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Discretization of Hydrologic Models

THEODORE V. HROMADKA II

Williamson & Schmid
Tustin, CA 92680
and
Department of Mathematics
California State University
Fullerton, CA

RICHARD H. McCUEN

Department of Civil Engineering
University of Maryland
College Park, MD 20742

ROBERT J. WHITLEY

Department of Mathematics
University of California
Irvine, CA 92717

CHUNG-CHENG YEN

Williamson & Schmid
Tustin, CA 92680

ABSTRACT

In all phases of hydrologic modeling, including calibration, design, and watershed evaluation, analyses on all except the smallest watersheds involve watershed subdivision, or discretization. In spite of the frequency in which subdivision is necessary and the lack of guidelines for proper discretization, little attention has been paid to the effect of this practice on the accuracy of model results. Due to the sparse rainfall-runoff data typically available at a watershed, it may be questionable whether the subdivision of the catchment into subareas, when there is no data to calibrate subarea hydrologic model parameters, is a "better" approach to modeling the catchment response. In this paper, the subject of catchment discretization is examined. It is noted that for a linear hydrologic model based on unit hydrographs (for various levels of discretization into subareas) and translation for channel routing, a discretized model is equivalent to a simple one-subarea model, and calibration of the simple model accounts for the variation of effective rainfall over the watershed that is not accounted for by the discretized model.

Key Words: Catchment discretization, calibration and variance of Hydrologic Model

INTRODUCTION

The typical procedure for using a unit hydrograph (UH) type model, such as TR-20 and HEC-1, is to subdivide a watershed into subareas that are linked by channel or detention basin routing. Parameters (e.g., losses, timing, etc.) are then assigned to the several UH subarea submodels and the routing models. Even though the catchment is virtually homogeneous, it is often discretized into several subareas that are linked by routing elements. For example, it is not uncommon for a nearly homogeneous (almost uniform development, drainage patterns, etc.) 1 or 2-square mile catchment to be subdivided into one or two dozen (or more) subareas.

The questions arise as to whether (1) such subdivision practices results in "better" models, and (2) such highly discretized models are more accurate for use in design than the simpler models based on a minimum number of subareas, perhaps even without subdivision. In this paper the UH modeling approach is used to examine the effects of subdivision, or discretization, of a watershed where a design or evaluation is required.

The first step of this study is to derive mathematical relationships for the simple (single area) and complex (multi-subarea) models. By using only a few modeling simplifications, Voltera integral equations for the runoff hydrograph are derived for the simple and complex models. It is then shown that the parameters of the simple model account for the important variation in effective rainfall over the catchment as correlated to the available rain gauge data; however, the complex model reduces to a simple UH model and neglects this important consideration. Using the derived integral equations, the effect of model calibration and watershed discretization can be examined in detail.

THE EFFECT OF DISCRETIZATION ON MODEL OUTPUT VARIANCE

In order to develop a mathematical analysis of the model discretization process, the considered modeling approach for the discretized representation of the catchment is to use m subareas linked together by a channel routing process which is pure translation. Thus subarea runoff hydrographs are developed, combined at confluence points, and then routed by translation to the next downstream subarea confluence point.

Let a catchment be divided into m subareas, where the discharge from the j-th subarea at time t is $q_j(t) = q_j(t,P_j)$, P_j denoting the vector parameters. The discharge $q_j(t,P_j)$ is a random variable with respect to the parameter values P and their associated probabilities.

Let $q_c(t) = q_c(t,P)$ be the discharge from the catchment at time t, considered as a "complex" model, which is composed of m discharges from the individual subareas. It is an informative simplification to approximate the effects of routing by translation, i.e., subarea discharges are added and channel routing is modeled as translation in time by the amount τ_j for q_j. In this case

$$q_c(t,P) = \Sigma\, q_j(t-\tau_j,P_j) \tag{1}$$

The maximum discharge M_c, is defined by

$$M_c = \max\, q_c(t,P) \tag{2}$$

is attained at a point t' which may vary with P; however t' does not generally vary much and so belongs to one of only a few unit intervals (or timesteps) t_p, where subscript p satisfies $m1 \leq p \leq m2$, and where (m2-m1) is small. Then

$$\mathrm{var}(M_c) = \mathrm{var}[\max(q_c(t_p,P): m1 \leq p \leq m2)] \tag{3}$$

The variance of the peak discharge is thus seen to depend on only a small number of the variances $\mathrm{var}(q_c(t_p,P))$ and these variances will now be analyzed.

By the independence of the $q_j(t'-\tau_j,P)$,

$$\mathrm{var}\, q_c(t_p,P) = \Sigma\, \mathrm{var}\, q_j(t'-\tau_j,P). \tag{4}$$

The discharge of the catchment when modeled as a simple model with only one subarea will be denoted by $q_s(t,P)$. The maximum discharges, which will be used for statistical analysis purposes, are given by

$$\begin{aligned} M_s &= \max\, q_s(t,P) \\ M_j &= \max\, q_j(t,P_j) \end{aligned} \tag{5}$$

where M_s is the maximum discharge for the simple model, M_j is the maximum discharge from subarea j, and both M_s and M_j are random variables in P. When P is taken to have the design values P_d (such as determined from the policy/hydrology manual), the design estimates for the maximum discharge are obtained:

$$\begin{aligned} \mu_s &= \max\, q_s\, (t,P_d) \\ \mu_j &= \max\, q_j\, (t,P_d). \end{aligned} \tag{6}$$

