values A and B above. Using Eq. 1, the values in Table 2 can be used to compute the probabilities of the groups into which the data have been grouped. These probabilities multiplied by the total number of data points give the expected number of observed events. Table 3 shows the results for the data from basin A. The value of chi-square is $\chi^2 = \Sigma$ {(observed − expected)^2/expected} = 2.34, the same value (to three decimals) for both A1 and A2 data. There are 3 data groups and so 3 − 1 = 2 degrees of freedom, and so the number 2.34 is to be compared with the tabulated values of χ^2 for these 2 degrees of freedom. From a table of χ^2, e.g. Table 2 in Breiman (1973), $\chi^2 < 1.39$ with probability 0.5 and $\chi^2 < 2.77$ with probability 0.75. Therefore the value 2.34 is not an unusual value, the observed numbers of T-year events is consistent with the predicted number of events, and this data passes this test. Table 4 shows the results for data from basin B. Then $\chi^2 = 2.32$, and there are 3 degrees of freedom. Since $\chi^2 < 2.37$ with probability 0.5, we again conclude that the observed number of events is consistent with the predicted number of events.

It is interesting to see how sensitive this analysis is to the precise definition of a "T-year event." Another reasonable definition of a T-year event is one whose magnitude X lies in the range $M_{T-0.5} < X \leq M_{T+0.5}$. If $T > 1.5$ this event has probability

TABLE 3. Statistical Comparisons of Observed Data and Expected Results for A Data (Eq. 2)

Return period (1)	Observed A1 (2)	Observed A2 (3)	Expected (4)
1 year	22	17	17.50
2–5 years	9	15	11.67
≥6 years	4	3	5.83

TABLE 4. Statistical Comparisons of Observed Data and Expected Results for B Data (Eq. 1)

Return period (1)	Observed (2)	Expected (3)
1 year	21	25.00
2–5 years	19	16.67
6–10 years	6	3.79
≥11 years	4	4.54

TABLE 5. Probability of T-year Flood (Eqs. 3 and 4)

$T = 1$ (1)	2 (2)	3 (3)	4 (4)	5 (5)	6 (6)	7 (7)	8 (8)	9 (9)	10 (10)	≥11 (11)
0.333	0.267	0.114	0.064	0.040	0.028	0.021	0.016	0.012	0.010	0.095

TABLE 6. A1 Data (Definition 2 of a T-year Event)

Return period (1)	Observed (2)	Expected (3)
1 year	22	11.67
2–5 years	9	16.97
≥6 years	4	6.36

$$P(M_{T-0.5} < X \leq M_{T+0.5}) = \frac{1}{T^2 - 0.25} \quad (3)$$

For $T < 1.5$, since $P(X < M_1) = 0$ the probability of a T-year event is

$$P(M_1 \leq X \leq M_{T+0.5}) = 1 - \frac{1}{T + 0.5} \quad (4)$$

in particular the probability of a one year flood is 1/3 (see Table 5).

These probabilities are numerically close to the usual definition if T is large, but not if T is small. If you use these probabilities in computing the expected number of T-year events (see Table 6) then $\chi^2 = 13.76$. For χ^2 with 2 degrees of freedom, the probability that $\chi^2 < 10.6$ is 0.995; in this case the computed value of χ^2 is too large and the data are not consistent with the expected values. The point of this example is that the definition to use is the one used in making up the regional data base, which is indeed the first definition. And, with this example in mind, any user of this test should also use the usual definition of a T-year event when compiling the data to be tested by chi-square.

Conclusions

Often only the stream gauge data are used to derive a flood-frequency curve for peak flow rates. It is then of interest to also evaluate the likelihood that the resulting flood frequency curve is appropriate for the specific catchment. One such evaluation uses the rainfall for the catchment's critical duration and estimates the probability of the occurrence of the record precipitation sequence using the chi-square test.

Appendix. References

Water Resources Council Hydrology Committee (1981). "Guidelines for determining flood-flow frequency." *Bulletin #17B*, Washington, D.C.

Stedinger, J. R. (1983). "Confidence intervals for design events." *J. Hydr. Engrg.*, ASCE, 109, 13–27.

Department of Water Resources (1981). "Rainfall depth-duration-frequency for California." Div. of Planning, Dept. of Water Resour., Calif.

Kendall, M., and Stuart, A. (1979). *The advanced theory of statistics II*, Griffin.

Breiman, L. (1973). *Statistics: with a view towards applications*. Houghton-Mifflin, Boston, Mass.