In our discussion it is assumed that there is only a single rain gauge and stream gauge for data analysis. Consequently, the modeling parameters developed from the data are typically assumed to apply not only to the whole catchment, but to subareas. That is, it is implicitly assumed that the random variables $Y_j = M_j/\mu_j$ and $Y = M_s/\mu_s$ have approximately the same distribution. And in fact, a similar assumption is necessary if one wants to make statistical inferences about the subarea when only the statistics of the entire catchments are known. That Y and Y_j have the same distribution implies, of course, that var(Y) = var(Y_j), or equivalently,

$$var(M_j/\mu_j) = var(M_s/\mu_s). \quad (7)$$

The random variable $q_j(t'-\tau_j, P_j)$ occurring in Eq. (4) has a variance that can be expressed as a proportion, λ_j, of the var(M_j) when $t' = t_p$ by

$$var(q_j(t_p-\tau_j, P_j)) = \lambda_j \; var(M_j) \quad (8)$$

Combining Eqs. (4) and (8) gives

$$var\; q_c(t_p, P) = \Sigma \; var\; q_j(t_p, P_j)$$

$$= var(Y)\; \mu_s^2 \; \Sigma \lambda_j (\mu_j/\mu_s)^2$$

$$= var(M_s)\; \Sigma \lambda_j (\mu_j/\mu_s)^2 \quad (9)$$

and

$$var(M_c) = var(M_s)\; \Sigma \lambda_j (\mu_j/\mu_s)^2 \quad (10)$$

Two extreme cases of Eq. (10) help point out the implications. First, consider the case of parallel subareas, all with the same discharge q_j and adding together (with the same lag) to produce q_c. In this case $\lambda j = 1$ and $m\mu_j = \mu_s$ for $j = 1, \ldots, m$; and the maximum occurs at a point t' where all of the q_j have their maxima. Thus, it follows that

$$var(M_c) = (1/m)\; var(M_s) \quad (11)$$

More generally, it will follow that the variance of M_c is smaller than the variance of M_s if the following conditions hold: (1) if the model schematic has a "large somewhat parallel component" in the sense that many q_j's contribute to the max q_c so that the μ_j/μ_s is small; (2) under the assumptions that Eq. (8) holds and that λ_j is not too large, and (3) the maximum of q_c only occurs in a few intervals.

Second consider the case where subareas are routed together in a linear series, all with the same discharge $q = q_j$ and with the lags chosen so that the second area contributes only after the first area is completely discharged, the third contributes only after the second has completely discharged, and so on. In this case var(M_c) = var(M_s) with no reduction in variance. (Here the value given by Eq. (10) is too large, unless the scale factors λ_j are considered to be functions of t' with one of them 1 while all the others are 0, or a slightly different deviation is followed.)

The final formulation includes the effect of the peak flow rate occurring in (m2-m1) unit intervals, and λ_j not varying for $m1 \leq p \leq m2$.

$$\mathrm{var}(M_c) = (m2 - m1)\ \mathrm{var}(M_s)\ \Sigma \lambda_j (\mu_j/\mu_s)^2 \qquad (12)$$

The μ_j/μ_s are the design peak flow rate values computed for each subarea and the single area (simple) model, respectively, using the appropriate single design parameter set P_d. Then it is seen that generally $\mathrm{var}(M_c) \neq \mathrm{var}(M_s)$. In fact, for a "small" watershed it is noted that $\mathrm{var}(M_c) \ll \mathrm{var}(M_s)$ as the number of subareas increase, whereas for a "large" watershed where $\mu_j > \mu_s$, $\mathrm{var}(M_c) \gg \mathrm{var}(M_s)$. But the $\mathrm{var}(M_s)$ reflects the true variation in peak Q estimates at the single stream gauge based on the single rain gauge information; hence, the $\mathrm{var}(M_c)$ departing from $\mathrm{var}(M_s)$ must indicate a form of modeling error introduced by the watershed discretization process which can only be removed by a supply of runoff data for each subarea used in the complex model.

That is, stream gauge data must be available for each subarea, j, in order to develop best fit parameter sets (for each subarea) to correlate individual subarea runoff data to the available (single) rain gauge data. Thus a best fit parameter set for a m-subarea complex model would be composed of m optimized parameter sets, one set for each subarea. This "ultimate" best fit parameter set would correlate the stream gauge measured runoff hydrograph to the available rain gauge data obtained from the considered rain gauge, for the specific storm event reconstituted. Given a set of such ultimate best fit parameter sets for $q_c(t)$ and the corresponding best fit parameters for $q_s(t)$, then $\mathrm{var}(M_c) \cong \mathrm{var}(M_s)$. Should rain gauge data not be supplied in each subarea used for $q_c(t)$, then the subarea model parameter correlations could be based on subarea data rather than the single rain gauge, resulting in a true drop in the $\mathrm{var}(M_c)$ as a result of a better correlation between rainfall-runoff data.

SINGLE AREA AND DISCRETIZED UH MODEL VOLTERA INTEGRAL REPRESENTATIONS

Insight into the hydrologic modeling process can be gained by considering the linear unit hydrograph method with translation for channel routing. That is, as in the derivation above, subarea hydrographs are added directly at confluences and channel routing is modeled as a simple translation in time. (Should peak attenuation be considered, the relationship of Eq. (12) still applied but the λ_j typically are closer to unity in value.)

For a single area model of a free-draining, near homogeneous catchment without significant flow detention effects (e.g., detention basins, exceptionally wide channels, etc.), having unit hydrograph $\psi(t)$, the simple runoff hydrograph is given by $q_s(t)$ where

$$q_s = \int_{s=0}^{t} e_g^{*}\ (t-s)\ \psi(s)\ ds \tag{13}$$

where $e_g^{*}(t)$ is a "representative" effective rainfall distribution used for calibration purposes. Typically, only one rain gauge and one stream gauge are available for calibration.

As an alternative to the model in which the watershed is not subdivided, let the catchment be subdivided into m subareas with areas $A_j (j=1,2,\ldots,m)$, each with an area-centered (rain gauge measured) effective rainfall distribution $e_j(t)$. In considering the discretized model, a filter function U_j can be defined for each subarea A_j by

$$U_j(t) = \begin{cases} 1, & t \geq \tau_j \\ 0, & t < \tau_j \end{cases} \tag{14}$$

Assuming subarea A_j has an associated unit hydrograph $\phi_j(t)$, then the effect of channel routing (assuming a uniform translation time of τ_j and no hydrograph peak attenuation) results in m-subarea runoff hydrograph complex model $q_c(t)$ given by

$$q_c(t) = \sum_{j=1}^{m} e_j(t-s)\ U_j(s-\tau_j)\ \phi_j(s-\tau_j)\ ds \tag{15}$$

The product $U_j(s-\tau j)\ \phi_j(s-\tau_j)$ gives the unit hydrograph response from subarea A_j with respect to the stream gauge location, and includes the effect of translation channel routing with a time offset of τ_j. Thus, zero flow contribution from subarea A_j occurs at the stream gauge until model time τ_j, when the subarea unit hydrograph $\phi_j(s)$ initiates. The above $q_c(t)$ represents (i) a distributed effective rainfall, $e_j(t)$, based on measured rainfall for each subarea j; (ii) translation channel routing with travel time τ_j assigned to subarea j as the translation time of the hydrograph to the stream gauge; and (iii) a unique unit hydrograph $\phi_j(s)$ for each subarea j.

In most calibration studies, only one rain gauge is available for study purposes, resulting in a "representative" effective rainfall distribution $e_g^{*}(t)$, which is used to correlate model parameters to the measured runoff hydrograph data from the stream gauge. Note that $e_g(t)$ is the actual point effective rainfall distribution whereas $e_g^{*}(t)$ is a representative effective rainfall distribution determined by calibration of the model parameters from a calibration study. It will also be assumed that there are constants c_j, such that the individual subarea effective rainfalls $e_j(t)$ can be related to the single effective rainfall distribution $e_g(t)$, which is based on rainfall data measured at a single gauge, by

$$e_j(t) = c_j \; e_g(t) \tag{16}$$

Then the convolution model for the complex model is

$$\begin{aligned} q_c(t) &= \sum_{j=1}^{m} \int_{s=0}^{t} c_j \; e_g(t-s) \; U_j(s-\tau_j) \; \phi_j(s-\tau_j) \; ds \\ &= \int_{s=0}^{t} e_g(t-s) \sum_{j=1}^{m} c_j U_j(s-\tau_j) \phi_j(s-\tau_j) \, ds \\ &= \int_{s=0}^{t} (W \; e_g(t-s)) \left(\sum_{j=1}^{m} c_j \; U_j(s-\tau_j) \phi_j(s-\tau_j)/W \right) ds \end{aligned} \tag{17}$$

where $W = \Sigma \; c_j \; Aj / \Sigma \; A_j$.

Model Comparisons

In comparing $q_s(t)$ and $q_c(t)$ and since

$\int_0^{\infty} \psi(s) ds = 1$ unit-area, the following equality is necessary:

$$\psi(s) = \sum_{j=1}^{m} c_j U_j(s-\tau_j) \; \phi_j(s-\tau_j)/W \tag{18}$$

The c_j are unknown in the calibration process because there is only a single rain gauge. Additionally, the several $\phi_j(s)$ are unknown and, even if the $\phi_j(s)$ functions were known, the scale factors c_j cannot be determined using only a single gauge and stream gauge pair for rainfall-runoff data. In comparison, the simple model $q_s(t)$ of Eq. (13) enables the direct calibration of its single area UH, $\psi(s)$. One consequence of the calibration of the single area model UH is the development of a "probable" runoff distribution UH that includes not only the variability in the individual subarea UH, $\phi_j(s)$, and the effects of channel routing, U_j, but also the variation in the unknown rainfall distribution (represented in this discussion by the m constants, c_j).

By definition

$$\int_{s=0}^{\infty} \psi(s)\ ds = \int_{s=0}^{\infty} \left(\sum_{j=1}^{m} c_j U_j(s-\tau_j)\ \phi_j(s-\tau_j)/W \right) ds = 1 \text{ unit-area} \tag{19}$$

and, therefore, the final discrepance in runoff yield and other losses between the true distribution of effective rainfall at the rain gauge site, $e_g(t)$, and the modeled distribution of effective rainfall must be absorbed in the representative effective rainfall. Hence, when correlating the available runoff hydrograph information with the available rain gauge information for model calibration purposes, it is possible to find that yields greater than unity and even negative loss rates gives the best fit with the available rainfall-runoff data.

An example of the importance of the rainfall factors c_j is given in the recent paper by Schilling and Fuchs (1986). They show that the variation in recorded precipitation over an 1800-acre catchment as obtained with a nine rain-gauge system when compared with a single centered rain gauge resulted in variations in runoff volumes and peak flows "...well above 100-percent over the entire range of storm, implying that the spatial resolution of a rainfall has a dominant influence on the reliability of computed runoff." They also noted that "errors in the rainfall input are amplified by the rainfall-runoff transformation..." so that "...a rainfall depth error of 30-percent results in a volume error of 60-percent and peak flow error of 80-percent." These conclusions of Schilling and Fuchs point out the significance of the error due to the assumed precipitation distribution over a catchment, which has been correlated by other studies such as Hornberger et al. (1985), and Garen and Burges (1981).

Application

A demonstration of the above Voltera integral model is given by the example problem shown in Fig. 1. In this example, the effective rainfall, $e_j(t)$, for each of the nine subareas are assumed related to the gauged site effective rainfall, $e_g(t)$, by the effective rainfall ratios, c_j, given in Table 1. Individual subarea unit hydrographs are all assumed as "exactly" known. In this example, five of the nine subareas' runoff hydrographs are offset in arrival time, τ_j, at the stream gauge by 3-hours due to translation channel routing which forces a double peak Q. Figure 2 shows the $e_g(t)$ effective rainfall distribution and the runoff hydrograph from the 9-subarea model, $q_c(t)$.

Figure 3 shows the S-graph for $q_c(t)$ developed from the 9-subarea model with a constant effective rainfall intensity and with the tabulated parameters and c_j factors. The weighting factor for the simple model, W, is calculated from Eq. (19) to be

$$W = 0.85 \tag{20}$$

TABLE 1. 9-SUBAREA PARAMETERS (Figure 1)

SUBAREA NO.	T_c (hrs)	t_j (hrs)	c_j	c_j
1	0.80	3.7	1.20	(1.20)
2	0.85	3.65	0.90	(0.90)
3	0.90	3.60	0.80	(0.80)
4	0.95	0.05	0.85	(5.00)
5	1.00	0.0	0.60	(5.00)
6	1.00	0.0	0.85	(5.00)
7	0.85	3.65	0.90	(0.90)
8	0.90	3.60	0.80	(0.80)
9	0.95	0.05	0.80	(5.00)

Notes:

1 τ_j = translation offset (hrs) to stream gauge

2 c_j = effective rainfall ratio to gauged site $e_g(t)$

3 (c_j) = c_j values for second test

4 T_c = assumed time to concentration for subarea

Using the above S-graph representation of the simple model unit hydrograph $\psi(s)$ and also the value of W calculated above, the plots of $q_c(t)$ and $q_s(t)$ are shown in Fig. 2.

A comparison showing the importance of the variation in effective rainfall over the watershed is given by using the second set of c_j factors listed in Table 1. With these new c_j values, W is now

$$W = 2.81 \tag{21}$$

The new S-graph is shown in Fig. 4. A comparison between $q_c(t)$ and $q_s(t)$ is shown in Fig. 5. It is noted that the resulting double peaked runoff hydrograph is of an entirely different shape than developed in Fig. 2. Also it is noted that in both cases, the single area model represented the double peaked runoff hydrograph even though the effective rainfall pattern is single peaked (Figs. 2 and 5).

The above two examples demonstrate the significance of the roles played in the simple model S-graph (or unit hydrograph, $\psi(s)$) by (i) the individual subarea unit hydrographs, $\phi_j(s)$; (ii) the translation routing time increments, τ_j: and (iii) the distribution of effective rainfall over the watershed.

Discussion

The factors c_j for each subarea A_j account for precipitation effects. For example, should one portion of a catchment have a high probability for having lower precipitation intensities ($c_j<1$) while another part of the catchment has a high probability of high intensities ($c_j>1$), both relative to the one available rain gauge, the catchment hydraulic response could contain closer or quicker responses from portions of the catchment that would not be represented by the usual assumption of uniform effective rainfall distribution over the entire catchment. Other variations in precipitation include offsets in rainfall timing and storm pattern shape variations.

In addition, the typical procedure in practice is to assume that the UH for each subarea has the same functional form (when normalized with respect to lag): usually, this hydrograph is assumed to be described by the total catchment normalized unit hydrograph as calibrated to the available stream gauge or similar regionalized data. From the above equations, it is seen that such assumptions are a possible source of error. That is, the UH is better envisioned as a "probable" runoff distribution as correlated to the identified effective rainfall distribution. Hence, a portion of the catchment that is less runoff productive would have a reduced UH ($c_j<1$). Thus, the probable runoff distribution from a subarea A_j is likely to have different characteristics than those of the total catchment. Additional error is also introduced by incorporating routing approximations (e.g., timing offsets τ_j). How the confluences of the stream network is approximated also introduces error. And as stated in the above, the factors c_j are all set equal to one in the discretized model.

Based on the above discussion, it appears that a highly discretized model of a catchment (with the previous assumptions) loses a portion of the randomness that is inherent in catchment hydrology. As shown in the previous section, the discretization process often results in a significant change in model output variance (e.g., variance of the peak flow rate) whereas the true catchment hydrologic output variance cannot be changed unless considerably more data are obtained to identify the several unknowns in the hydrologic system such as, for example, represented by the c_j terms which occur in the single area UH but which are forced to all be 1 in the application of a highly discretized model. As a result, the single area UH, $\psi(s)$, which represents the probable runoff distribution as correlated to the effective rainfall distribution (applied to the total catchment), includes a significant source of uncertainty that is very likely transferred to the other model variables when the catchment is discretized.

SINGLE AREA AND DISCRETIZED UH MODEL CALIBRATIONS

As discussed in the previous section, let the true runoff hydrograph be given by the complex model representation of Eq. (15). In calibration, only a single rain gauge is generally available which is represented by the actual point effective rainfall distribution $e_g(t)$. Using correction factors, c_j, the true runoff hydrograph can be rewritten as (see Eq. (16))

$$q_c(t) = \int_{s=0}^{t} (W\ e_g(t-s)) \left(\sum_{j=1}^{m} c_j U_j(s-\tau_j)\ \phi_j(s-\tau_j)/W \right) ds \tag{22}$$

For calibration purposes, the c_j are unknown and in the complex model the c_j are defined by $c_j = 1$, $j = 1,2,\ldots,m$. Thus, the calibration version $\hat{q}_c(t)$ of $q_c(t)$ is given by

$$\hat{q}_c(t) = \int_{s=0}^{t} e_g^{**}(t-s) \sum_{j=1}^{m} U_j(s-\tau_j)\phi_j(s-\tau_j)\ ds \tag{23}$$

where $e_g^{**}(t)$ is the representative effective rainfall distribution, the parameters of which must be calibrated in the complex model. For the simple model of Eq. (12), $e_g^{*}(t-s)$ typically differs from $e_g^{**}(t-s)$.

The requirements for calibration or both $\hat{q}_c(t)$ and $q_s(t)$ to the assumed exact model with exact parameters, $q_c(t)$, can now be determined. For a simple model, the representative effective rainfall, $e_g^{*}(t-s)$, must satisfy

$$e_g^{*}(t-s) = (W)\ e_g(t-s) \tag{24}$$

and the unit hydrograph, $\psi(s)$, is directly calibrated by

$$\psi(s) = \sum_{j=1}^{m} c_j U_j(s-\tau_j)\phi_j(s-\tau_j)/W \tag{25}$$

In contrast, the complex model calibration effort focuses upon the representative effective rainfall $e_g^{**}(t-s)$ which must satisfy

$$e_g^{**}(t-s) = \frac{e_g^{**}(t-s)\sum_{j=1}^{m} c_j U_j(s-\tau_j)\phi_j(s-\tau_j)}{\sum_{j=1}^{m} U_j(s-\tau_j)\phi_j(s-\tau_j)}$$

$$= e_g^{*}(t-s)\ \psi(s)/\psi^{**}(s) \tag{26}$$

where $\psi(s)$ follows from the simple model unit hydrograph, and $\psi^{**}(s)$ is the rigid unit hydrograph of the complex model, with all c_j's = 1:

$$\psi^{**}(s) = \sum_{j=1}^{m} U_j(s-\tau_j)\phi_j(s-\tau_j) \tag{27}$$

Discussion

The above model calibration requirements indicate the hydrologic interplay between the effective rainfall distribution and watershed runoff/hydraulic characteristics on the one hand, and the ability of the simple model unit hydrograph versus the complex model link-node schematic to represent the watershed runoff tendencies for a specific storm event. For the simple case considered where each subarea is assumed subject to constant multiple (or fraction) of the rain gauge measured effective rainfall, $e_g(t)$, (represented by the factors c_i in Fig. 6), and "equivalent" watershed can be envisioned where all other subarea parameters remain in the same, but each subarea size is modified by its "effective contribution area adjustment" factor, c_i. Figure 6 illustrates such a construction. For this specific storm, subarea #1 is reduced in area by 20-percent, subarea #2 and #4 are reduced in area by 10-percent, and subarea #9 is increased in area by 150-percent.

Calibration of the simple model, $q_s(t)$, will result in an S-graph (or unit hydrograph) which represents the modified watershed. Hence, as is commonly noted in practice, there is a different optimum unit-hydrograph for each storm considered.

Hence it appears that modeling error is not reduced by discretizing the watershed (such as considered herein) beyond the knowledge of the distribution of rainfall over the watershed. Obviously, if rain gauges were available in each subarea, then the representation of subareas by "adjusted" subareas is no longer needed, and the complex model formulation, $\hat{q}_c(t)$, has all of its c_j factors properly defined, as given by $q_c(t)$.

IS DISCRETIZATION BETTER?

In order to better explain the interplay between the several previous discussions, consider the following thought problem: Let the watershed be essentially homogeneous, nearly free-draining, with a stream gauge and an offsite rain gauge. (The rain gauge is assumed offsite to emphasize a point.) Suppose a set of 100 severe storms occur such that the measured rainfall, $p^i(t)$, and measured effective rainfalls, $e_g{}^i(t)$, at the rain gauge (which has soil similar to the watershed) is known, for i = 1,2,...,100. Furthermore, suppose the effective rainfall distribution over the watershed (which is entirely unknown) is related to the measured data as in Eq. (16). Finally, suppose each $p^i(t)$ and $e_g{}^i(t)$ is identical and equal to a single pattern, $p(t)$, $e_g(t)$, respectively, for each storm, while the $\{c_j\}$ in Eq. (16) differ for each storm. Therefore even though each $p^i(t)$ and $e_g{}^i(t)$ are identical, the runoff hydrograph $Q^i(t)$ is different for every i. Let $Q^i{}_p$ be the peak flow rate for each $Q^i(t)$. Then from a frequency distribution plot of the $Q^i{}_p$, the $\mathrm{Var}(Q_p)$ can be computed as correlated to the available rainfall data (at the rain gauge site), and for the sample set of $\{Q^i{}_p\}$.

By calibrating the simple model, $q_s(t)$, to each $Q^i(t)$, a set of optimized parameters P_i are developed for each i = 1,2,...,100. Since the rainfall distribution at the rain gauge, $p^i(t) = p(t)$, is fixed, the var(Q) from $q_s(t)$ due to the variation in optimized parameter sets, $\{P_i\}$, approximately equals the $\mathrm{var}(Q_p)$.

In contrast, when discretizing the catchment, var(Q) produced by $\hat{q}_c(t)$ typically decreases significantly with the level of discretization, when mostly parallel routing is used, and when using the parameters developed from the $q_s(t)$ calibration. Thus the $\hat{q}_c(t)$ model output (of peak flow rate, Q) does not show the true variance between the rain gauge and stream gauge correlations.

Should the complex $\hat{q}_c(t)$ be supplemented with sufficient runoff data (e.g., a stream gauge for each subarea) such as optimize each subarea UH model and c_j factor for each storm, then $\hat{q}_c(t)$ is approximately $q_c(t)$ as expressed by Eq. (15) which reduces to the simple model formulation of Eq. (13) under the conditions of Eq. (16). Thus with sufficient rainfall-runoff data in the watershed, the $q_c(t)$ produced var(Q) equals the $q_s(t)$ produced var(Q) which equals the true var(Q) with respect to the available (single) rain gauge. Therefore, if a criteria for model comparison is that the model replicates the $\mathrm{var}(Q_p)$ with respect to the available rain gauge data, then $q_s(t)$ does this better than the discretized model of $\hat{q}_c(t)$.

An important fact is that, in general, a single stream gauge and a single rain gauge is used to calibrate the simple UH model. For each storm included in the calibration, optimized parameters are obtained that best "correlate" the rain gauge and stream gauge data. This is only a "correlation" because the available rain gauge data oftentimes provides only a gross estimate of the actual rainfall distributions over the entire watershed. As more storms are analyzed and "best fit" model parameters are developed, a set of value are obtained for each model parameter set that can be arranged into a frequency distribution. This frequency distribution or parameter sets displays the statistical correlation between the available rainfall-runoff data, and the assumed model. Consequently, the var(Q) due to the variation in the model parameter sets reflects the variation in the correlation between the available rainfall-runoff data. However for the discretized ("complex") model, the var(Q) decreases (typically) as the number of subareas increase where now the var(Q) represents the model variance in the complex model peak flow rate as the optimized model parameter sets vary independently in each subarea. This drop in model output variance is usually advocated by modelers to represent an increase in modeling accuracy, and beneficial to the calibration of model parameters. But obviously the knowledge of the effective rainfall distribution over the watershed does not increase due to an increased model complexity by adding more subareas. Rather, the decrease in the var(Q) for a complex model reflects a departure of the modeling results from the true catchment hydrologic behavior.

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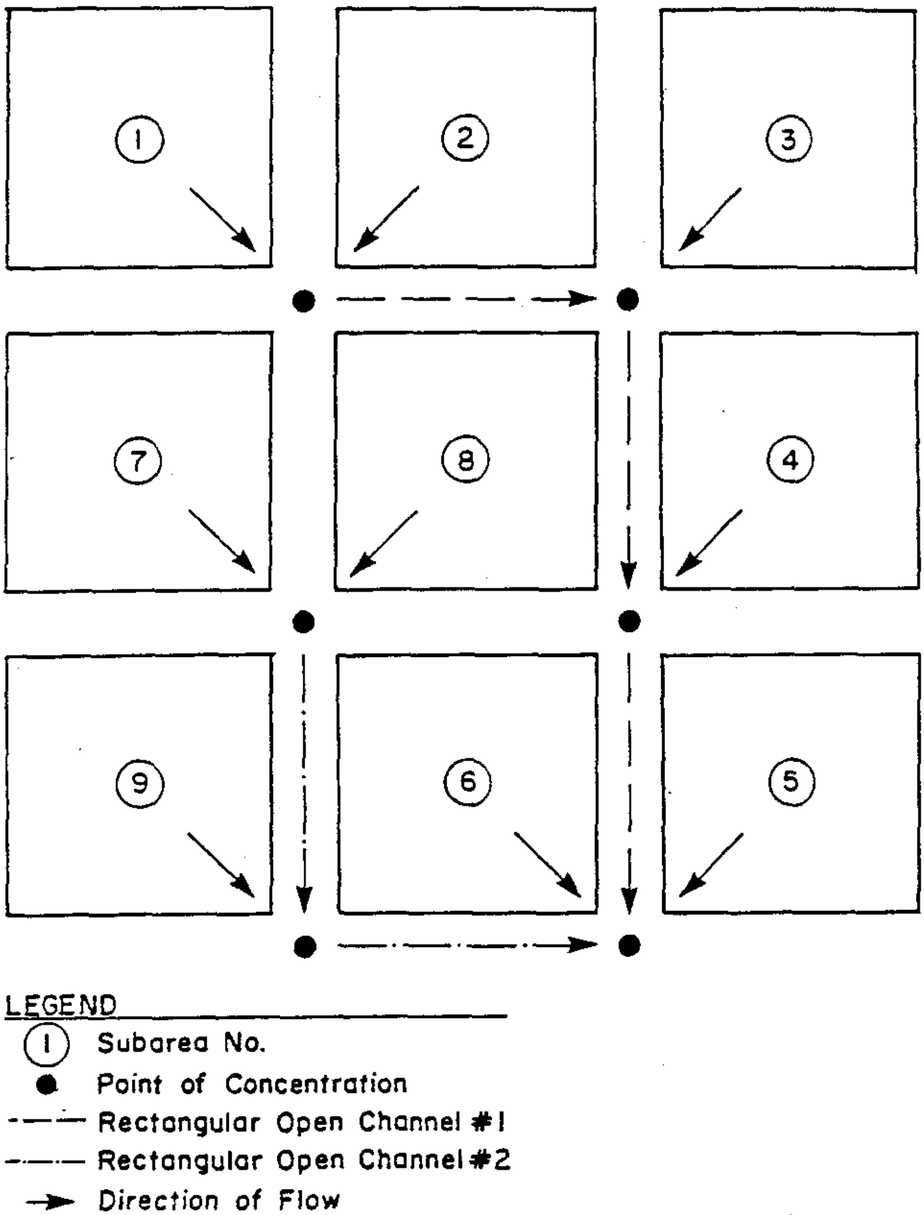


Figure 1. Test Watershed Schematic of 9-Subarea Link-Node Model

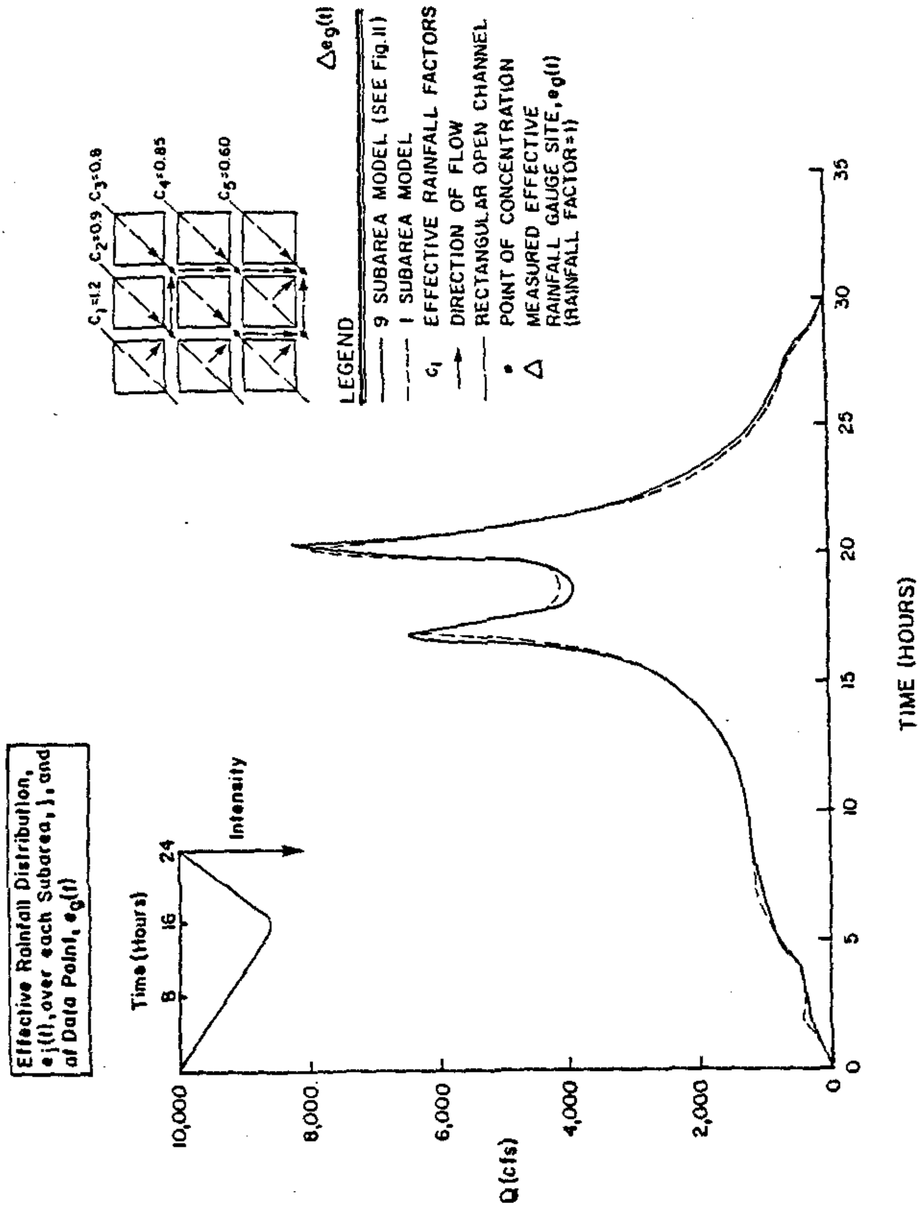


Figure 2. Outflow Hydrograph for W = 0.85

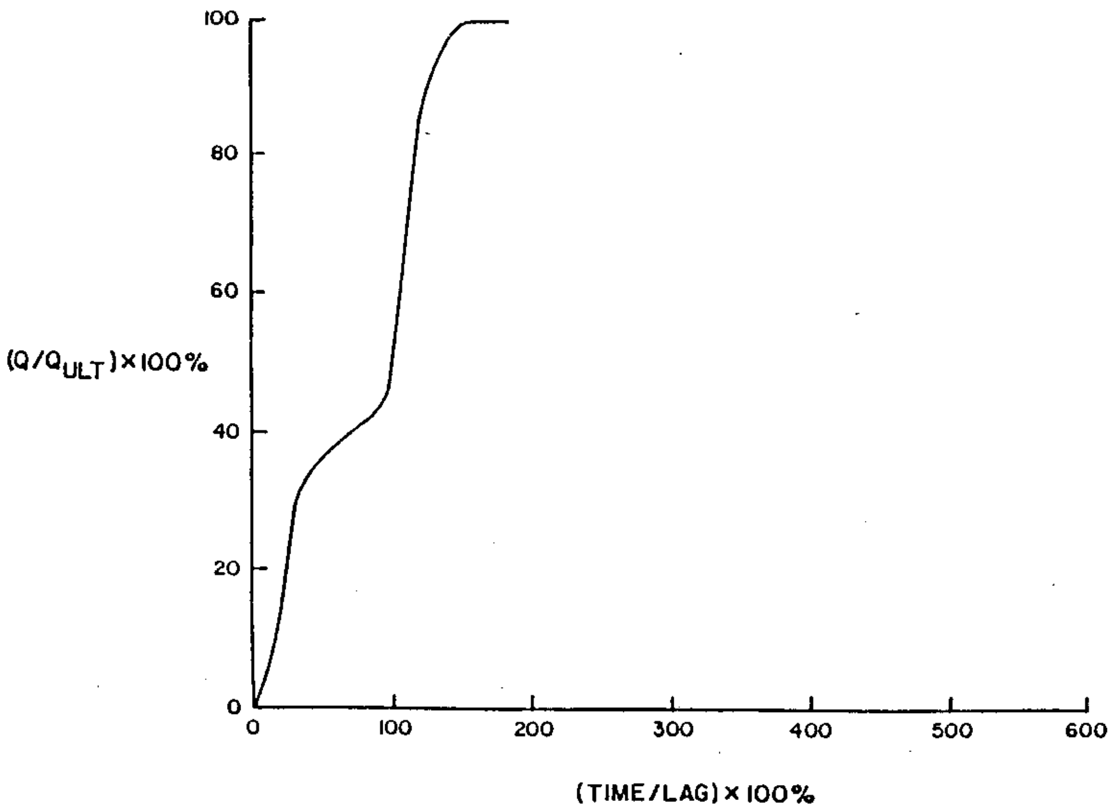


Figure 3. S-Graph for W = 0.85

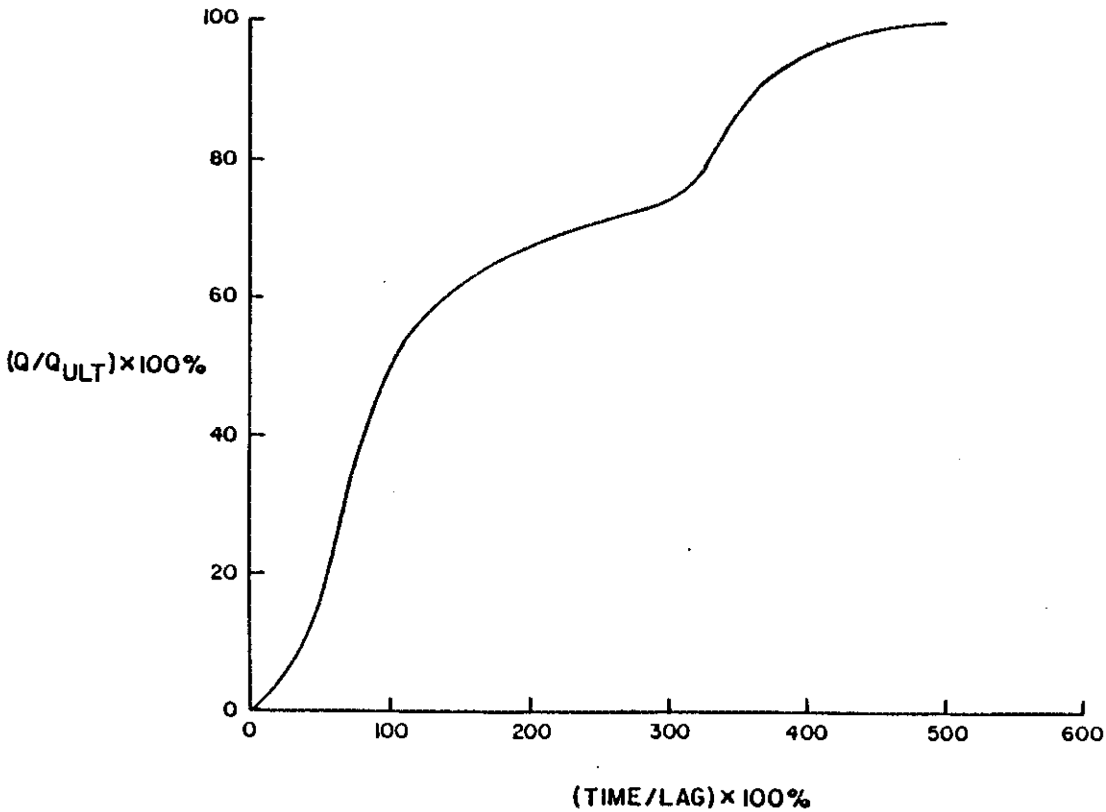


Figure 4. S-Graph for W = 2.81

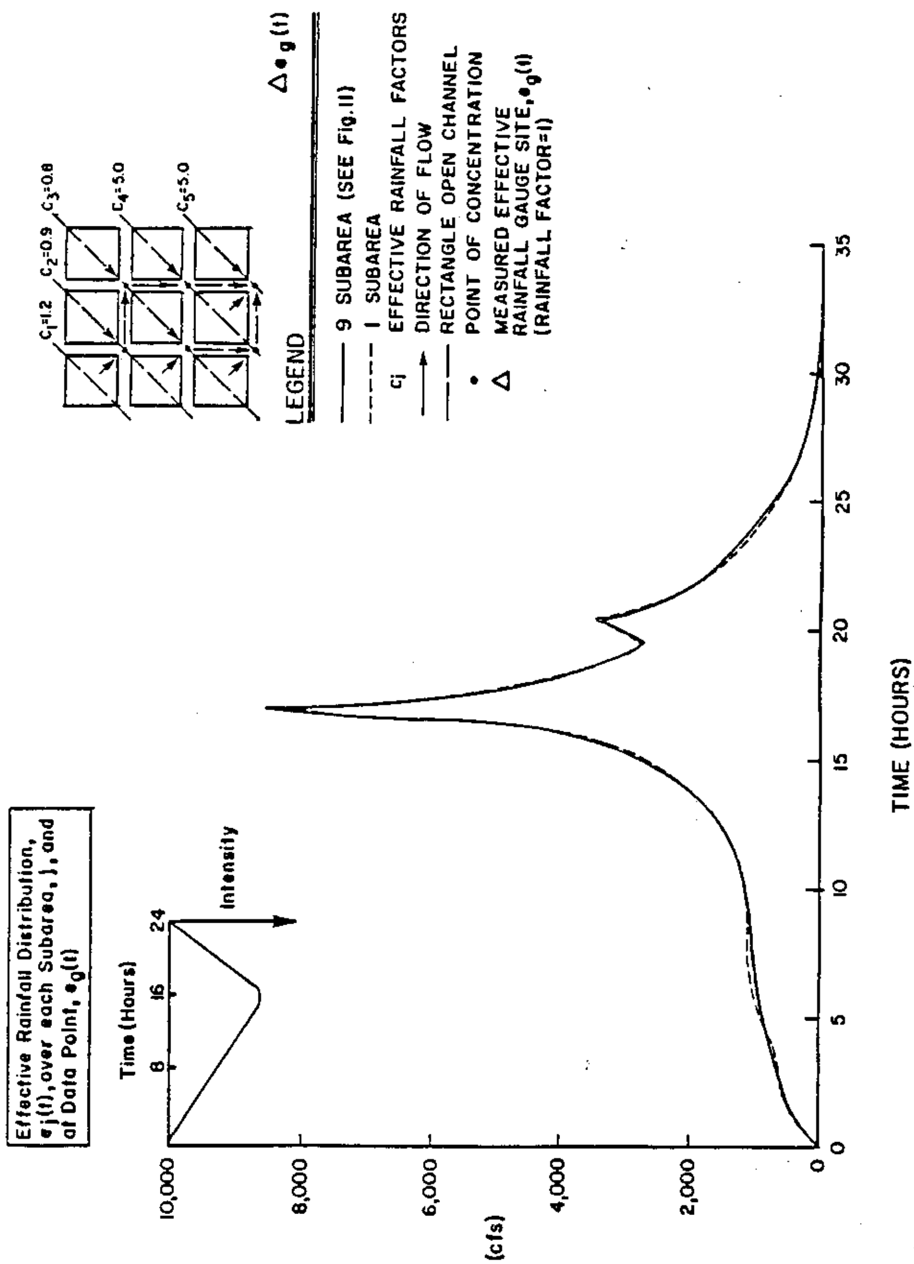
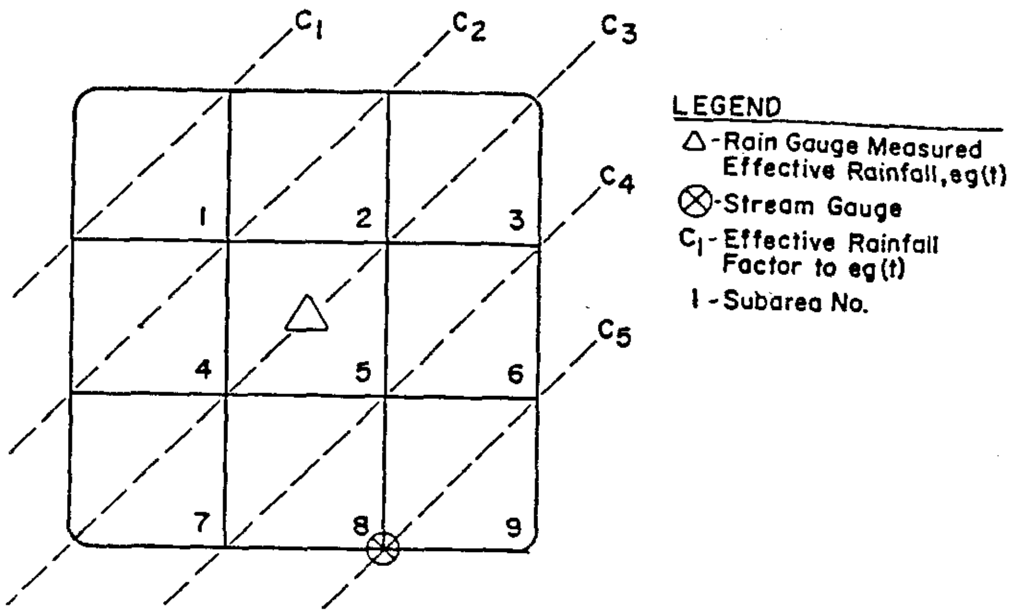
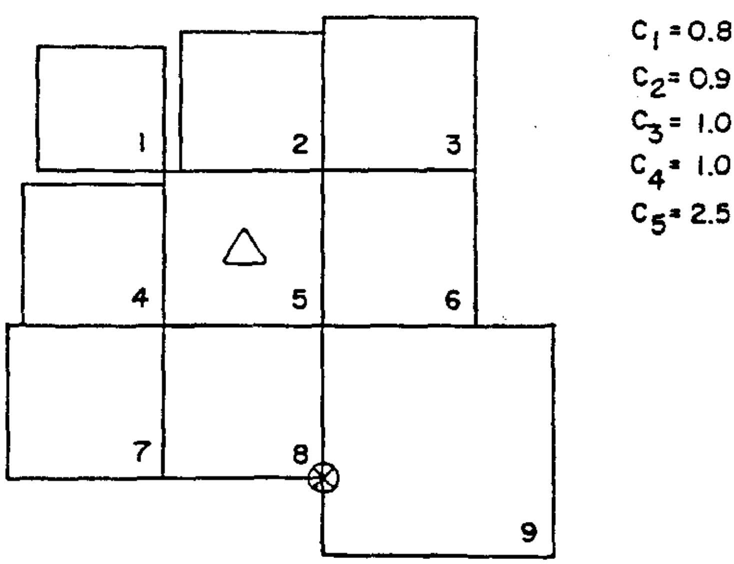


Figure 5. Outlfow Hydrograph for W = 2.81



(a) TRUE DISTRIBUTION OF EFFECTIVE RAINFALL WITH RESPECT TO THE RAIN-GAUGE SITE



(b) ADJUSTED AREA REPRESENTATION OF WATERSHED WITH RESPECT TO THE RAIN GAUGE DATA

Figure 6. Adjusted Subarea Sizes due to Effective Rainfall Factors, c_i